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Safety Factors for Probabilistic Slope Design[a]

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Abstract: Inconsistencies in present solution methods for slope stability problems under undrained conditions are noted, and a first-order, second-moment, solution technique with a probabilistic base is suggested. The proposed procedure examines these problems with regard to the separate variables involved and utilizes partial safety factors which are proportional to the coefficient of variation of the pertinent parameters. For simplicity, a circular arc failure mechanism is assumed, and design acceptability is based on the derived value of a reliability measure, the safety index, which reflects the probability of occurrence of the assumed failure mechanism. Statistical data for the required load, resistance, and bias variables are presented. Using these, the safety index associated with current design techniques is determined, and the implications of its magnitude are examined. Sensitivity studies, performed to determine which variables have the greatest effect on design results are also described. Finally, partial safety factors are proposed for design corresponding to a desired failure probability.

Introduction and Scope

The goal of slope stability analysis is to avoid shear failure and the downward movement of soil within the slope. Since the problem's governing variables, e.g., loads due to unbalanced soil weight and soil shearing resistance, are random rather than deterministic in nature, every slope will have a finite failure probability associated with its particular geometry. Methods have been developed for assessing the failure probability of a slope with defined geometry (1,9,39,41,42). Alternatively, this paper presents a rapid method for determining a slope geometry possessing a preselected desired failure probability under given soil conditions.

The probabilistic procedure used is of first-order, second-moment nature. For simplicity, the method will be developed for slope stability in fine-grained soil under undrained conditions. Under this circumstance, the soil's shearing resistance is independent of the slope geometry, and, thus, total stress ($\phi = 0$) analysis may be applied. In some problems, this may not be the critical situation;

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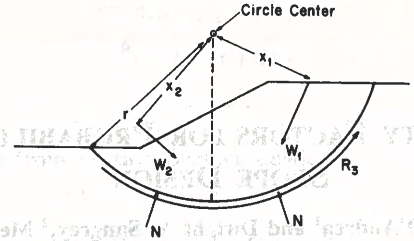

FIG. 1.—Forces Acting on Trial Failure Section

therefore, the analysis refers solely to that condition that occurs prior to significant dissipation of the pore water pressures produced by slope construction. The probabilistic procedure described in this paper can be applied to any slope geometry and undrained strength profile. To facilitate the technique's explanation, the slopes examined are assumed to be of constant inclination and made up of soils whose strength-depth relationship is uniform. This situation has considerable practical significance, and its deterministic solution in chart form was originally presented by Taylor (36).

A major application of reliability methods is cost-based decision analysis. Although this extension is not addressed in the paper, the methodology has been developed for this purpose.

Deterministic Practice.—In analysis of slopes using the $\phi = 0$ method, trial failure surfaces, generally circular for mathematical ease, are assumed until the "critical" surface (that with minimum value of a single overall safety factor) is determined. Iteration of slope geometry follows until the safety factor coincides with a preselected desired value. Use of design charts or computer programs facilitates the process considerably.

To examine inconsistencies in the traditional deterministic single safety factor approach, consider the trial failure surface shown in Fig. 1. N indicates forces normal to the failure surface. W_1 represents the resultant of all forces, including soil weight, surcharge loading, and water forces due to slope submergence of water filled tension cracks, which act to produce overturning about the circle center. W_2 represents a similar resultant for restoring forces. The moment arms about the circle center of the forces W_1 and W_2 are x_1 and x_2, respectively. R_3 is the resisting force that acts along the failure surface, and its maximum value is a function of the soil's undrained shearing resistance, S_u. A number of definitions for single safety factor have been proposed. One commonly used for circular surfaces defines safety factor, F, as the ratio of the stabilizing to overturning moments about the center of rotation. Upon defining M_1, M_2, and M_3 as moments about the circle center due to W_1, W_2, and R_3, two forms of equation for F are then possible. The inconsistency occurs because M_2 may be considered as reducing the total overturning moment or increasing the resisting moment. In the first case, the safety factor, F_1, becomes

$$F_1 = \frac{M_3}{M_1 - M_2} \quad \text{(1)}$$

TABLE 1.—Traditional Overall Safety Factors for Slope Stability

Source (1)	Suggested F (2)	Comments (3)
Bjerrum (6)	1.30	for use with field vane strengths that have been corrected for strain rate and anisotropic effects
Bowles (8)	1.25	
Gedney and Weber (4)	1.25–1.50	highest F if high failure consequence, poor construction, or highly uncertain strength
Hansen (18)	1.50	
Meyerhof (28)	1.30–1.50	
Sowers (35)	1.30–1.40	
Terzaghi (37)	1.50	
	1.25–1.3	if temporary loading conditions or end of construction is critical
U.S. Dept. of the Navy DM-7 (13)	1.50	for permanent or sustained conditions

However, should M_2 be considered to increase the resisting moment, the alternatively defined safety factor, F_2, is

$$F_2 = \frac{M_3 + M_2}{M_1} \quad \text{(2)}$$

In general, F_1 and F_2 are equivalent only under conditions of impending failure.

A more customary, and preferable, interpretation of safety factor defines F as the ratio of the strength available to the strength required for equilibrium. For $\phi = 0$, this yields results identical to Eq. 1. Unfortunately, since F is applied to strength only, this definition implies that soil strength is the only random variable associated with the problem and that effects of external loads may be represented by precise deterministic values.

Assuming that the definition of F given by Eq. 1 is accepted practice, Table 1 illustrates that design methodology will still be inconsistent due to the range of F values which have been recommended by various engineers.

The technique proposed in this paper will overcome these inconsistencies in two ways: through the use of partial safety factors and through selection of safety factor values based on probability theory. The partial safety factors, (2,14,18), are applied to an equilibrium equation by dividing the resistance terms by safety factors and multiplying loading terms by their own safety factors, e.g., if W_1 and W_2 in the previously described problem were due solely to soil weight, the design equation, regardless of interpretation of M_2, becomes

$$\frac{M_3}{\theta_3} - M_1\theta_1 + M_2\theta_2 = 0 \quad \text{(3)}$$

in which θ_1, θ_2, and θ_3 are partially safety factors applied to M_1, M_2, and M_3, respectively. As a result, upon establishing appropriate partial safety factors, a consistent design will be achieved for any statically valid analysis.

The second advantage of the proposed technique lies in the fact that magnitudes of the partial safety factors may be determined by invoking probability theory so that they will be functions of both the appropriate failure probability and the random nature of the variable on which they are applied. Consequently, the respective contributions of each variable to the overall unreliability of the problem may be adequately weighted.

Partial Safety Factor Format.—A partial safety factor format has been described by Ditlevsen (14). Although it may appear that the detailed separation of sources of variable randomness proposed by Ditlevsen is more elaborate than the slope stability problem requires, this format was chosen because of its adaptability to all forms of undrained stability problems, including that of building foundations.

The procedure consists of first defining a failure function for the assumed mechanism. The variables within the function include not only stochastic variables contributing to the total resistance and stochastic variables inducing load, but also bias factors, B. Bias factors may or may not be treated deterministically.

Consider the simple slope (Fig. 2), which is not submerged or subjected to external loads. Assuming that all geometry may be treated deterministically, the failure function states that failure occurs when

$$B_A B_S M_3 - (M_1 - M_2) = 0 \quad \text{(4)}$$

in which B_S and B_A = bias factors reflecting, respectively, the errors in evaluating shearing resistance and the errors associated with the analysis procedure used.

Partial safety factors are then applied to produce the design function, G:

$$\frac{B_A B_S M_3}{\theta_{B_A} \theta_{B_S} \theta_S} - \theta_\gamma (M_1 - M_2) = G = 0 \quad \text{(5)}$$

in which θ_{B_A} and θ_{B_S} are partial safety factors reflecting uncertainty in evaluating analysis technique bias and strength estimation bias. θ_{S_u} and θ_γ are partial safety factors applied to soil strength variability and the soil's total unit weight variability.

To create a format readily applicable to other stability problems, the two sources of randomness have been categorized as results of "variability" and "uncertainty" in a manner suggested by Cornell (10). For instance, variability

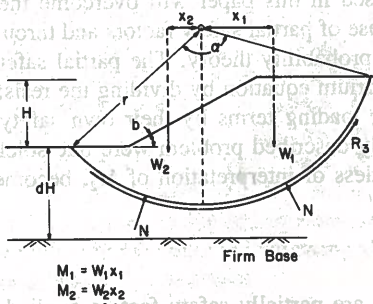

FIG. 2.—Failure Mechanism for Simple Slope

describes the inherent unavoidable deviations in the shearing resistance due to the soil's nonhomogeneous nature. Uncertainty, in contrast, reflects the differences between the actual resistance to overturning and the design estimate of resistance made by the engineer. An important difference between "variability" and "uncertainty" is that control is possible for the latter but not the former. Through various options available for sampling, testing, and mathematical modelling, an engineer can limit uncertainty. In contrast, he cannot influence variability, although he can reduce the error associated with its estimate. In summary, the random nature of the actual resisting moment is assumed to be due to three distinct sources: bias, uncertainty, and variability. Each source possesses a number of contributing agents reflected in the partial safety factors.

The shear strength bias, B_S, is defined as the ratio of the maximum shearing resistance, which may be actually mobilized at a point in situ and that point's measured shearing resistance. The origins of this bias are disturbance during sampling and test preparation, O_d; differences between test and field loading rates, O_r; use of intact specimens as related to size and in situ spacing of fissures, O_s; and the combined effects of soil strength anisotropy and laboratory shearing modes, O_a. Depending upon the manner of strength testing employed, B_S will be estimated directly (as with field vane shear) or as the product of the bias origins via

$$\bar{B}_S = (\bar{O}_d)(\bar{O}_r)(\bar{O}_s)(\bar{O}_a) \quad \text{(6)}$$

(a bar placed over the variable indicates the mean value). The format is a second-moment approximation in that it is assumed that all variables are completely defined by their respective means and standard deviations. The most useful second moment stochastic parameter is the coefficient of variation, V, which is the variables standard deviation divided by its mean. When specific data on the origins of strength bias are available, a first-order approximation to the coefficient of variation of B_S is

$$V_{B_s} = (V_{O_d}^2 + V_{O_r}^2 + V_{O_s}^2 + V_{O_a}^2)^{1/2} \quad \text{(7)}$$

Similarly, analysis bias, B_A, is defined as the ratio of the actual resistance to overturning provided by material strength and the moment resistance provided by material strength calculated via the applied analysis technique. Analysis bias is assumed to be the product of the following agents: plane strain simplification, a_s; tension crack phenomena, a_t; rupture surface shape simplications, a_c; and assumptions of plastic behavior, a_p. Statistical parameters of B_A are determined via

$$\bar{B}_A = (\bar{a}_s)(\bar{a}_t)(\bar{a}_c)(\bar{a}_p) \quad \text{(8)}$$

$$\text{and} \quad V_{B_A} = (V_{a_s}^2 + V_{A_t}^2 + V_{a_c}^2 + V_{a_p}^2)^{1/2} \quad \text{(9)}$$

Suggested values for these variables will be presented independently from the partial safety factor format development in subsequent sections.

Returning to the design equation, Eq. 5, the partial safety factors, θ_i, are defined by

$$\theta_i = e^{\nu V_i} \quad \text{(10)}$$

in which e = the base of natural logarithms; V_i = the coefficient of variation of the parameter on which θ_i is placed; and ν = a random variable which will be used to assess the reliability of the state expressed by the design equation:

Some significant observations concerning this definition of partial safety factor are

1. All partial safety factors are greater than or equal to 1.0, with resistances, or stochastic bias factors that contribute to resistance, being divided by their partial safety factor, and loads, or bias factors contributing to instability, being multiplied by the pertinent safety factor.
2. If the coefficient of variation of the parameter is zero (indicating exact knowledge of that parameter's value, or that the parameter is deterministic in nature), the partial safety factor applied to that parameter is 1.0.
3. As the coefficient of variation increases (indicating the value of the parameter is more random in value) the value of the partial safety factor increases exponentially.

Although other alternative definitions of θ as a function of ν may be acceptable, the aforementioned definition was chosen since it handles subsequent mathematical steps with relative ease.

The design equation, Eq. 5, has now become a stochastic relation in terms of the basic reliability parameter ν, which is itself a random variable. The value of ν depends upon the original random variables (undrained shear strength, S_u; total unit weight, γ; and the bias factors). Values of ν less than zero indicate failure states; $\nu = 0$ represents impending failure; and positive values of ν denote "safe" design. By varying load and resistance terms in a trial design, the engineer would note increasing conservatism in subsequent trials if the associated values of ν increased. Each design alternative for a particular problem will have an associated distribution of values for the random variable ν. By determining the characteristics of the distribution of ν for a particular design, conclusions concerning that design's acceptability may be reached.

The distribution of ν can be characterized by its mean and standard deviation. These characteristics will be useful in formulating a measure of reliability, the safety index. To evaluate the mean of ν, $\bar{\nu}$, mean values and coefficients of variation of the original random variables (S_u, γ, B_A, and B_S) are substituted into the expected value of the design equation, i.e.

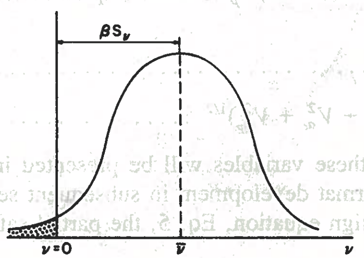

FIG. 3.—Frequency Distribution of ν

$$\frac{\bar{B}_A \bar{B}_S \bar{M}_3}{\bar{\theta}_{B_A} \bar{\theta}_{B_S} \bar{\theta}_S} - \bar{\theta}_\gamma (\bar{M}_1 - \bar{M}_2) = G = 0 \quad \text{in which} \quad \bar{\theta}_i = e^{\bar{\nu}} V_i \quad \ldots\ldots (11)$$

Assuming the mean and standard deviations of all of the original variables are known, Eq. 11 may be solved for ν.

Since the load, strength, bias factors, and ν variables are not correlated, the standard deviation of ν, S_ν, may be approximated by neglecting terms of second and higher orders via a technique described by Hahn and Shapiro (17). After implicit differentiation, the resulting equation is

$$S_\nu = \left[\left(\frac{\frac{\partial G}{\partial B_A}}{\frac{\partial G}{\partial \nu}} \right)_m^2 S_{B_A}^2 + \left(\frac{\frac{\partial G}{\partial B_S}}{\frac{\partial G}{\partial \nu}} \right)_m^2 S_{B_S}^2 + \left(\frac{\frac{\partial G}{\partial S_u}}{\frac{\partial G}{\partial \nu}} \right)_m^2 S_{S_u}^2 + \left(\frac{\frac{\partial G}{\partial \gamma}}{\frac{\partial G}{\partial \nu}} \right)_m^2 S_\gamma^2 \right]^{1/2} \quad (12)$$

In Eq. 12, m subscripts indicate evaluation of the partial derivatives at mean values of the variables involved, and S_i indicates the standard deviation of variable i. This first-order approximation coupled with the assumption that all variables may be adequately defined by their respective means and standard deviations results in this technique being termed of first-order, second-moment format.

After solving Eqs. 11 and 12 for a given design geometry, the design's reliability may be assessed quantitatively by a safety index, β, defined as

$$\beta = \frac{\bar{\nu}}{S_\nu} \quad \ldots\ldots (13)$$

Fig. 3 shows the frequency distribution of the random variable ν, with $\bar{\nu}$ being β standard deviation units greater than $\nu = 0$ (the failure condition). Thus, the area under the curve for $\nu > 0$ (shown shaded) indicates the probability of failure, P_f. Although the exact shape of this curve is unknown, since ν can be written as the sum of a number of random variables, the central limit theorem, (4), indicates that the distribution of ν is approximately normal. If so, values of β of 1.29, 2.32, 3.09, and 3.72 correspond to P_f values of 10^{-1}, 10^{-2}, 10^{-3}, and 10^{-4}, respectively. Although the normality assumption will be invoked throughout the rest of this paper, it should be noted that even if no particular distribution was assumed, various designs or trial failure modes could be compared since the lowest value of β represents the least safe condition.

The design conclusions noted previously are based on a P_f value reflecting the probability of failure on the critical surface. Catalan and Cornell (9) have demonstrated that due to spatial strength variability this value must be equal to or less than the slopes overall probability of failure in the shearing mode. However, their results indicate that the differences are small unless the horizontal spatial variation in strength is unusually large. For the method presented herein, a conservative control on this possibility would be based on using "worst hole" values to determine the statistical parameters of S_u.

This partial safety factor technique may be applied to determine the β associated with a given design. The method is more useful when manipulated to derive values of partial safety factors pertaining to specific desired β values.

These methods can be explained more clearly after a summary of methods for evaluating statistical parameters of the pertinent variables.

Statistical Parameters of Soil Properties.—After a finite number of strength and total unit weight measurements have been performed on sample elements, standard statistical techniques are used to determine the means. These mean values are used to compute $\bar{M}_1$, $\bar{M}_2$, and $\bar{M}_3$ since it is assumed that the mean of the spatial average of the strength mobilized along a shear surface (or unit weight within the surface) equals the mean of the individual sample strengths (or sample unit weights). The sources of the random nature of these variables will be considered individually as those due to nonhomogeneity and those due to insufficient sampling with the following procedure advocated with respect to strength statistics. If n samples are tested for strength, conventional statistical theory indicates that the coefficient of variation of the strength of the sample-sized elements may be estimated as

$$V_{S_n} = \frac{\left[\frac{1}{n-1}\sum_{i=1}^{n}(S_{u_i} - \bar{S}_u)^2\right]^{1/2}}{\bar{S}_u} \quad (14)$$

V_{S_n} typically ranges between 0.1 and 0.48 (19,23). V_{S_n} is not necessarily the parameter needed for evaluating a field failure surface, however. A more meaningful characteristic would be the measurement-based estimate of coefficient of variation of the spatial average strength of the many sample-sized elements comprising a typical failure surface, V_{S_a}. Complicated methods of relating V_{S_a} and V_{S_n} have been suggested by Yuceman, Tang, and Ang (42), and Vanmarcke (38). In lieu of these, Wu (40) used Chicago Clay data to estimate V_{S_a} equal to two tenths V_{S_n}, and this simplification, which is thought to be conservative, has been used in this paper for all clays.

Yuceman, et al. (42) indicate that the estimation uncertainty will decrease with increasing n, and may be accounted for via an additional coefficient of variation on strength, V_{S_e}, of

$$V_{S_e} = \frac{V_{S_n}}{n^{1/2}} \quad (15)$$

Combining the nonhomogeneous and insufficient sampling effects yields

$$V_S = (V_{S_a}^2 + V_{S_e}^2)^{1/2} = \left[(0.2\,V_{S_n})^2 + \frac{V_{S_n}^2}{n}\right]^{1/2} \quad (16)$$

Strength variability may then be accounted for via the partial safety factor

$$\theta_S = e^{\nu V_S} \quad (17)$$

Similar arguments could be made for an involved derivation of V_γ. However, due to the relatively small value of the coefficient of variation of the total unit weight of the sample-sized elements, V_{γ_n}, V_γ may be conservatively approximated as 0.04 without pronounced effect on the end results. Thus

$$\theta_\gamma = e^{0.04\nu} \quad (18)$$

TABLE 2.—Suggested Values of O_d

Clay sensitivity (1)	Open tube sample (2)	Piston sample (3)	Block sample (4)
1.0–2.0	1.35	1.20	1.03
2.0–4.0	1.40	1.25	1.05
Above 4.0	1.75	1.30	1.07

Bias Factor for Shear Strength.—When the source of strength data is field vane test results, B_S and V_{B_S} may be calculated directly rather than considering individual contributions and applying Eqs. 6 and 7. Results reported by Bjerrum (6) supplemented by those of Ladd and Foott (25) were manipulated via standard statistical methods (12) to yield

$$\bar{B}_S = 1.082 - 0.0052\,I_p \quad (19)$$

in which I_p = the soil's plasticity index, as a percentage. The scatter of assembled data points about this line resulted in a calculated value of V_{B_S} of 0.20. These statistical parameters are valid within the range of given data (I_p below 110%), and it is suggested that B_S should not exceed 1.0. Dascal and Tournier (11) and Graham (16) have advocated values lower than those of Eq. 19 if the soil is highly sensitive. However, if the lower B_S is due to strain softening exhibited by such sensitive soils, the proposed format reflects this decreased moment resistance in analysis bias, as described in the following section.

When compression tests are the primary vehicle for establishing the soil's strength, Eqs. 6 and 7 should be applied using the statistical parameters of the four bias origins summarized later.

Disturbance reduces the laboratory measured strength below that which an in situ sample would exhibit under similar conditions (25). This results in O_d values greater than 1.0 which increase with increasing soil sensitivity. Available data on disturbance and sampling method (20,28,41,42) were used to derive the $\bar{O}_d$ values listed in Table 2. Furthermore, V_{O_d} of 0.14 is subjectively suggested for soils whose sensitivity is less than 4.0 with V_{O_d} = 0.2 for soils of sensitivity greater than 4.0 (12).

A second significant strength bias origin, O_r, reflects the differing strain rates imposed during shear in the field and laboratory. Previous studies (16,25,31, 33,42) have demonstrated that undrained strength decreases as loading rate decreases. The strength difference has been shown (12) to be a function of plasticity index and sensitivity;

$$O_r = 1.0 - y \log\left(\frac{t_f}{t_t}\right) \quad (20)$$

in which t_f and t_t = field and test failure times, respectively; $y = 0.04 + 0.0007\,I_p \leq 0.15$, if sensitivity < 4; and $y = 0.08 + 0.0007\,I_p \leq 0.15$, if sensitivity ≥ 4. Because of limited data, these values are somewhat subjective, as are the V_{O_r} values indicated in Table 3. Skempton and Hutchinson (34) imply that sample size may produce a third origin of strength bias if the soil mass is fissured. Additional evidence supports the conclusion that testing standard size specimens

TABLE 3.—Recommended V_{O_r} Values

Time differential (1)	Sensitivity < 4 (2)	Sensitivity ≥ 4 (3)
$\log t_f/t_r > 2.0$	0.05	0.10
$2.0 \le \log t_f/t_r \le 3.0$	0.12	0.17
$\log t_f/t_r > 3.0$	0.20	0.25

overpredicts the strength of fissured clays (22,42), and it would seem most prudent to test specimens of fissured clays large enough to contain representative fissures. However, should that be impossible, $O_S = 0.7$ and $V_{O_S} = 0.15$ should be applied to results from 1.5-in. (38-mm) diam by 3-in. (76-mm) high cylindrical specimens. For intact clays, or fissured clays for which large specimens have been tested, $O_S = 0.95$ and $V_{O_S} = 0.04$ are suggested to account for minor structural discontinuities (12).

The final strength bias, O_a, originates from strength anisotropy due to both inherent anisotropy which occurs during soil formation and induced anisotropy resulting from principal stress rotation during shear. This directional strength dependence appears to decrease with increasing plasticity index. Furthermore, the soil elements on a toe circle associated with a steep slope undergo much less principal stress rotation than elements on a base circle of a shallow slope. Consequently, O_a is a function of both plasticity index and slope angle. Available information on this factor (24,25,26,41) has been summarized to yield the statistical parameters presented in Table 4.

Bias Factor on Analysis.—Any analysis method based on simplifications will produce erroneous estimates of resisting moment even if the soil strength and strength bias are precisely known. The bias factor on analysis accounts for this, with Eqs. 8 and 9 invoked to compute the statistical parameters of the bias factor. The four agents contributing to analysis bias will also be considered separately. In contrast to strength bias, their value depends on final design geometry and requires initial assumptions of slope geometry and possible subsequent iteration.

TABLE 4.—Statistical Parameters for O_a

I_p, as a percentage (1)	Deep-seated slope failure surfaces (2)	Toe failures of steep slopes (3)	Vertical cuts (4)
0	$\bar{O}_a = 0.67$	$\bar{O}_a = 0.75$	$\bar{O}_a = 1.0$
	$V_{Oa} = 0.09$	$V_{Oa} = 0.05$	$V_{Oa} = 0.03$
20	$\bar{O}_a = 0.78$	$\bar{O}_a = 0.84$	$\bar{O}_a = 1.0$
	$V_{Oa} = 0.09$	$V_{Oa} = 0.05$	$V_{Oa} = 0.03$
40	$\bar{O}_a = 0.86$	$\bar{O}_a = 0.9$	$\bar{O}_a = 1.0$
	$V_{Oa} = 0.09$	$V_{Oa} = 0.05$	$V_{Oa} = 0.03$
60	$\bar{O}_a = 0.95$	$\bar{O}_a = 0.96$	$\bar{O}_a = 1.0$
	$V_{Oa} = 0.09$	$V_{Oa} = 0.05$	$V_{Oa} = 0.03$
80	$\bar{O}_a = 0.97$	$\bar{O}_a = 0.98$	$\bar{O}_a = 1.0$
	$V_{Oa} = 0.09$	$V_{Oa} = 0.05$	$V_{Oa} = 0.03$
100	$\bar{O}_a = 0.99$	$\bar{O}_a = 1.0$	$\bar{O}_a = 1.0$
	$V_{Oa} = 0.09$	$V_{Oa} = 0.05$	$V_{Oa} = 0.03$

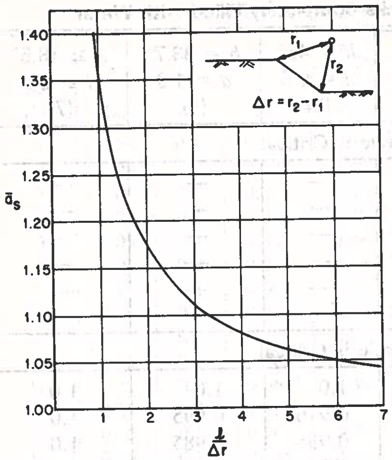

FIG. 4.—Analysis Bias from Plane Strain Assumption for Slopes

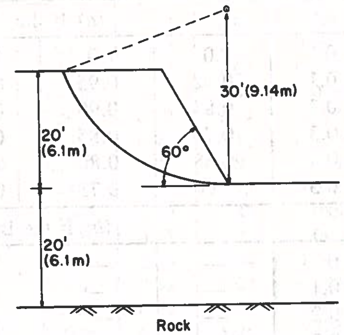

FIG. 5.—Geometry of Slope Used in Sensitive Studies

When a slope's plan length, l, is relatively small compared to height, additional shearing resistance supplied at the slope's ends should be added if plane strain conditions were assumed. The bias element produced by this resistance increase, a_s, may be calculated using the results of Baligh and Azzouz (3) as indicated in Fig. 4. This factor should be used in conjunction with a coefficient of variation given by:

$$V_{a_s} = \frac{\bar{a}_s - 1.0}{4} \qquad (21)$$

If l is not defined, or the slope's lateral extent is large, $\bar{a}_s = 1.0$ and $V_{a_s} = 0.0$.

The presence of tension cracks along a slope's crest will generally decrease stability. Although the proposed format could be modified to treat the depth of water, within such cracks, stochastically, lack of data prompts the following semideterministic approach for soils whose strength is independent of depth. First, the designer must choose to assume either no cracks or cracks to some deterministic depth. When no cracks are assumed present, $\bar{a}_t = 1.0$ and $V_{a_t} = 0.0$. When cracks are assumed to some given depth and completely filled with water, the results of Janbu (21) produced the a_t values suggested in Table 5. $V_{a_t} = 0.04$ should be used in conjunction with such a_t values to account for errors in the approximations used to derive them. Note that this implies that the stability decrease is mainly due to a loss in resisting moment, and the partial safety factors on increased loading due to water and the decrease in soil weight loading are 1.0.

The third analysis bias agent, a_c, reflects the error obtained by assuming a circular arc-shaped failure surface. Booker and Davis (7) have shown that for slope angles, b, greater than 20°, the circular arc provides identical results to those of more exact analysis techniques. Based on their findings, it is suggested that if $b \ge 20°$, $a_c = 1.0$ should be employed; otherwise

TABLE 5.—$\bar{a}_t$ Values Assuming Cracks Completely Filled with Water

Crack depth (1)	b = 90° (2)	b = 60° (3)	b = 45° (4)	b = 45° d = 0.5 (5)	b = 33.7° d = 1.0 (6)	$b \geq$ 18.5° $d \geq$ 2.0 (7)
(a) If the Toe Circle is Critical						
0	1.0	1.0	1.0	—	—	—
0.1	0.92	0.95	0.97	—	—	—
0.2	0.84	0.90	0.93	—	—	—
0.3	0.76	0.85	0.90	—	—	—
0.4	0.68	0.80	0.86	—	—	—
0.5	0.60	0.75	0.82	—	—	—
(b) If the Base Circle is Critical						
0	—	—	—	1.0	1.0	1.0
0.1	—	—	—	0.976	0.995	1.0
0.2	—	—	—	0.956	0.985	1.0
0.3	—	—	—	0.944	0.976	1.0
0.4	—	—	—	0.916	0.963	1.0
0.5	—	—	—	0.887	0.95	1.0

Note: b and d is defined on Fig. 2.

$$\bar{a}_c = 0.67 + 0.2b^{1/6} \quad \ldots\ldots\ldots (22)$$

in which b is in degrees. A V_{a_c} of 0.02 may be used regardless of b magnitude.

Since the limit equilibrium technique assumes a simultaneous mobilization of shearing resistance equal to peak shear strength at all points along the failure surface, the nonuniform nature of failure surface displacement produces an overprediction of resistance if the soil exhibits strain softening. The result is a fourth analysis bias agent, a_p, which accounts for nonplastic behavior and may be significantly below 1.0 for soils with high at-rest pressure coefficients as well as those which strain soften. Bishop (5) has suggested brittleness index as a measure of its strain-softening tendencies. Based on his observations, plus other data (27,32,42), it is suggested that for soils whose brittleness index is below 0.3 and whose liquidity index exceeds 0.5, $\bar{a}_p$ = 0.95 and V_{a_p} = 0.06 should be used in conjunction with peak shearing resistance values.

Sensitivity Studies.—This section describes studies performed to assess the sensitivity of a slope's reliability to variations in the statistical parameters of the resistance variables. As an example, consider a 20-ft (6.1-m) high slope which is to be cut into a soil whose strength versus depth profile is uniform to a depth of 40 ft (12.2 m), then underlain by bedrock. The soil's mean total unit weight, $\bar{\gamma}$, and mean corrected strength, $\bar{B}_s \bar{S}_u$, are 104 pcf (1,666.1 kg/m^3) and 517 psf (24.75 kPa), respectively. Using deterministic methods (36), the maximum slope inclination and critical circle for an overall safety factor of 1.30 are shown in Fig. 5.

It is further assumed that the slope failure is long, $\bar{a}_s$ = 1.0; tension cracking is negligible, a_t = 1.0; and the soil's brittleness index is low and its liquidity index exceeds 0.5 ($\bar{a}_p$ = 0.95). Since the slope's inclination is 60°, $\bar{a}_c$ = 1.0; thus, Eq. 8 yields $\bar{B}_A$ = 0.95. With V_γ fixed at a constant value of 0.04, V_S, V_{B_S}, and V_{B_A} were varied in Eqs. 11 and 12 over ranges consistent with previous comments. The results are presented in Table 6 where the failure probabilities indicated assume a normal distribution of ν.

TABLE 6.—Results of Sensitivity Studies

Case (1)	V_S (2)	V_{BS} (3)	V_{BA} (4)	$\bar{B}_s\bar{S}_u$ = 517 psf (25 kPa) $\bar{\nu}$ (5)	β (6)	P_f (7)	$\bar{B}_s\bar{S}_u$ = 596 psf (28 kPa) $\bar{\nu}$ (8)	β (9)	P_f (10)
1	0.05	0.1	0.06	0.86	1.61	0.0537	1.42	2.68	0.0037
2	0.05	0.1	0.085	0.78	1.46	0.0721	1.29	2.44	0.0073
3	0.05	0.2	0.06	0.61	0.98	0.1635	1.02	1.63	0.0516
4	0.05	0.2	0.085	0.57	0.94	0.1736	0.95	1.57	0.0582
5	0.05	0.3	0.06	0.48	0.68	0.2483	0.79	1.14	0.1271
6	0.05	0.3	0.085	0.45	0.67	0.2514	0.75	1.12	0.1314
7	0.15	0.1	0.06	0.61	1.10	0.1357	1.02	1.83	0.0336
8	0.15	0.1	0.085	0.57	1.05	0.1469	0.95	1.75	0.0401
9	0.15	0.2	0.06	0.48	0.82	0.2061	0.79	1.37	0.0853
10	0.15	0.2	0.085	0.45	0.80	0.2119	0.75	1.33	0.0918
11	0.15	0.3	0.06	0.39	0.62	0.2676	0.65	1.04	0.1492
12	0.15	0.3	0.085	0.37	0.61	0.2709	0.62	1.02	0.1539
13	0.25	0.1	0.06	0.48	0.77	0.2206	0.79	1.28	0.1003
14	0.25	0.1	0.085	0.45	0.75	0.2266	0.75	1.25	0.1056
15	0.25	0.2	0.06	0.39	0.65	0.2578	0.65	1.08	0.1401
16	0.25	0.2	0.085	0.37	0.64	0.2611	0.62	1.07	0.1423
17	0.25	0.3	0.06	0.33	0.54	0.2946	0.55	0.90	0.1841
18	0.25	0.3	0.085	0.32	0.53	0.2981	0.53	0.89	0.1867
19	0.35	0.1	0.06	0.39	0.58	0.2810	0.65	0.96	0.1685
20	0.35	0.1	0.085	0.37	0.57	0.2843	0.62	0.95	0.1711
21	0.35	0.2	0.06	0.33	0.52	0.3015	0.55	0.87	0.1922
22	0.35	0.2	0.085	0.32	0.52	0.3015	0.53	0.86	0.1949
23	0.35	0.3	0.06	0.29	0.46	0.3228	0.47	0.76	0.2236
24	0.35	0.3	0.085	0.28	0.45	0.3264	0.46	0.76	0.2236

In a second study, the soil's mean corrected strength was assumed to be 596 psf (28.54 kPa), with unit weight and geometry as before. A deterministic analysis of this case gives an overall safety factor of 1.5 and a critical circle identical to that of the initial case. Similar variations in V values produced the additional data of Table 6.

These results indicate that slopes designed deterministically with any chosen overall safety factor will be of inconsistent reliability. Furthermore, a slope designed with a conventional safety factor (even as high as 1.5) may have a surprisingly high associated failure probability.

Another sensitivity study considered the effects of change in the significant variables on the mean corrected strength required to maintain a particular slope with a specific reliability. Three β values (1.29, 2.32, and 3.09) were selected corresponding to failure probabilities (assuming ν normally distributed) of 0.1, 0.01, and 0.001, respectively. The slope geometry was the same as in Fig. 5, as were the mean total unit weight of 104 pcf; B_A of 0.95; and V_γ of 0.04. V_S, V_{B_A}, and V_{B_S} were then varied, with the results shown in Table 7. Note that for β = 1.29 the range in mean corrected strengths required would have produced corresponding traditional overall safety factors varying from a low of 1.25 (for Case 1) to a high of 1.93 (for Case 12). Similar F ranges are from

TABLE 7.—Necessary Mean Corrected Strength to Yield Required β

Case (1)	V_{BS} (2)	V_{BA} (3)	V_S (4)	$\bar{B}_s\bar{S}_u$, in pounds per square foot, required for β = 1.29 (5)	β = 2.32 (6)	β = 3.09 (7)
1	0.10	0.06	0.05	496	568	630
2	0.10	0.06	0.20	565	720	863
3	0.10	0.06	0.35	674	987	1,314
4	0.10	0.085	0.05	504	586	656
5	0.10	0.085	0.20	571	733	883
6	0.10	0.085	0.35	678	999	1,334
7	0.30	0.06	0.05	625	862	1,097
8	0.30	0.06	0.20	671	980	1,300
9	0.30	0.06	0.35	762	1,232	1,765
10	0.30	0.085	0.05	629	874	1,116
11	0.30	0.085	0.20	675	991	1,320
12	0.30	0.085	0.35	766	1,243	1,786

1.43 (Case 1) to 3.13 (Case 12) for β = 2.32, and from 1.59 (Case 1) to 4.50 (Case 12) for β = 3.09. These results are in reasonable agreement with the conclusions of Meyerhof (29) and Yuceman, Tang, and Ang (42).

Recommended Partial Safety Factors.—Since conventional deterministic overall safety factor analysis produces designs whose reliability varies over such a range, it is impossible to derive single values of partial safety factors that result in designs of the same reliability as conventional analysis. However, it is possible, and more desirable, to provide partial safety factors which yield a design of specified failure probability.

Because of the fluctuation in failure consequence associated with slopes, no single value of desired failure probability applies to all situations. As a result, partial safety factors corresponding to P_f values of 0.1 (β = 1.29), 0.01 (β = 2.32), and 0.001 (β = 3.09) are given, assuming that this range will adequately cover most practical situations.

Since sensitivity studies prove a relatively small ν fluctuation for a particular β and V_γ is comparatively small, unique values of θ_γ of 1.03, 1.06, and 1.08, corresponding to P_f values of 0.1, 0.01, and 0.001, respectively, are suggested. However, the large range of V_{BA}, V_{BS}, and V_S precludes the derivation of unique values of θ_{BA}, θ_{BS}, or θ_s. Alternatively, Fig. 6 presents values of the product $\theta_{BA}\,\theta_{BS}\,\theta_S$ as a function of V_S, V_{BA}, and V_{BS} for various β values, and the following detailed slope design will demonstrate that their use in this form is as expedient as conventional deterministic analysis.

Assume that it is desired to design a cut slope of height 20 ft (6.1 m) and a failure probability of 0.01 in a clay whose strength is uniform with depth to a level of 30 ft (9.14 m) at which point bedrock is encountered. Further assume that appropriate testing and applications of Eqs. 16 and 7 result in $\bar{B}_s\bar{S}_u$ = 710 psf (34 kPa); V_s = 0.15; V_{B_s} = 0.2; γ = 112.26 pcf (1,798.4 kg/m^3); and the soil is of low brittleness index with a liquidity index exceeding 0.5; thus, $\bar{a}_p$ = 0.95 and V_{a_p} = 0.06. If the slope is of great extent, a_s = 1.0 and V_{a_s} = 0.

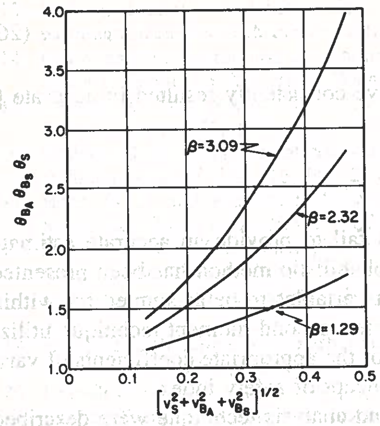

FIG. 6.—Partial Safety Factors for Slopes of Various β

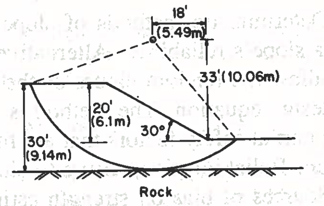

FIG. 7.—Critical Circle for Demonstration Problem

Neglecting tension cracking, a_t = 1.0 and V_{a_t} = 0. If it is temporarily assumed (and subsequently verified by the final design) that the slope's inclination exceeds 20°, $\bar{a}_c$ = 1.0 and V_{a_c} = 0.02. Applying Eq. 8

$$B_A = (1)(1)(1)(0.95) = 0.95 \quad \text{(23)}$$

and Eq. 9

$$V_{BA} = [0 + 0 + (0.02)^2 + (0.06)^2]^{1/2} = 0.063 \quad \text{(24)}$$

The slope may now be designed as follows: Since for P_f = 0.01, θ_γ = 1.06, model the slope as consisting of a soil whose unit weight is ($\gamma\theta_\gamma$ = (112.26)(1.06) = 119 pcf (1,906.4 kg/m^3). Further, for

$$(V_S^2 + V_{BS}^2 + V_{BA}^2)^{1/2} = [(0.15)^2 + (0.2)^2 + (0.063)^2]^{1/2} = 0.258 \quad \text{(25)}$$

and P_f = 0.01, (i.e., β = 2.32), Fig. 6 indicates $\theta_{B_A}\theta_{B_s}\theta_s$ = 1.73. Consequently, the soil within the slope will be modeled with a strength of $\bar{B}_A(\bar{B}_s\bar{S}_u)/(\theta_{B_A}\theta_{B_s}\theta_s)$ = 0.95(710)/1.73 = 389.9 psf (18.67 kPa). Finally, a deterministic analysis (via either charts or circle iteration) is performed to determine the maximum angle of inclination of a 20-ft high slope whose critical circle has a traditional overall safety factor of 1.0 for a 30-ft thick layer of unit weight 119 pcf and strength of 389.9 psf. The solution is the 30° slope shown with its critical circle in Fig. 7. The slope has been designed for a failure probability of 0.01 without expending any more effort than if the slope has been designed deterministically for some overall safety factor exceeding 1.0 with the unfactored unit weight of 112.26 pcf and unfactored strength of 710 psf.

To verify the validity of this solution, the appropriate values may be substituted into Eqs. 11 and 12 resulting in S_ν = 0.5759 and ν = 1.3366. Substitution into Eq. 13 then produces

$$\beta = \frac{\bar{\nu}}{S_\nu} = \frac{1.3366}{0.5759} = 2.321 \quad (26)$$

Other problems similar to this example have consistently resulted in accurate β prediction.

Summary and Conclusion

Deterministic methods of slope analysis fail to provide an accurate estimate of a slope's reliability. Alternatively, a probabilistic method has been presented to allow the random nature of the pertinent variables to be accounted for within a design equation. The method is a first-order, second-moment technique utilizing partial safety factors that are functions of the appropriate coefficients of variation. Reliability is measured using the concept of safety index.

Sources of bias on strength estimation and analysis technique were described and treated stochastically. To illustrate the relative importance of all variables, several parameter sensitivity studies were presented and led to the conclusion that bias on strength estimation as well as variation in spatial average strength influence design most significantly. Finally, partial safety factors were suggested for rapid slope design at a given reliability.

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Use of Cyclic Element Tests to Assess Scale Models

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Abstract: Large scale laboratory model tests have been performed to aid the design of offshore gravity platforms. The paper shows that undrained conditions, necessary for correct model simulation, cannot be achieved if the full field loading program and remolded site material are used. It is theoretically possible to shorten the time of the model event by increasing cyclic load levels and reducing the number of cycles or increasing the frequency or using a dynamically weaker leaner model clay, or by a combination. Cyclic triaxial data are presented for three remolded clays of differing plasticities tested at various frequencies. The results show that for all practical purposes, even if gross model modifications are considered, similarity between model and field events is impossible. It is concluded that laboratory model tests are only indirectly helpful. They may be used to investigate a foundation's response in its weakest fully softened state or to assess a design technique by applying it to a model subjected to a brief arbitrary storm event.

Introduction

Considerable interest in the behavior of soils under cyclic loading has been stimulated by the foundation design problems associated with offshore gravity structures, particularly those in the North Sea where storm conditions are severe.

Because of the great size of the structure and the relatively short period of a storm, the amount of drainage likely to occur in a predominantly clay foundation during that time is negligible. Accordingly, for the purpose of design, it is reasonable and conservative to assume that completely undrained conditions occur. This has important implications for any physical model studies, such as those conducted by Rowe (8); Rowe, Craig and Procter (14); and Andersen, Selnes, Rowe, and Craig (2). If essentially undrained conditions are to occur in a model foundation during a simulated storm, then Rowe (8) has shown that the event must be completed within a time factor, $T = C_v t/L^2$, of approximately 5×10^{-4}. If a remolded clay, with a plasticity index of the order of 20%, is used for a model bed, a C_v value of the order of lm^2/year would seem appropriate from Table 1. With typical model dimensions of 0.45 m–0.6 m, maximum testing times of 50–90 minutes are indicated. Since the design storm may occupy a

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