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TECHNICAL NOTES
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SAFETY FACTORS FOR PROBABILISTIC SLOPE DESIGN[a]

By Robert A. D'Andrea[1] and Dwight A. Sangrey,[2] Members, ASCE

ABSTRACT: Inconsistencies in present solution methods for slope stability problems under undrained conditions are noted, and a first-order, second-moment, solution technique with a probabilistic base is suggested. The proposed procedure examines these problems with regard to the separate variables involved and utilizes partial safety factors which are proportional to the coefficient of variation of the pertinent parameters. For simplicity, a circular arc failure mechanism is assumed, and design acceptability is based on the derived value of a reliability measure, the safety index, which reflects the probability of occurrence of the assumed failure mechanism. Statistical data for the required load, resistance, and bias variables are presented. Using these, the safety index associated with current design techniques is determined, and the implications of its magnitude are examined. Sensitivity studies, performed to determine which variables have the greatest effect on design results are also described. Finally, partial safety factors are proposed for design corresponding to a desired failure probability.

INTRODUCTION AND SCOPE

The goal of slope stability analysis is to avoid shear failure and the downward movement of soil within the slope. Since the problem's governing variables, e.g., loads due to unbalanced soil weight and soil shearing resistance, are random rather than deterministic in nature, every slope will have a finite failure probability associated with its particular geometry. Methods have been developed for assessing the failure probability of a slope with defined geometry (1,9,39,41,42). Alternatively, this paper presents a rapid method for determining a slope geometry possessing a preselected desired failure probability under given soil conditions.

The probabilistic procedure used is of first-order, second-moment nature. For simplicity, the method will be developed for slope stability in fine-grained soil under undrained conditions. Under this circumstance, the soil's shearing resistance is independent of the slope geometry, and, thus, total stress ($\phi = 0$) analysis may be applied. In some problems, this may not be the critical situation;

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Stability of Slopes by Method of Characteristics

By A. Siva Reddy[1] and K. N. Venkatakrishna Rao[2]

Introduction

The assumption made in the method of characteristics that each and every point is at critical equilibrium is not physically appealing and it is reasonable to visualize that only the points along the failure surface are in critical equilibrium, and at other points above the failure surface, the mobilized shear strengths are less than those at critical equilibrium. In this analysis, by assuming that mobilized shear strength varies with depth, and treating the whole soil mass as a series of layers, factors of safety of a given slope at different heights are arrived at. The results are compared with those obtained by friction circle method.

Analysis

Referring to Fig. 1, for a slope having surcharge, p, on the surface, for critical equilibrium, the slope angle, β_o, at point 0 is given by Sokolovsky (2):

$$\beta_o = \frac{\cot \phi}{2} \ln \left[\frac{(p + H)(1 - \sin \phi)}{H(1 + \sin \phi)} \right] \quad \text{(1)}$$

in which β_o = angle between the x-axis and tangent at point 0 to the contour of slope; ϕ = angle of internal friction of soil; $H = c \cot \phi$; and c = cohesion of soil. The factor of safety, F_s, is defined as

$$F_s = \frac{c}{c_d} = \frac{\tan \phi}{\tan \phi_d} = \frac{c + \sigma_n \tan \phi}{c_d + \sigma_n \tan \phi_d} \quad \text{(2)}$$

in which c_d, ϕ_d = cohesion and angle of internal friction developed at a point; and σ_n = normal stress. For a given β_o and nondimensional surcharge, $p' = p/c$, ϕ_d at point 0 can be obtained from

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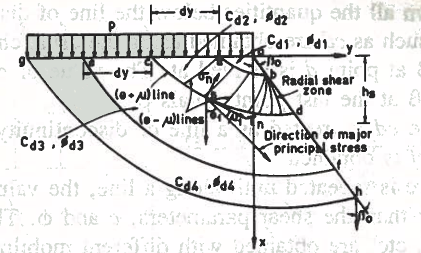

FIG. 1.—General Scheme Followed in Stability Analysis by Method of Characteristics

$$\beta_o = \frac{\cot \phi_d}{2} \ln \left[\frac{(p' + \cot \phi)(1 - \sin \phi_d)}{\cot \phi(1 + \sin \phi_d)} \right] \quad \text{(3)}$$

In the analysis presented herein, the soil is assumed to consist of a series of layers as shown in Fig. 1, and within each layer, the values of c_d and ϕ_d are assumed to be constant and the analysis consists of the following steps:

1. To start with, a point a (see Fig. 1) is chosen very close to point 0, and the value of β at point b is determined by the usual procedure using c_d and ϕ_d obtained at point 0. Point a being very close to point 0, the value of β at point b will be practically equal to β_o. The quantities x', y', ξ, η, θ, and σ' along the line ab are obtained by using the characteristic equations ($x' = x/l$, $y' = y/l$, $l = c/\gamma$, γ = unit weight of soil, $\xi = \chi + \theta$, $\eta = \chi - \theta$, $\chi = \cot(\phi d/2) \ln(\sigma')$, $\sigma' = \sigma/c$, $\sigma = \sigma_1 + \sigma_2/2 + H$, $H = c \cot \phi = c_d \cot \phi_d$, σ_1, σ_2 = major and minor principal stresses, respectively, and θ = angle between the major principal stress and x-axis measured in the counterclockwise direction.

2. The line ab is treated as a line of discontinuity along which c_{d1}, ϕ_{d1} (c and ϕ developed in the region $0ab$) changes to c_{d2}, ϕ_{d2} when the line is crossed. At any point on the line of discontinuity such as S in Fig. 1, the normal and tangential stresses should satisfy the conditions

$$\sigma_{n1} = \sigma_{n2} \quad \text{(4)}$$

$$\tau_{n1} = \tau_{n2} \quad \text{(5)}$$

in which σ_{n1}, τ_{n1} = normal and tangential stresses in the top layer; and σ_{n2}, τ_{n2} = normal and tangential stresses in the lower layer. Using Eqs. 4 and 5, quantities σ', θ ξ, and η below the line ab are determined.

3. At a point such as point a and b (Fig. 1), in addition to satisfying discontinuity conditions, boundary conditions are to be satisfied. These two conditions are difficult to be satisfied fully and thus the following approximations are made: At a point such as a, there should not be any shear stress, and thus θ necessarily has to be zero, though it violates the condition $\sigma_{n1} = \sigma_{n2}$. At point b, to satisfy boundary conditions, σ'_n and τ'_n should be equal to zero. At this point, while a radial shear zone is introduced for a change in θ to satisfy Eqs. 4 and 5 to β_o, σ'_n and τ'_n are made equal to zero at the end, and σ' corresponding to this condition is taken as σ' for the part on the slope after the last radial line. Similar approximations have been made by Stragnov (3).

4. Having known all the quantities below the line of discontinuity, *ab*, quantities along a line such as *cd* are obtained using the usual characteristic equations and the value of β at point *d* is arrived at. The value of c_{d2} and ϕ_{d2} are varied until the value of β at the last point equals β_o.

5. Now, the line *cd* is treated as a line of discontinuity similar to *ab* and a new line such as *ef* is obtained.

6. The procedure is repeated until along a line, the values of c_{d2} and ϕ_{d2} are equal to or greater than the shear parameters, *c* and ϕ. Thus, a series of lines such as *ab*, *cd*, *ef*, etc. are obtained with different mobilized shear strength.

In arriving at the numerical values of the quantities, care is taken to use a sufficiently finer mesh size. A plot of the F_s versus nondimensional height is plotted.

TABLE 1.—Comparison of Factors of Safety from Method of Characteristics and Friction Circle Method

p' (1)	ϕ, in degrees (2)	β_o, in degrees (3)	h_s (4)	F_s (Method of Characteristics) (5)	F_s (Friction Circle Method) (6)	Difference, as a percentage (7)
3	10	45	0.4801	1.3476	1.42	5.1
			0.9628	1.2412	1.33	6.7
			1.5238	1.1405	1.27	10.2
			2.4993	1.0047	1.14	11.9
			1.2362	1.4811	1.55	4.4
3	20	45	2.3628	1.3105	1.44	9.0
			6.3492	0.9997	1.15	13.1
			3.0201	1.5483	1.68	7.8
3	30	45	8.0218	1.2208	1.36	10.2
			16.3597	1.0128	1.16	12.7
			3.8957	1.8172	1.96	7.3
3	40	45	7.8542	1.5633	1.69	7.5
			13.1831	1.3894	1.55	10.4
			0.7530	1.1897	1.22	2.5
5	20	45	2.0568	1.0996	1.15	4.4
			3.8912	0.9998	1.06	6.6
			3.1071	1.3259	1.37	3.2
5	30	45	7.0201	1.1659	1.25	6.7
			15.2616	0.9946	1.10	9.6
			0.6010	1.5567	1.64	5.1
3	10	60	1.4449	1.3987	1.49	9.1
			3.2477	1.1617	1.28	9.2
			2.2401	1.6855	1.78	5.3
3	20	60	7.6889	1.2738	1.42	10.3
			9.0974	1.2181	1.36	10.4
			11.5936	1.1349	1.28	11.3
			0.2050	1.2033	1.23	2.2
3	10	30	0.3958	1.1531	1.20	3.9
			0.7824	1.0666	1.16	8.1

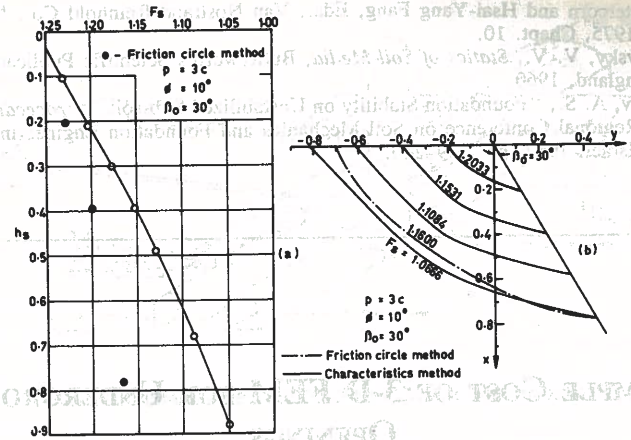

FIG. 2.—(a) h_s versus F_s for $p = 3c$, $\phi = 10°$, and $\beta_o = 30°$; and (b) Failure Surfaces and Factors of Safety for $p = 3c$, $\phi = 10°$, and $\beta_o = 30°$

Results and Analysis

Based on the analysis presented, a few results have been obtained. Table 1 shows the factor of safety obtained by the present method and friction circle method for a few heights. The safety factors obtained by the friction circle method are with respect to shear strength. The friction circle method has been used for convenience and for homogeneous slopes, and the results of the friction circle method, method of slices, and limit analysis are almost the same (1). Fig. 2(*a*) shows a typical graph of F_s versus nondimensional height, h_s $(=h/l)$. Fig. 2(*b*) shows the slip surfaces obtained for few heights.

As can be seen from Table 1, the safety factors obtained by the present method are smaller than those obtained by the friction circle method for the corresponding heights and thus the values are more conservative. But the feature of analysis presented herein is that for a given set of parameters, a series of lines with different mobilized shear strengths (safety factors) can be obtained in a single analysis, and a plot of F_s versus h_s can be obtained. It can also be seen from Fig. 2(*b*) that the failure surfaces obtained by the present method are not circular.

The results presented are for a limited number of parameters and the results are encouraging for application to more general cases like a surface with zero *p*.

Conclusions

The method of characteristics with some simplifying assumptions is made applicable for analyzing a given straight slope. The factors of safety obtained by the present method are lower than those obtained by friction circle method.

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Example Cost of 3-D FEM for Underground Openings

By Charles W. Schwartz,[1] Amr S. Azzouz,[2] Associate Members, ASCE, and Herbert H. Einstein,[3] M. ASCE

Introduction

The capabilities of three-dimensional (3-D) finite element techniques for analyzing the stresses and deformations around underground openings with complicated geometries has been well-established in past literature. However, the use of these analytical tools in practice has been resisted, partially because of their supposed high costs, both for computer time and man-power. This technical note will attempt to put a more quantitative perspective on this objection by examining the costs from one particular but typical 3-D finite element study.

Description of the Problem

The purpose of this study was to predict the excavation-induced stresses and displacements in the rock mass surrounding the Peachtree Center Station, a section of the Metropolitan Atlanta Rapid Transit Authority's new subway system. Of special interest were the effects of the main station excavation on an experimental research chamber at the southern end of the complex (Fig. 1). The rock movements in this region were to be monitored by extensometers and inclinometers during construction. For details regarding the chamber and the measurement program, the reader is referred to other readings (5).

Kulhawy (5) has previously analyzed the influence of the running tunnel excavation on the research chamber using 2-D, plane strain finite element techniques. His analysis considered a transverse vertical plane through the midpoint of the research chamber. However, for the purposes of our study a full 3-D finite element analysis was required for two principal reasons. First, we were interested in analyzing the effects of the main station cavern excavation on the research chamber. This is impossible using any sort of two-dimensional simplification of the geometry, as is clearly shown in Fig. 1. Second, a 3-D solution allowed us to analyze movements in the longitudinal direction (along the *z*-axis, Fig. 1). These movements were expected to be relatively large since the in situ longitudinal stresses are very high.

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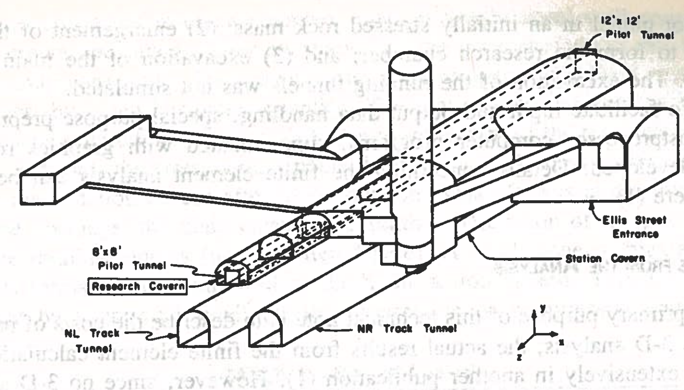

FIG. 1.—Simplified View of Peachtree Center Station Area (5)

Finite Element Representation

The analyses in this study were performed using the finite element program ADINA (2,3). The finite element mesh was designed to represent only the major openings (the pilot tunnel, research chamber, running tunnels, and station cavern) and consisted of 1,093 3-D isoparametric elements connected at 2,915 nodal points. This mesh was sufficiently fine to give smooth stress and displacement variations around the openings. Features of the analysis to be noted include the following:

1. The subsurface investigation program (4) found the rock surrounding the research chamber to be an excellent quality (RQD > 90%) gneiss without any major open discontinuities. The rock mass was accordingly modeled as a linearly elastic, isotropic, homogeneous material in the analysis.
2. The assumed vertical (σ_y), lateral (σ_x), and longitudinal (σ_z) in situ stresses were based on data from in situ tests in the pilot tunnel (4). These stresses were applied as pressure loads at the boundary of the mesh. The variation of in situ stress over the height of the opening was neglected.
3. No supports were modeled in the analysis, since the actual supports in the test chamber were installed several months after excavation.
4. The construction sequence was modeled in three steps: (1) Excavation of