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[a]Discussion period closed for this paper. Any other discussion received during this discussion period will be published in subsequent Journals.

George A. Thompson aided in interpretation of the gravity data. Appreciation and gratitude is extended to those who assisted in the project.

Appendix I.—References

1. Dobrin, M. B., "Gravity Field Measurements and Reductions," *Introduction to Geophysical Prospecting*, McGraw-Hill Book Co., Inc., New York, N.Y., 1960, pp. 218-241.
2. Parasnis, D. S., "Gravitational Methods," *Principles of Applied Geophysics*, John Wiley and Sons, Inc., New York, N.Y., 1962, pp. 31-64.
3. "Soil Investigation for the Proposed Engineering Unit I Building University of California, Santa Cruz, California," Woodward-Clyde and Associates, Oakland, Calif., 1968.

Appendix II.—Notation

The following symbols are used in this paper:

G = universal gravitation constant, 6.67×10^{-11} *SI*;
$\vec{g}(r)$ = gravitational field;
g_z = vertical component of gravitational field;
m_1, m = mass;
R = radius of sphere;
r = distance between mass and field point (Fig. 1);
$\mathbf{r}$ = unit vector directed between mass and field point;
t = body thickness;
x = horizontal distance between mass and field point (Fig. 1);
z = vertical distance between mass and field point (Fig. 1);
θ = angle between vertical and line joining mass and field point (Fig. 1);
ρ = density; and
ϕ = angle at field point subtended by mess.

JOURNAL OF THE GEOTECHNICAL ENGINEERING DIVISION

Soil Arching in Slopes

By Wen L. Wang,[1] M. ASCE and Bing C. Yen,[2] A. M. ASCE

Introduction

In the field of soil mechanics the term arching has been generally used to describe the phenomenon of stress transfer through the mobilization of shear strength in soils, such as the familiar problems of stress concentration or relief around underground conduits (22,23,26) and shelters (1,2,9,18). Existing theories of arching usually assume a horizontal semi-infinite soil mass. From this assumption, various approaches have been developed to calculate either stress acting on the structure or the stability of the surrounding soil. A full spectrum of these studies can be found in the "State-of-the-Art" report by Allgood, et al. (3,4). However, little is known of soil arching when the soil mass is inclined instead of horizontal. The mechanism of arching when both stress and strain are considered is quite complicated. Many approaches in the past have considered the soil as an arch or beam. The approach used herein is a direct application of Terzaghi's definition (24,25) assuming soils are rigid-plastic.

An understanding in the mechanism of soil arching in slopes is of significance to practicing soil engineers. An example is the use of large reinforced concrete cylinders placed vertically into active or potential failure slopes which act as pins or a contilever wall to arrest the slope failure (10). Recent examples of the successful use of such piles include the freeway on Potrero Hill in San Francisco (19), which consisted of a series of 30 pile bents placed along 240 ft (73.2 m) of wall formed by paired holes 4 ft in diameter, reinforced by 36 WF 230 and concreted in place. Large cylinder piles were successfully used in downtown Seattle at required cuts ranging from 40 ft (12.2 m) to 50 ft (15.25

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m), extending into heavily overconsolidated, fractured, and jointed lacustine silty clay. Piles, 3 ft (0.915 m) in diameter, were used in Turkey to stabilize a landslide on a natural slope (20). Drilled caissons, generally with diameters of about 2 ft (0.61 m) have been used occasionally on hillside areas to arrest movement or slides in Southern California, resulting in a mixture of success and failure (17). To the writers' knowledge the arching condition was not considered in the placement of these caissons.

In order to assess the validity of slope stabilization by piles and subsequent attempts to analyze the soil arching in slopes caused by the presence of a series of cylindrical piles, a set of relationships between soil properties and pile geometry in an infinite slope at critical equilibrium is developed in this paper. Although practical problems usually involve slopes of various geometry, the solution of an infinite slope at critical equilibrium provides the fundamental understanding of arching. The infinite slope at critical equilibrium is therefore, examined herein. The assumptions employed in the analytical relationships are reviewed in order to assess the limitations of the relationships. The analytical relationships, plotted in dimensionless charts, experimental results, and illustrative examples are included in the paper.

Review of Previous Works

Previous work on soil arching can generally be classified into three categories: (1) The shear plane method as pioneered by Terzaghi (24,25); (2) the elastic approach as examined by Finn (8) or Chelapati (6); and (3) the approach of soil-structured model studies (9,27).

In 1936, Terzaghi performed experiments on sands with a yielding trap door. A condition of shear failure was assumed. The shear plane method was subsequently proposed in 1943 by Terzaghi. The method is based on the stability of the soil mass bounded between the potential vertical shear planes above the underground structures. These planes separate the soil mass settling with the structure from the surrounding soil. The analysis involves studying the equilibrium of a horizontal element of a soil mass. A similar approach but in axial symmetry, has been investigated by Jenike (13) and by Jenike and Yen (14) in the study of arching and piping of gravity flow in the storage bins. The shear plane concepts of approach were the basis of Marston (16) and Spangler's (22) method for calculating pressures on buried conduits. The main disadvantages of this approach are that the correct shape of the shear plane is overlooked and that no stress-strain relationship prior to failure is known. Since the orientations of the ground surface and conduits are different from the problem considered, results from this approach cannot be used to evaluate the soil arching in slopes.

The theory of elasticity assumes the soil around and above the buried structure as a semi-infinite elastic medium. This approach has two disadvantages: first, the ideal elastic soil properties are needed; and second, complicated calculations are required for even simple cases.

Most of the model studies on soil arching are made in the configuration of underground shelters motivated by the impetus of dynamic surface loadings. Based on several soil and structure parameters the model studies, in general, permitted a sufficiently accurate description of loads on the structure. This model approach appears to be a qualitative but powerful tool in the understanding of soil arching. Research by the writers did not uncover published works on model studies involving soil arching in slopes as presented analytically herein.

Theoretical Considerations

Basic Assumptions.—The basic assumptions are:

1. Slope Geometry: Infinite slope—In Fig. 1(*a*) the plane view of a series of piles in a semi-infinite plane is shown. A typical cross section, AA', is shown in Fig. 1(*b*), and generic element with all the forces shown is presented in Fig. 1(*c*). Soil arching action within the slope above an assumed failure plane is investigated. The piles are assumed to be embedded firmly into a sound layer below.

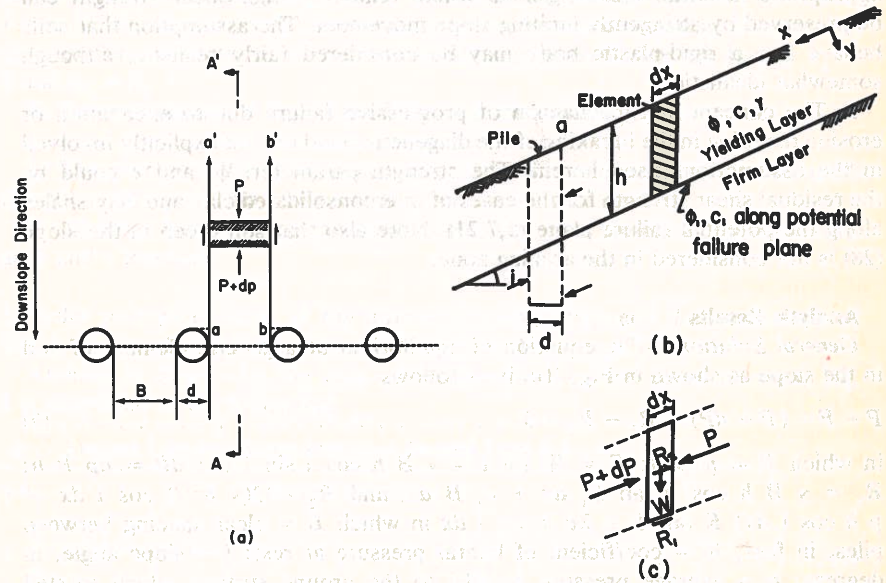

FIG. 1.—Plan View of Series of Piles: (*a*) on Slope; (*b*) Cross Section; (*c*) Generic Element

2. The soil behaves as a rigid-plastic solid and its strength can be described by

$$\tau = c + \sigma \tan \phi \quad \text{(1a)}$$

in which τ = shearing strength, in pounds per square foot; c = cohesion, in pounds per square foot; σ = normal stress, in pounds per square foot; and ϕ = angle of internal friction, in degrees. It is further assumed that the shearing strength along the potential failure can be described by

$$\tau = c_1 + \sigma \tan \phi_1 \quad \text{(1b)}$$

Review of Assumptions.—The implications and limitations of the previous two assumptions are as follows:

1. An infinite slope assumption is obviously an oversimplification. Therefore, the insight gained from such an analysis is only an approximation of the field conditions which are a two or three-dimensional problem. Note the assumed shear failure planes in Fig. 1(*a*). Due to the fact of symmetric arrangement of piles, the straight shear planes can be considered reasonable.
2. The assumption of rigid-plastic soil behavior limits the soil arching effect to the stress condition at the state of incipient plastic flow. Thus, unlike the elastic approach, the stress-strain relationship of the soil before failure is not considered.

In reality, drilled caissons have the advantage of being installed without significantly decreasing slope stability during construction. They are particularly appropriate in strain-softening soils where relatively high shear strength can be preserved by stringently limiting slope movement. The assumption that soils behave like a rigid-plastic body may be considered fairly realistic, although somewhat idealistic.

3. The concept and mechanism of progressive failure due to excavation or erosion resulting in the breaking of the diagenetic bond are not explicitly involved in the assumptions used herein. The strength parameter, ϕ, and c could be the residual shear strength for the cases of overconsolidated clay and clay shales along the potential failure plane (5,7,21). Note also that soil creep in the slope (28) is not considered in the arching zone.

Analytic Results

General Solution.—The equation of equilibrium of a generic element of soil in the slope as shown in Fig. 1(*c*) is as follows:

$$P + F - (P + dP) - R_1 - R_2 = 0 \quad \text{(2)}$$

in which $P = p\,B\,h$; $F = W \sin i = \gamma\,B\,h \cos i \sin i\,dx$; $dP = dp\,B\,h$; $R_1 = \gamma\,B\,h \cos^2 i \tan \phi_1\,dx + c_1\,B\,dx$; and $R_2 = 2(\gamma\,h^2/2 \cos i\,dx + p\,h \cos i\,dx)\,K \tan \phi + 2c\,h \cos i\,dx$ in which B = clear spacing between piles, in feet; K = coefficient of lateral pressure at rest; i = slope angle, in degrees; p = average pressure parallel to the ground surface which existed within the soil element of depth h and located at a distance of x ft from where the arching effect ends; and γ = effective unit weight of soil, in pounds per cubic foot. Eq. 2 can be reduced to a differential equation:

$$\frac{dp}{dx} + K_1\,p = K_2 \quad \text{(3)}$$

in which $K_1 = 2K/B \cos i \tan \phi$; $K_2 = \gamma \cos i \sin i - (K\,\gamma\,h/B \cos i \tan \phi + 2\,c/B \cos i + \gamma \cos^2 i \tan \phi_1 + c_1/h)$. The boundary conditions are $x = 0$; and $p = k\,\gamma\,h/2$. The solution of Eq. 3 for the boundary condition given is as follows:

$$p = \frac{K_2}{K_1}(1 - e^{-K_1 x}) + \frac{K}{2}\gamma\,h\,e^{-K_1 x} \quad \text{(4)}$$

Let $B/h = m$; and $x/B = n$; and taking K_1 and K_2 into consideration, then Eq. 4 can be changed into a dimensionless form

$$\frac{p}{\gamma h} = \frac{\left(m \cos i \sin i - K \cos i \tan \phi - \dfrac{2c}{\gamma h} \cos i - m \cos^2 i \tan \phi_1 - \dfrac{c_1}{\gamma h} m\right)}{2\,K \cos i \tan \phi}$$

$$\times (1 - e^{-2Kn\cos i \tan \phi}) + \frac{1}{2} K\,e^{-2Kn\cos i \tan \phi} \quad \text{(5)}$$

Eq. 5 is the general solution which describes the average soil pressure developed in a slope due to the presence of piles. It is of interest to point out that Terzaghi's results describing the soil arching caused by a horizontal yielding door (25) is a special case of Eq. 5. This can be proved by: (1) Letting $i = 0°$ for all the terms of Eq. 5 involving cosine functions and normal stresses; (2) letting $i = \pi/2$ for all the terms involving sine functions and shearing stresses; (3) ignoring lateral frictional stress due to sloping ground; and (4) letting $c_1 = 0$ and $\phi_1 = 0$.

In the examination of Eq. 5, the following observations can be made:

1. The average soil arching pressure, p, increases exponentially (in the uphill direction) to a maximum value equal to the pressure at rest.
2. All other factors being the same, arching is more prominent (p decreases) as ϕ and c increase.

In order to explore some of the inter-relationships between the parameters of Eq. 5, sand and clay slopes are analyzed in the following.

Arching in Sand.—In the case of sand, c and c_1 are equal to zero, Eq. 5 is simplified to the following:

$$\frac{p}{\gamma h} = \frac{(m \cos i \sin i - K \cos i \tan \phi - m \cos^2 i \tan \phi_1)}{2\,K \cos i \tan \phi}(1 - e^{-2Kn\cos i \tan \phi})$$

$$+ \frac{1}{2} K e^{-2Kn\cos i \tan \phi} \quad \text{(6)}$$

Physically, Eq. 6 may be considered as a simulation of a long slope of talus or weathered granular material ($c = 0$, $c_1 = 0$) of thickness h overlying a potential sliding plane along which the residual cohesive resistance c_1, is estimated to be zero (21). Clearly, from Eq. 6 the average soil pressure caused by soil arching for a given slope having certain properties varies with pile spacing m and location n. When the pile spacing is too wide, arching will not develop. Therefore, it would be interesting to determine the critical spacing, m_{cr}, for a given slope and soil properties. At $m = m_{cr}$ the arching does not exist, i.e., the average soil pressure, P, is independent of n, i.e., $\partial p/\partial n = 0$. Taking the partial derivative of Eq. 6 with respect to n and solve for the critical spacing, m_{cr} is obtained as follows:

$$m_{cr} = \frac{K(K + 1) \tan \phi}{\cos i (\tan i - \tan \phi_1)} \quad \text{(7)}$$

Figs. 2 and 3 show the relationship between m_{cr}, slope, and other soil parameters. The coefficient of lateral earth pressure at rest K is assumed to be $1 - \sin \phi$ as suggested by Jaky (12). It is believed that for granular soil at initial loading this assumption appears to be satisfactory by experiments and field observations (11,15). The ϕ_1 values shown in Fig. 2 ranges from 8° to 16°. This range was chosen to represent a realistic range of residual internal angle of friction existing along the sliding plane (5,21).

Piles placed in slopes, with spacing larger than the critical values, will be of little use for stabilization. Fig. 4 shows the arching development along a 1-1/2:1 slope with varying pile spacing. As the pile spacing approaches $m_{cr} = 1.2$, the arching ceases to function in this particular case. On the other hand, as the spacing becomes smaller than m_{cr}, the arching becomes more effective, i.e., p decreases. The p value in Eq. 6 may even become negative. However,

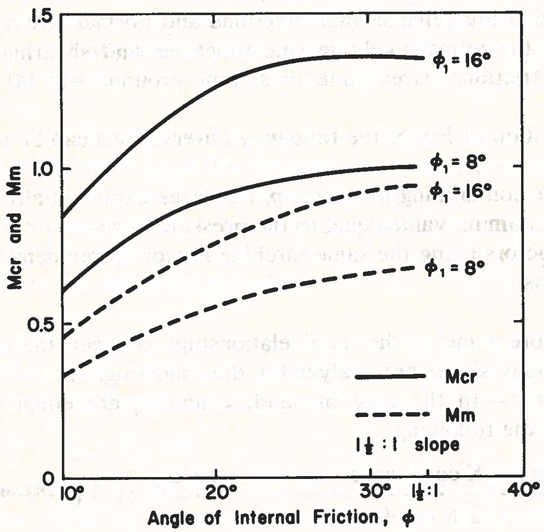

FIG. 2.—Relationship Between Pile Spacing and Soil Properties; 1-1/2:1 Slope

no tension can be developed in the cohesionless soils; therefore, the arching zone for this situation can be otained by letting $p = 0$. The corresponding n is

$$n_0 = \frac{\ln\left(1 - \dfrac{K}{2\alpha}\right)}{2K \cos i \tan \phi} \quad \text{(8)}$$

in which $$\alpha = \frac{m \cos i \sin i - K \cos i \tan \phi - m \cos^2 i \tan \phi_1}{2K \cos i \tan \phi} \quad \text{(9)}$$

In order to have a meaningful n_0 value, the following two conditions must be satisfied: (1) $1 - K/2\alpha > 0$; or (2) $K/2\alpha < 0$. No. 1 yields $m < m_{cr}$ which is unrealistic. No. 2 yields $m < m_m$ in which

$$m_m = \frac{K \tan \phi}{\sin i - \cos i \tan \phi} = \frac{m_{cr}}{1 + K} \quad \text{(10)}$$

This is the smallest pile spacing to create no tensile stress in zone n_0. The same result can also be obtained from Eq. 6 by letting $p = o$ as $n \to \infty$.

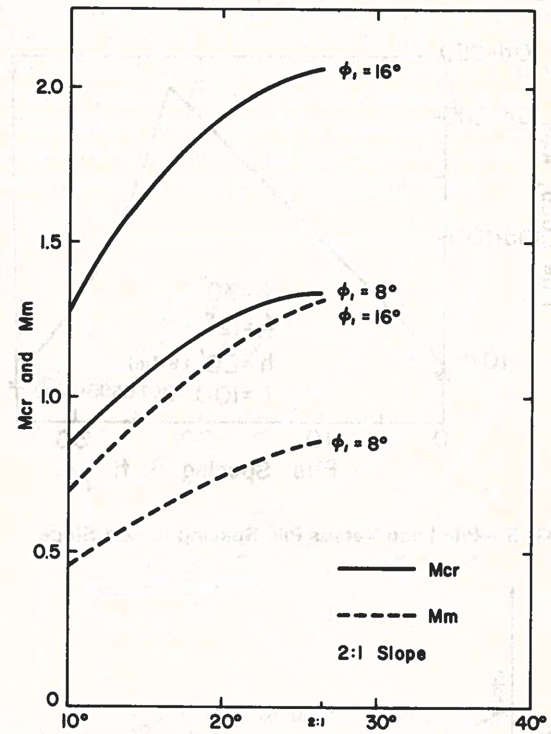

FIG. 3.—Relationship Between Pile Spacing and Soil Properties; 2:1 Slope

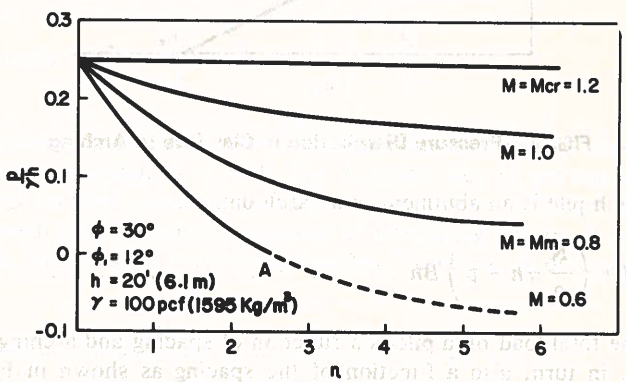

FIG. 4.—Pressure Distribution in Sand Due to Arching

The $p/\gamma h$ versus n curve with $m = m_m$ is shown in Fig. 4. The m_m values are also plotted in Figs. 2 and 3.

It is interesting to investigate the pile load in a sandy slope. The load on each pile embedded in sandy slopes is the summation of two loads, one from the pressure at rest, acting on the pile, similar to the lateral pressure on a retaining wall. The other is the soil arching pressure transferred to the adjacent

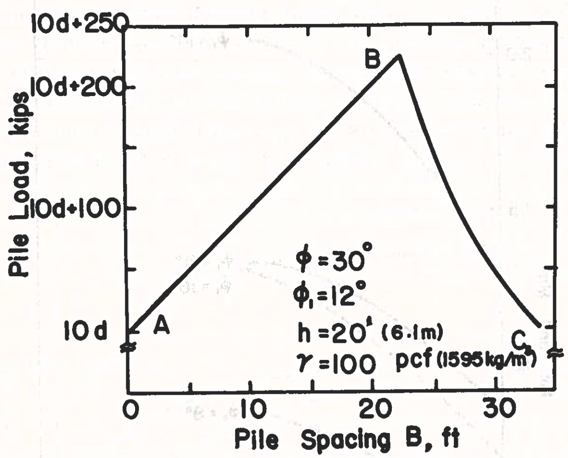

FIG. 5.—Pile Load Versus Pile Spacing for 2:1 Slope

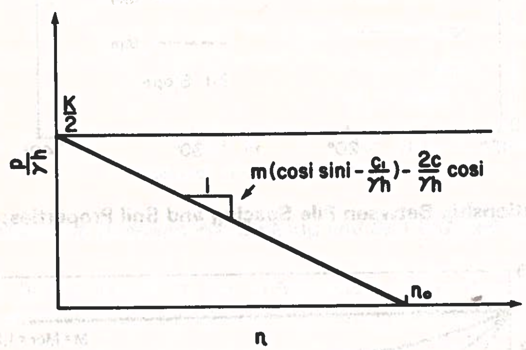

FIG. 6.—Pressure Distribution in Clay Due to Arching

piles as if each pile is an abutment of an arch dam.

$$P = \frac{K}{2}\gamma h^2 d + \left(\frac{K}{2}\gamma h - p\right)Bh \quad (11)$$

Note that the total load on a pile is a function of spacing and arching, pressure p, which is, in turn, also a function of the spacing as shown in Fig. 4. For example, a 2:1 sandy slope with $\phi = 30°$; $\phi_1 = 12°$; $h = 20$ ft (6.1 m); and $\gamma = 100$ pcf (1,590 kg/m³), the critical and most effective spacings are m_{cr}

$= 1.69$ and $m_m = 1.12$ according to Eqs. 7 and 10, respectively (Fig. 3). Therefore, the critical and effective spacings, B_{cr}, and B_m, are 33.8 ft (10.3 m) and 22.4 ft (6.8 m), respectively. The total load per pile at different widths is shown in Fig. 5. At point A, with zero pile spacing the load, represents the pressure at rest alone. The transferred load increases linearly with respect to the spacing between A and B, where the spacing is equal to $m_m h$. Beyond point B, at which the spacing equals $m_m h$, the arching becomes less effective and only a portion of the soil pressure is transferred to the piles. As a result, the total load decreases, although spacing increases. At point C, at which m is equal to m_{cr}, the arching ceases to exist. The load between B and C are calculated, using Eq. 6, assuming a long slope. The necessary n to develop arching fully is approximately five to six, as can be seen from Fig. 4. Slopes with a selected pile spacing are considered long, if n is greater than six.

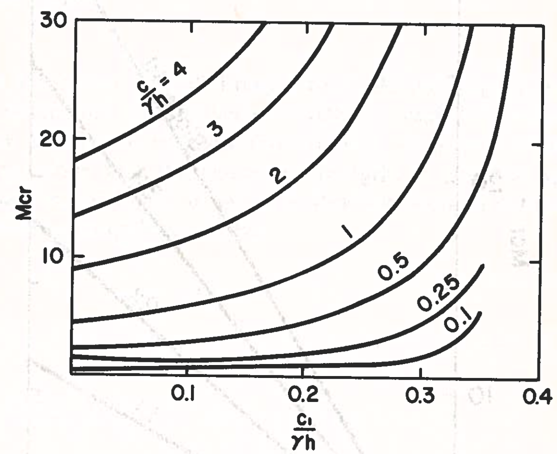

FIG. 7.—Critical Spacing of 2:1 Cohesive Slope

Arching in Clay.—In the case of clay, ϕ and ϕ_1 are assumed to be zero. Eq. 5 is simplified to the following:

$$\frac{p}{\gamma h} = n\left(m\cos i \sin i - \frac{2c}{\gamma h}\cos i - m\frac{c_1}{\gamma h}\right) + \frac{K}{2} \quad (12)$$

Eq. 12 describes the arching effect within an undrained clayey slope stabilized by piles with a spacing, $B = m\,h$. Fig. 6 shows Eq. 12 schematically. Note that the soil arching pressure, p, varies with n; $n = x/B$ and $p = 0$ at $n = n_0$. Although cohesive soils are capable of developing tension, it is reasonable to discard it, since tension cracks may develop. Therefore, n_0 defines the arching zone

$$n_0 = \frac{K}{2\left[2\frac{c}{\gamma h}\cos i + m\left(\frac{c_1}{\gamma h} - \cos i \sin i\right)\right]} \quad (13)$$

When the pile spacing is too wide apart, arching will not develop in the uphill slope. From Eq. 12, this is the condition when p is independent of n, i.e., $\partial p/\partial n = 0$ which is

$$m \cos i \sin i - \frac{2c}{\gamma h} \cos i - m \frac{c_1}{\gamma h} = 0 \quad (14)$$

Solving for m and let $m = m_{cr}$ in which m_{cr} = the critical spacing

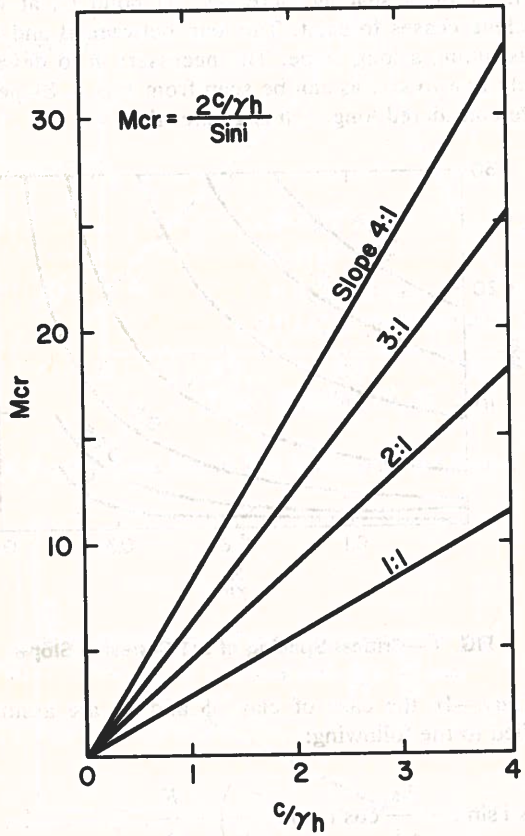

FIG. 8.—Critical Spacing of Piles in Various Slopes; $c_1 = 0$

$$m_{cr} = \frac{2 \dfrac{c}{\gamma h} \cos i}{\cos i \sin i - \dfrac{c_1}{\gamma h}} \quad (15)$$

Fig. 7 shows the relationship between critical spacing, $c/\gamma h$ and $c_1/\gamma h$ for a 2:1 slope. For slopes other than 2:1 the relationship can be calculated and plotted by using Eq. 15. For a given clayey slope, this spacing is controlled

not only by the strength in the clay slope (c) in which the arching action develops but also by the strength along the potential failure plane (c_1).

Taking into consideration Eqs. 13 and 15, the arching zone may be redefined in terms of pile spacing, slope angle, cohesion, and the coefficient of earth pressure as follows:

$$n_0 = \frac{K}{2(m_{cr} - m)\left(\cos i \sin i - \dfrac{c_1}{\gamma h}\right)} \quad (16)$$

There is no infinite long slope in reality. Therefore, it is desirable to install piles at a location where the uphill slope is greater or equal to $n_0 B$ in order to transmit the entire load to piles. Moreover, the arching will be fully developed within the uphill slope.

Special Case 1

Special Case 1 is when $c_1 = 0$. Frequently, field investigations unveil seams of weak soils, practically with no shear resistance, such as a seam of bentonitic clay. The value of $c_1 = 0$ is justifiable for this situation. Depending on the steepness of the slope and the cohesive strength, c, the pile spacing should be kept less than m_{cr} of Fig. 8 in order to develop arching action in the uphill slope. The critical spacing then will be

$$m_{cr} = \frac{2 \dfrac{c}{\gamma h}}{\sin i} \quad (17)$$

This is plotted in Fig. 8.

Special Case 2

Special Case 2 is when $c \leqq c_1$. In the case of a normally consolidated clay slope of depth h underlaid by a competent stratum, whose strength at the contact is greater or equal to the undrained cohesive strength of the normally consolidated clay, i.e., $c \leqq c_1$, the pile spacing should be less than m_{cr} of Eq. 18 in order to develop the arching action in the uphill slope, i.e.

$$m_{cr} = \frac{2 \dfrac{c}{\gamma h} \cos i}{\cos i \sin i - \dfrac{c}{\gamma h}} \quad (18)$$

Eq. 18 with $c = c_1$ is plotted in Fig. 9.

The load exerted on each pile embedded in clay slopes is the summation of two loads as in the sandy slopes; one from the pressure at rest acting on the pile, similar to the lateral pressure on a retaining wall; the other is the soil arching pressure transmitted to the adjacent piles as if each pile is an abutment of an arching dam. Therefore, the total load per pile in the downhill direction is

$$P = \frac{K}{2} \gamma h^2 (d + B) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (19)$$

For example, in a 2:1 slope with the following soil properties: c = 1; c_1 =

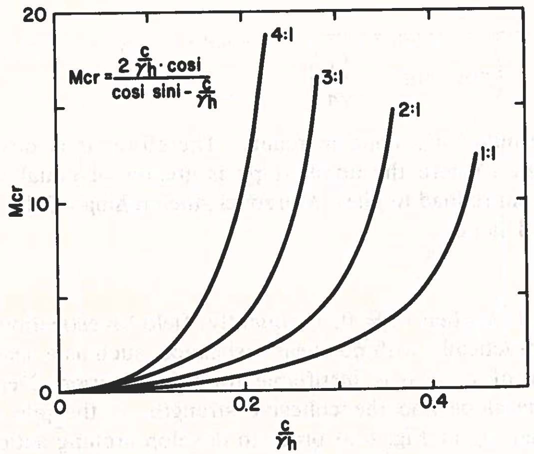

FIG. 9.—Critical Spacing of Piles in Various Slopes; $c_1 = c$

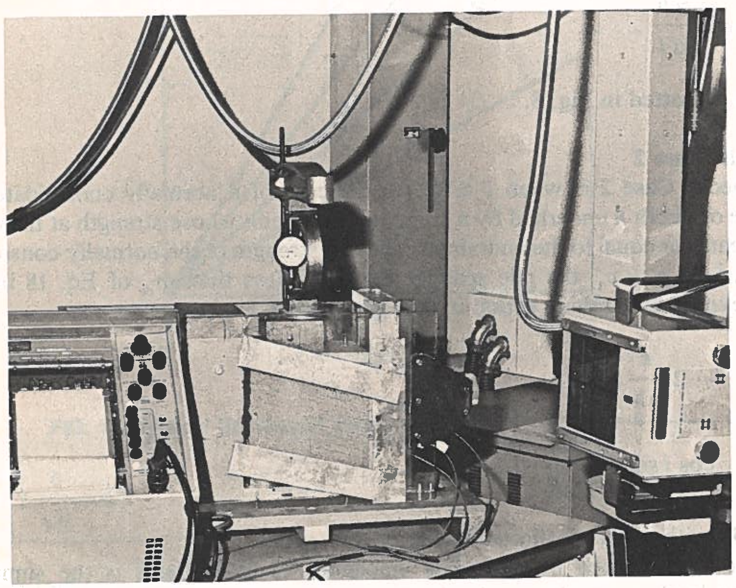

FIG. 10.—General Set-Up

0.4 ksf (19.16 kN/m²); h = 20 ft (6.1 m); γ = 0.1 kcf (1,590 kg/m³); and K = 0.9. The total load per pile is 117 kips (570 kN) if the spacing, B, is

5 ft (1.5 m) between piles of 1-1/2 ft (10.44 m) diam. For comparison, it is 171 kips (761 kN) per pile if B is 8 ft (2.44 m). For this example, the arching zone extends 2.7 ft (0.82 m) uphill for B = 5 ft (1.53 m) [$n_0 B$ = 0.532 × 5 ft (1.53 m)], and it extends 4.5 ft (1.37 m) uphill for B = 8 ft (2.44 m) ($n_0 B$ = Q.563 × 8 ft = 4.5 ft). From Fig. 7, or Eq. 15, the critical spacing, m_{cr} = 4.5, B_{cr} = 90 ft (27.5 m) can be obtained. Although it is impractical to have such a wide pile spacing, it is interesting to point out the arching effect can theoretically exist in clays even at a rather wide pile spacing.

Experiment

Experiments were carried out to verify the principle of stress transfer by arching action. The main goals were to examine the configuration of the arching

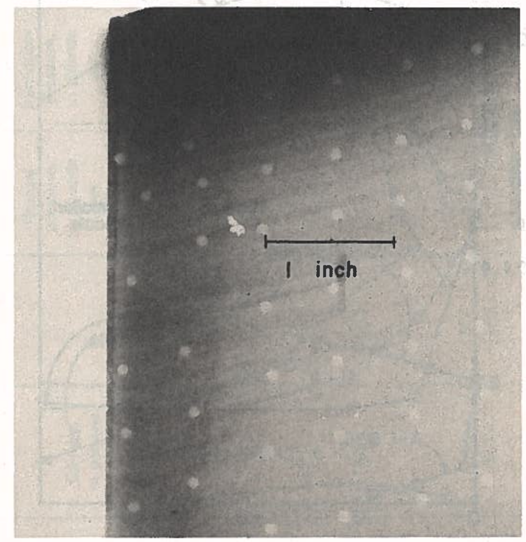

FIG. 11.—X-Ray Photo

zone in the soil, and measure the pressures on the yielding vertical gate and fixed piers on both sides of the gate, thus measuring the arching effect. Two plexiglas boxes, A and B, were fabricated. Box A, 6 in. (15.24 cm) wide, 24 in. (60.96 cm) long, 12 in. (30.5 cm) deep, consisted of 1/2-in. (1.27 cm) thick plexiglas plates held together and reinforced by screws and clamps. Wooden bottoms and loading heads with various slope angles were placed in the box. Blast sand was uniformly compacted and loaded by tightening a steel frame looped around the box. A proving ring held between the steel frame and the loading head registered the load. Pressure transducers, attached flush to the inner surfaces of the gate and the piers, monitored the pressure during the test. The gate movement was controlled by turning two knobs on a horizontal bar attached to the gate; displacements were thus, measured by a dial. Outward movement of the gate induced the arching in the soil. The general set-up is shown in Fig. 10. The sand possessed peak value of $\phi' = 41°$ at 75% relative density, $D_{10} = 0.19$ mm, and coefficient of uniformity = 1.7. By choosing variable parameters such as the slope angle, gate width, and soil density etc., a set of tests could be performed. The US Standard 30-40 Ottawa sand was used for Box *B*, which was loaded by a triaxial compression machine.

As previously stated an arching action occurs in which the soil dialates. Fig. 11, an X-ray photo taken on Box *A* through a lead window, shows the distinct change of tone indicative of density changes. For $i = 15°$, normal pressure = 6.1 psi (42 kN/m²), the calculated arching zone measured uphill from the inner face of the pier is 0.83 in. (2.11 cm). The 0.83 in. (2.11 cm) checks well with the picture shown in Fig. 4. The average arching zone measured

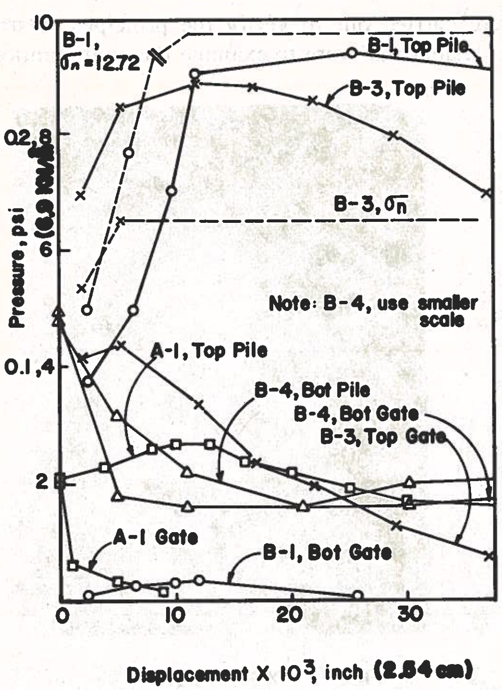

FIG. 12.—Pressure Versus Displacement

was 0.75 in. (1.91 cm). The narrower zone near the base, as shown in the X-ray, was caused by the presence of the sandy base slope. The dots are lead shots placed in the sand.

The results of transducer pressures, both on the piles and the gate, are plotted in Fig. 12 and listed in Table 1. It can be seen clearly that the pressure on the piers built up, while the gate pressure decreased as the gate was moved away from the original position. For various slopes and under different normal loads, the calculated pier pressures at full arching were in fairly good agreement with those recorded. In Test B-4, which has a B/h ratio very close to the critical spacing, m_{cr}, arching action did not materialize as the theory predicted.

TABLE 1.—Test Results

Test number (1)	Soil (2)	Slope (3)	Depth, in inches (4)	Density, in pounds per cubic foot (5)	ϕ, in degrees (6)	Normal pressure, in pounds per square inch (7)	Pile transducer pressure, in pounds per square inch (8)	Pile calculated pressure, in pounds per square inch (9)	Difference, as a percentage (10)	Remarks (11)
A-1	Blast sand	15°	5.5	105	41	6.10	2.72 top 3.00 bottom	4.45	37	K is calculated from the recorded pressure at rest under applied load in A series.
A-2	Blast sand	35°	5.5	105–	41	2.95	0.69 top	0.85	19	
A-3	Blast sand	35°	5.5	105	41	6.10	2.45 top 1.8 bottom	2.20	4	
B-1	Otawa sand	26.5°	9.12	101.0	35.2	12.72	9.40 top	13.30	29.3	$K = 1 - \sin\phi$ is used for B series. The piles are 7/8-in. (2.22-cm) thick plexiglas bars.
	US 30–40									
B-2	US 30–40	26.5°	9.20	99.5	34.0	12.72	9.80 top	13.40	26.9	
B-4	US 30–40	26.5°	5.70	99.5	34.0	0.17	0.06 bottom	—	—	Not applicable, near critical spacing (see text).

Note: 1 in. = 2.54 cm; 1 pcf = 15.95 kg/m³; 1 psi = 6.9 kN/m².

Summary and Conclusions

An analytical expression of soil arching in long slopes created by piles under idealized conditions and experimental results have been presented. The analysis allows the estimation of effective pile spacing for slope stabilization, and also estimates the loads on piles. Illustrative examples are presented. However, the writers wish to show that the results in this article should be considered as a research rather than a basis for practical design at present. The findings are itemized as follows:

1. The average arching pressure in cohesive granular soils increases exponentially to a maximum value equal to the pressure at rest in slopes.
2. All factors being the same, arching is more prominent for larger ϕ and c values, as can be seen in Eq. 5.
3. Both sandy and clayey slopes have the critical spacing (Eqs. 7 and 15) which is a function of soil properties and slope. For pile spacings larger than the critical value, no arching will develop.
4. For sandy slopes there exists a characteristic spacing m_m (Eq. 10). For pile spacings greater than this m_m, only part of soil pressure is transferred to the piles.
5. Both sandy and clayey slopes have the arching zone n_0 (Eqs. 8 and 13) within which soils are not governed by the uphill pressure.
6. The experiments conducted on two types of sands support the theory presented in the paper.
7. The piles, or piers, thus designed, required more reinforcement than those subjected to pressure at rest, directly pressing on their uphill surfaces. The size of piles, or piers, is not directly involved in the arching phenomenon, but it is tacitly assumed that the practical sizes will be sufficiently large and rigid so that the arching phenomenon can develop in the soil.

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Appendix II.—Notation

The following symbols are used in this paper:

B = clear spacing between piles;
B_{cr} = critical clear spacing between piles;
B_m = most effective clear spacing between piles;
c, c_1 = cohesion;
d = diameter of pile;
e = natural exponent base;
F = driving force;
h = depth of sliding plane;
i = slope;
K = coefficient of earth pressure at rest;
K_1, K_2 = combined terms;
$m = B/h$ = relative width;
m_{cr} = critical relative width;
m_m = most effective relative width;
$n = x/B$ = relative distance;
n_0 = arching zone;
P = total force;
p = average pressure;
R_1, R_2 = resistance;
W = soil weight;
x = distance along slope;
α = combined term;
γ = unit weight of soil;
ϕ = angle of internal friction;
σ = normal stress; and
τ = shear strength.

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DISCUSSION