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TECHNICAL NOTES

Proc. Paper 11684

INFORMATION RETRIEVAL

The key words, abstract, and reference "cards" for each article in this Journal represent part of the ASCE participation in the EJC information retrieval plan. The retrieval data are placed herein so that each can be cut out, placed on a 3 × 5 card and given an accession number for the user's file. The accession number is then entered on key word cards so that the user can subsequently match key words to choose the articles he wishes. Details of this program were given in an August, 1962 article in CIVIL ENGINEERING, reprints of which are available on request to ASCE headquarters.

[a]Discussion period closed for this paper. Any other discussion received during this discussion period will be published in subsequent Journals.

11697 SEISMIC INSTRUMENTATION OF DAMS

KEY WORDS: Accelerometers; Dams; Dynamics; **Earthquakes; Geotechnical engineering; Instrumentation; Seismic detection; Seismographs**

ABSTRACT: The desirability of installing seismic instruments on and near major dams is explained. Two types of instruments are required: (1) Strong-motion accelerographs for recording potentially destructive ground shaking and resulting dam vibrations; and (2) sensitive seismographs for determining the local seismicity. A minimum of two strong-motion accelerographs should be installed on the dam and a minimum of two should be installed in the immediate vicinity of the dam. Each accelerograph should record three components of motion, should have a natural frequency of approx. 20 Hz, a recording speed of approx 1 cm/s. The sensitive seismographs are intended to record the local seismicity in the vicinity of the dam site before construction, and to detect any changes in seismicity during reservoir filling. A minimum of three seismographs is recommended for installation in the vicinity of the dam site. A vertical-component seismometer (1 Hz - 5 Hz) with visual recorder and approx 10,000 magnification at 1 Hz is recommended.

REFERENCE: Bolt, Bruce A., and Hudson, Donald E., "Seismic Instrumentation of Dams," *Journal of the Geotechnical Engineering Division,* ASCE, Vol. 101, No. GT11, **Proc. Paper 11697**, November, 1975, pp. 1095-1104

11705 END EFFECTS ON STABILITY OF COHESIVE SLOPES

KEY WORDS: Boundary conditions; **Cohesive soils;** Equilibrium; Failure; **Geotechnical engineering;** Safety factor; **Slopes; Slope stability; Stability analysis**

ABSTRACT: The concept of the "two-dimensional" circular arc method of stability analysis is extended to three-dimensional slope stability problems. End effects on the stability of cohesive slopes due to failure along a finite length are evaluated by means of a computer program STAB3D developed for this purpose. As an application of the technique, two cases are considered: (1) The toe failure of a vertical cut where typical end effects are illustrated; and (2) the toe failure of a slope with any angle β when a finite length of failure is imposed. The stability of a test section embankment loaded to failure is analyzed.

REFERENCE: Baligh, Mohsen M., and Azzouz, Amr S., "End Effects on Stability of Cohesive Slopes," *Journal of the Geotechnical Engineering Division,* ASCE, Vol. 101, No. GT11, **Proc. Paper 11705**, November, 1975, pp. 1105-1117

11706 QUASI-STATIC DEEP PENETRATION IN CLAYS

KEY WORDS: **Clays;** Cones; Deformation; **Geotechnical engineering; Penetration; Penetration resistance;** Steady state; Strain tests; **Wedges**

ABSTRACT: Deformations caused by the steady-state penetration of a rigid rough wedge in clay are compared with theoretical predictions. It is found that the mechanism of sharp wedge penetration was consistent with the cutting process assumed by the theory. However, the larger the apex wedge angle the less accurate are the theoretical predictions. The mechanism of blunt wedge penetration is one of compression in which a rigid region of clay moves with the wedge, so that the deformation patterns are difficult to interpret. Measured penetration resistance is in reasonable agreement with the theory. The suitability of plasticity theory to resolve penetration problems is assessed and its deficiencies identified.

REFERENCE: Baligh, Mohsen M., and Scott, Ronald F., "Quasi-Static Deep Penetration in Clays," *Journal of the Geotechnical Engineering Division,* ASCE, Vol. 101, No. GT11, **Proc. Paper 11706**, November, 1975, pp. 1119-1133 maintained by the Seismic Engineering Branch, U.S. Geological Survey, 390 Main Street, San Francisco, Calif., 94105. It is recommended that original accelerograms recorded in the United States be deposited at this office. Copies of the original records can then be made available for research purposes to those organizations wishing to carry out studies in depth. In other countries it would also be advisable to maintain a central file of accelerograph records.

The USCOLD Committee on Earthquakes, upon request, will provide advice on studies that should be made of accelerograms.

Appendix II.—References

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JOURNAL OF THE GEOTECHNICAL ENGINEERING DIVISION

End Effects on Stability of Cohesive Slopes

By Mohsen M. Baligh[1] and Amr S. Azzouz[2]

Introduction

Stability is an important consideration in the design of dams, levees, breakwaters, embankments for transportation facilities, cut slopes, and excavations. The result of a failure can be costly, involving the loss of time and property and even lives. Wright (15) presents a thorough literature survey of existing methods of slope stability analysis. Most of these methods use limit equilibrium techniques and apply to plane-strain conditions. Rigorously speaking, the plane-strain analysis implies that an embankment failure should extend for an infinite distance along its axis. Practically, a "reasonably long" failure makes the plane-strain analysis "reasonably applicable" to the observed three-dimensional failures.

However, for such problems as the stability of high dams constructed in narrow rock-walled valleys, the end effects are important and thus the problem can no longer be treated by means of the plane-strain analysis. In contrast to the voluminous literature on two-dimensional slope stability, little work has been done regarding three-dimensional problems. Sherard, et al. (9) present a method of analysis for three dimensional problems which is specifically intended to evaluate end effects for high dams in narrow valleys. This method essentially gives a "weighted" average of the stability of various sections of the embankment. The length of the dam is divided into a series of segments of equal lengths. The average cross section of each segment is analyzed as a two-dimensional problem. The factor of safety is then defined as the ratio of the sum of the resisting forces to the sum of the driving forces for all segments of dam length.

This article extends the concept of the two-dimensional circular arc shear failure method to three-dimensional slope stability problems. The techniques presented herein can be considered a more organized and rational approach of the method presented by Sherard, et al. End effects can now be evaluated

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by means of a computer program, STAB3D, which was developed for that purpose.

To illustrate the three-dimensional (or end) effects on the stability of cohesive slopes due to failure along a finite length, two cases were considered: (1) The toe failure of a vertical cut where typical end effects are encountered; and (2) the toe failure of a slope with any angle β when a finite length of failure is imposed. In addition, the stability of a test section embankment loaded to failure is analyzed. Although STAB3D has to be used on a case by case basis, modifications in the program can be made to handle a variety of three-dimensional problems such as: (1) The stability of slopes subjected to concentrated loads; (2) the stability of embankments that are curved in plan; (3) the stability of embankments with a variable cross section, e.g., high dams built in narrow valleys, sloping embankments, etc.; and (4) the probabilistic estimate of the length of failure of an embankment due to the variability of soil properties along its axis. The length of failure is important since it determines the cost of repairs in the event a failure takes place.

Mechanism of Slope Stability

The stability of slopes has traditionally been treated as a limit equilibrium problem. The disturbing forces acting on a free body or a cut in the slope are compared to the resisting forces provided by the soil, and the margin of safety against a failure is obtained.

Two-Dimensional Problems.—A number of methods for the two-dimensional analysis of slope stability based on limit equilibrium techniques are presently available (1,2,6,7,8,14). Various methods differ from one another in: (1) The shape of the failure surface or the shape of the cut to be analyzed; and (2) the assumptions used to achieve statical determinancy. This is reflected in the magnitude of the shearing resistance, τ, of frictional soils ($\phi \neq 0$) and, in turn, on the factor of safety obtained.

The circular arc method of analysis is one of the oldest and yet one of the most widely used methods (11).

For a typical slope such as the one shown in Fig. 1, the circular arc method assumes that the shear surface consists of a cylinder with an infinite length. The mechanism of failure consists of a rigid body rotation of the cylinder about its axis 00′ (z-axis, Fig. 1). If we now consider a unit length along the z-axis, the driving moment, M_d^o, and the resisting moment, M_r^o, are

$$M_d^o = W a \qquad (1)$$

$$M_r^o = T R_{\max} \qquad (2)$$

in which W = weight of soil above the circular arc bd; a = moment arm of W about the cylinder axis (the z-axis); T = integral of the shearing stresses acting along arc bd; and $R_{\max}$ = the radius of the cylinder.

The factor of safety against plane strain failure F^o is therefore, given by

$$F^o = \frac{M_r^o}{M_d^o} = \frac{T R_{\max}}{W a} \qquad (3)$$

Since the rigid body rotation is a kinematically acceptable mechanism of failure,

a minimum upper-bound solution is sought. Due to the complex geometry and the variety of soils existing in actual problems, the search for $F^o_{\min}$ is usually performed numerically by computer programs. Presently, no less than 25 to 50 programs for slope stability are in common use (13). Different locations of the z-axis and different values of $R_{\max}$ are assumed, and the corresponding values of F^o are calculated according to Eq. 3. Repeated trials ultimately lead

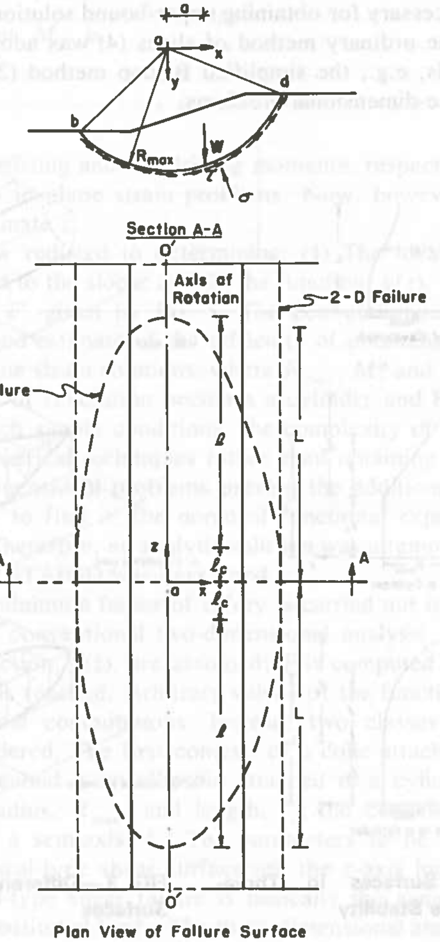

FIG. 1.—Typical Slope Failure

to the minimum value of the factor of safety, $F^o_{\min}$, which should exceed unity for the slope to be stable.

Three-Dimensional Problems.—In contrast to the voluminous literature on two-dimensional slope stability, little work has been done in three dimensions. Here, we consider the basic class of three-dimensional problems, in which the geometry of a slope and the corresponding soil properties do not vary along the axis. Extensions of the technique to more general conditions, as in the problems previously described, can be easily made once the fundamental mechanism has been established.

Three-dimensional analysis is treated herein as an extension of the circular arc method. Whereas the basic assumptions regarding this method are retained, the shear surface is not restricted to an infinitely long cylinder, but is taken as a surface of revolution extending along the ground surface for a finite length, $2L$ (Fig. 1). The rigid body motion at failure still provides an acceptable velocity field which is necessary for obtaining upper-bound solutions. To achieve statical determinancy, the ordinary method of slices (4) was adopted. However, other available methods, e.g., the simplified Bishop method (2), can be used in the treatment of three-dimensional problems.

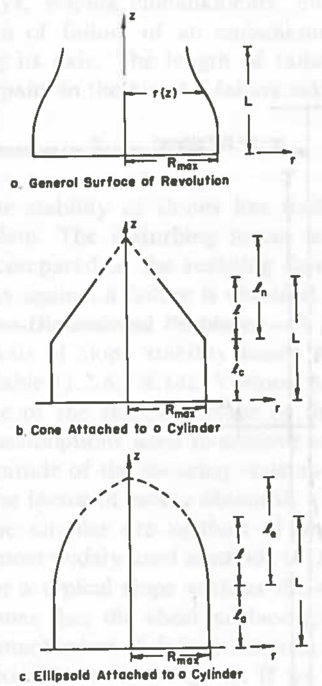

FIG. 2.—Failure Surfaces in Three-Dimensional Slope Stability

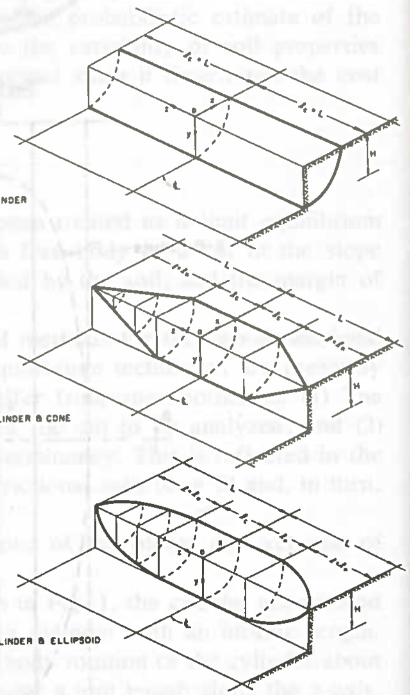

FIG. 3.—Different Geometries of Failure Surfaces

Let us consider the surface of revolution shown in Fig. 2(*a*) which is symmetric with respect to the plane $z = 0$ and has a generator defined by its radial distance, r, from the z-axis according to

$$r = g(z) \quad \ldots \quad (4)$$

If the shear surface lies between $z = -L$, and $z = L$, the factor of safety against rotation is

$$F = \frac{M_r}{M_d} \quad \ldots \quad (5)$$

in which the resisting moment, M_r, is

$$M_r = \int_0^L M_r^o \left(\frac{ds}{dz}\right) dz \quad \ldots \quad (6)$$

in which $$\frac{ds}{dz} = \sqrt{1 + \left(\frac{dg}{dz}\right)^2} \quad \ldots \quad (7)$$

and the driving moment, M_d, is

$$M_d = \int_0^L M_d^o \, dz \quad \ldots \quad (8)$$

M_r^o and M_d^o are the resisting and the driving moments, respectively, computed from Eqs. 1 and 2 as in plane strain problems. Now, however, M_r^o and M_d^o are functions of coordinate z.

The problem is now reduced to determining: (1) The location of the axis of rotation with respect to the slope; and (2) the function, $g(z)$, which minimizes the factor of safety, F, given by Eq. 5. The conventional two-dimensional analysis provides a good estimate of the difficulty of obtaining exact solutions to this problem. In plane strain solutions, where R_{max}, M_r^o and M_d^o are independent of z, the surface of revolution becomes a cylinder and Eq. 5 reduces to Eq. 3. Even under such simple conditions, the complexity of actual problems implies the use of numerical techniques rather than obtaining exact solutions. Furthermore, three-dimensional problems present the additional difficulty that $g(z)$ is unknown and, to find it, the nonlocal functional expressed by Eq. 5 has to be minimized. Therefore, no analytic solution was attempted, and instead, the computer program STAB3D was developed.

The search for the minimum factor of safety is carried out in STAB3D along the same lines as the conventional two-dimensional analysis. The location of the z-axis and the function, $g(z)$, are assumed; F is computed and the process is repeated until F_{min} is reached. Arbitrary values of the function, $g(z)$, cannot be treated in numerical computations. Instead, two classes of surfaces of revolution were considered. The first consists of a cone attached to a cylinder [Fig. 2(*b*)] and the second is an ellipsoid attached to a cylinder [Fig. 2(*c*)]. The cylinder has a radius, R_{max}, and length, l_c; the cone has a height, l_n, and the ellipsoid has a semiaxis, l_e. The parameters to be varied so as to minimize F for a conical-type shear surface are the z-axis location, R_{max}, l_c, and l_n. An ellipsoidal-type shear failure is basically the same as the conical one except that l_e is substituted for l_n. The three-dimensional analysis, therefore, introduces the two parameters, l_c and l_n, in addition to the ones used in the conventional two-dimensional analysis.

Three-Dimensional Effects on Stability of Cohesive Slopes

To illustrate the three-dimensional (or end) effects on the stability of cohesive slopes ($c = S_u$ and $\phi = 0$) due to failure along a finite length, concepts presented earlier were applied to two problems.

The Stability of Vertical Cut in Clay.—Let us consider the case of a vertical cut of height H in clay as shown in Fig. 3. The z-axis was taken at the crest of the cut and the cylinder to pass through the toe, i.e., $R_{max} = H$. Although not providing the minimum factor of safety, these conditions illustrate the fundamental three-dimensional effects and lead to results that could be checked analytically. The cylindrical shear surface assumed in two-dimensional analysis is shown in Fig. 3(*a*) where l_c is large compared to the height, H. Furthermore, Figs. 3(*b*) and 3(*c*) show the two classes of three-dimensional shear surfaces considered. Note that symmetry with respect to the plane $z = 0$ is assumed.

The factor of safety, F, in case of a conical shear surface attached to a cylinder is shown in Fig. 4(*a*) where the ratio of F/F^o is plotted versus l/H for various values of l_c/H (F^o is the plane strain factor of safety). The same information is given in Fig. 4(*b*) when the shear surface is an ellipsoid attached to a cylinder.

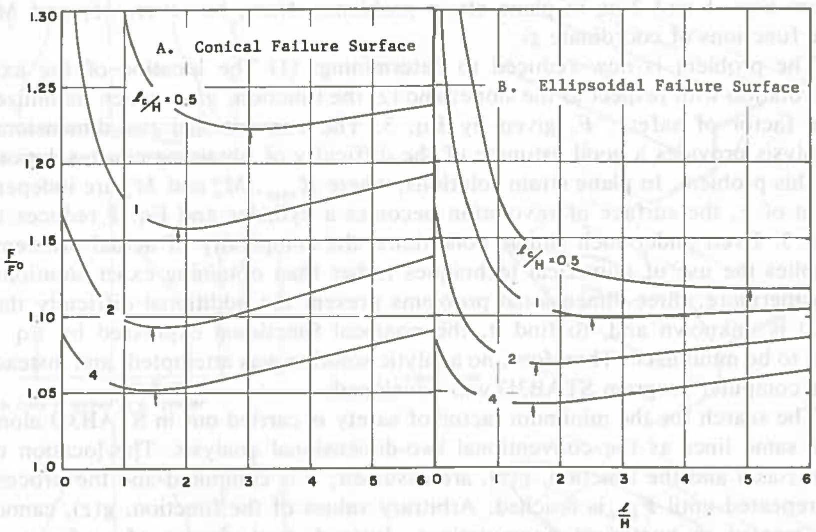

FIG. 4.—Effect of Shear Surface Geometry on Factor of Safety for Vertical Cut in Clays

The plots presented in Fig. 4 show that:

1. Since F/F^o exceeds unity, three-dimensional effects tend to increase the factor of safety. Slopes would therefore tend to fail for long distances provided the soil properties and the cross section of the embankment are independent of z.

2. As l_c/H increases, the value of F/F^o decreases. Failures having l_c/H in excess of four can be considered close enough to plane strain ($l_c/H = \infty$ and $F = F^o$).

3. For a fixed value of l_c/H, the factor of safety reaches a minimum at the critical value of l/H which determines the most likely length of failure. However, in the critical value region the curves are quite flat, especially for small values of l_c/H. This means that even if the factor of safety, F, can be predicted with reasonable accuracy, the length of failure, $2L$, is more difficult to predict.

4. When $l = 0$, the values of F are the same in both plots (Fig. 4) and correspond to a cylindrical shear surface when the end effects are taken into consideration. This solution is relatively easy to obtain and can be used to determine if a particular problem needs a three-dimensional analysis.

Toe Failure of Clay Slopes Having Finite Length.—To determine the critical shear surface and the corresponding factor of safety, F_{min}, of a clay slope of height H making an angle, β, with the horizontal and restricted to fail along a finite length, $2L$, by numerical techniques, the following steps are needed:

1. Choose a class of admissible shear surfaces. Here we consider a conical surface attached to a cylinder and an ellipsoid attached to a cylinder as previously described.
2. Assume the location of the z-axis. This is equivalent to assuming the location of the center of the circle in two-dimensional analysis.
3. Assume the radius of the cylinder, R_{max}. This is equivalent to assuming the radius of the circle in two-dimensional analysis.
4. Assume the length of the cylinder $l_c (0 \leq l_c \leq L)$ (Fig. 2).
5. Obtain the factor of safety, F, against a shear surface failure defined by steps 1, 2, 3, and 4.
6. Repeat steps 2, 3, 4, and 5 to get another factor of safety and, after a sufficient number of trials, choose the minimum value, F_{min}, obtained. This will represent an upper bound on the actual factor of safety of the slope.

When accurate results are required, many trial solutions are needed and the computational effort becomes substantial. This is particularly true in this case because F_{min} is needed for different values of L. The number of trials was thus reduced and only toe failures were considered. Once the z-axis location is assumed, the value of R_{max} for a shear surface passing through the toe of the slope is determined and the need to vary R_{max} independently is thus eliminated.

The results of computations showed that elliptic shear surfaces consistently gave a lower factor of safety than conical ones and are thus more likely to simulate the geometry of actual slope failures of the type considered. The end effects imposed by a limited length of failure, L, of the slope are illustrated by the results presented in Fig. 5, in which $(F/F^o)_{min}$ is plotted versus $L/\Delta R$ for slope angles $\beta = 10°$ and $90°$ ($\Delta R = R_{max} - R_{min}$, Fig. 5). Fig. 5 shows that:

1. When the length of failure, L, is large, a plane strain failure is approached (F/F^o approaches unity).

2. For any given value of $L/\Delta R$, the slope angle, β, has little influence on F/F^o, i.e., on the increase of factor of safety due to end effects. The case of $\beta = 40°$ was also considered and results were found to lie within the narrow band provided by $\beta = 10°$ and $\beta = 90°$ shown in Fig. 5.

3. The ratio, $L/\Delta R$, determines whether end effects in a particular problem are important and if a thorough three-dimensional analysis is needed. Given the slope cross section and the length of failure, L, it is therefore important

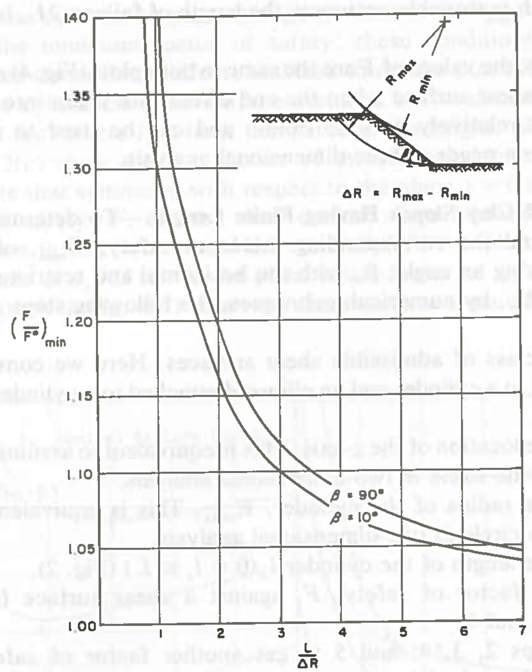

FIG. 5.—End Effects on Factor of Safety of Clayey Slopes

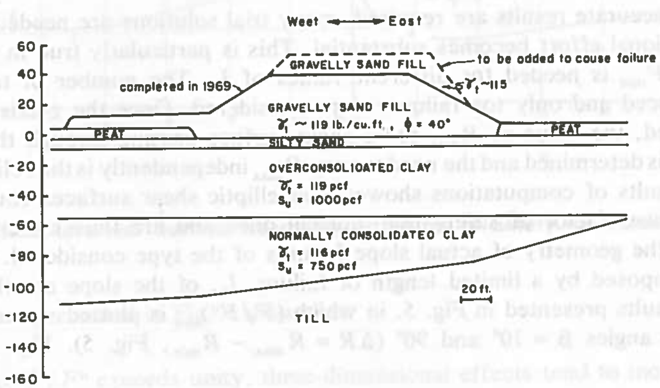

FIG. 6.—Cross Section of I-95 Embankment

to determine ΔR. This can be achieved by performing a conventional two-dimensional slope stability analysis to obtain the minimum factor of safety, F°_{min}, the location of the center of the slip circle (the z-axis), and its radius R_{max}. For the simple case of homogeneous clay slopes as treated herein, the z-axis and R_{max} and F°_{min} are given in Ref. 12. The value of R_{min} is the radius of the circle with the same origin as the slip circle and touching the slope. The value of ΔR is then computed as the difference between R_{max} and R_{min} (Fig.

5). For values of $L/\Delta R \geq 3$, the increase in the factor of safety due to end effects is less than 10% and three-dimensional analysis would not, in many cases, be warranted. However, for $L/\Delta R \leq 3$, a three-dimensional analysis including end effects might be considered if the savings resulting from a more accurate treatment justify this effort.

4. Strictly speaking, Fig. 5 should only be used for the toe failure of uniform clay slopes, However, it provides a rough estimate of the end effects for more general slope conditions after the conventional two-dimensional analysis has been performed.

Case History: I-95 Embankment Failure

In 1965, construction was begun on a 3-mile extension to Interstate Highway I-95 north of Boston. Fig. 6 shows a cross section of the embankment as well as the subsurface soil profile. Peat and sand overlie a thick deposit of Boston blue clay. The top 10-ft layer of peat was removed and replaced by sand and gravel. In the upper 50 ft-60 ft, the clay is overconsolidated, probably due to desication, and is normally consolidated below. For a more complete description of the site and the corresponding soil properties, the reader is referred to Refs. 3 and 5.

In 1974, as a part of a research project conducted at the Massachusetts Institute of Technology (MIT) regarding slope stability on clays, a test fill was rapidly built on a 300-ft long section of the present embankment to cause large undrained deformations in the clay foundation. The fill was added to the top of the embankment in 1-ft lifts at an average rate of 1 ft/day-2 ft/day.

The failure of the test section took place when the fill elevation was at 56.5 ft. More detailed information of the failure, including the surveys before and after failure, will appear in an MIT Department of Civil Engineering Report to be published in 1975. One of the many surprising features of the failure, which in fact motivated this study, was the length of failure (in plan) which extended over a distance of 280 ft on one side of the 300-ft test section and 450 ft on the other. The total failure length was therefore along 1,030 ft while the loaded length was only 300 ft. An estimate of the end effects was thus necessary in order to interpret the results of the field tests and, in particular, backfigure the shearing strength of the clay.

The undrained failure of the embankment was first analyzed by the conventional two-dimensional circular arc technique using the soil properties given in Fig. 6. The simplified Bishop method (2) gave a minimum factor of safety, $F^{\circ}_{min} = 0.797$, whereas according to the ordinary method of slices (4), a value of 0.794 was obtained. The critical circle for both methods was roughly the same. This is due to the small contribution of the sand fill in the shearing resistance compared to the resistance of the underlying clay. A thorough treatment of the embankment stability is given in Ref. 10, where the Morgenstern-Price method (7) and simplified Bishop Method (2) were utilized and compared with the actual field results.

The end effects were considered next after the program STAB3D had been modified to account for the fact that in the loaded section, 300 ft, the fill elevation was at 56.5 ft while the original embankment elevation was at 37.8 ft. This change in slope cross section needed special consideration. The transition zone between the two elevations of the embankment, which had a length of

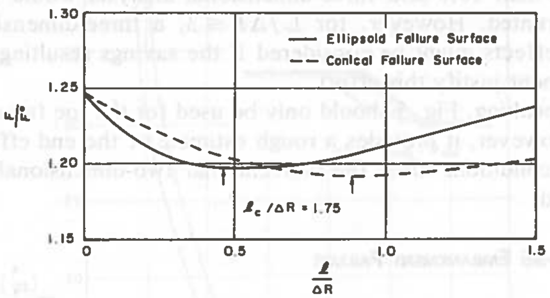

FIG. 7.—Effect of Shear Surface Geometry on Factor of Safety for I-95 Embankment

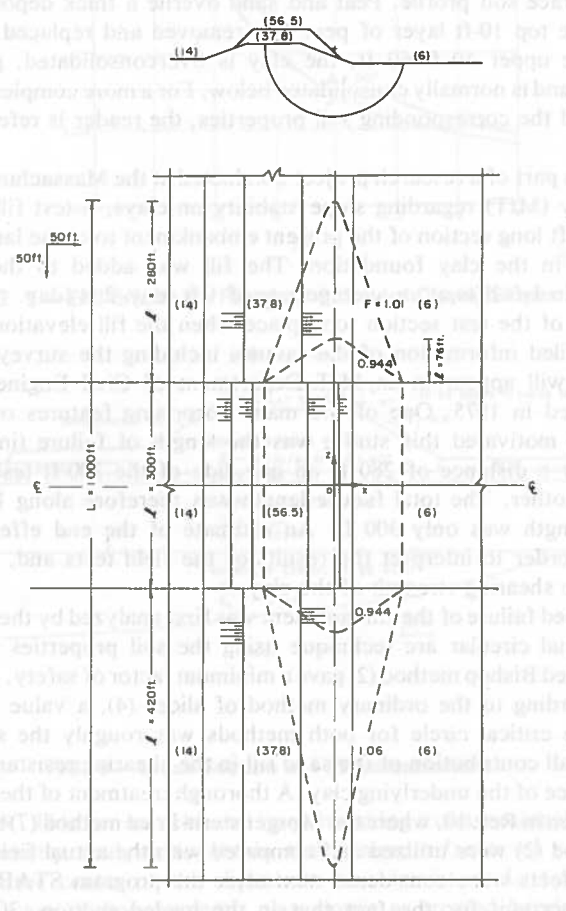

FIG. 8.—Plan View of Failure Surface—I-95 Embankment

about 20 ft in plan, was neglected in the analysis. The shear surface was first taken as a cone attached to a cylinder and then as an ellipsoid attached to a cylinder. The cylinder was assumed to extend along the loaded section, i.e., noting the symmetry in plan, l_c was taken equal to 150 ft. The same material properties were taken as for the two-dimensional analysis (see Fig. 6).

The results of different trial solutions are presented in Fig. 7, where the variation of F/F^o with $l/\Delta R$ is shown. The value of ΔR in this case equals 86 ft and l is the length of failure beyond the loaded section of the embankment. It can be seen from Fig. 7 that:

1. The conical failure surface is slightly more critical than the ellipsoidal surface since it gave a lower value of (F/F^o).
2. The value of $(F/F^o)_{min}$ is equal to 1.19, meaning that end effects caused a 19% increase in the factor of safety.
3. The critical value of $l/\Delta R$ corresponding to $(F/F^o)_{min}$ is equal to 0.88. Knowing that $\Delta R = 86$ ft, the critical length of failure beyond the loaded section, l, is thus equal to 76 ft.
4. The value of F/F^o changes slightly over a wide range of values of $l/\Delta R$. Therefore, even if the factor of safety can be predicted with reasonable accuracy, the length of failure surface is difficult to predict unless other factors neglected in the analysis, e.g., the variation of soil properties along the axis of the embankment are taken into consideration.

A plan view of the calculated critical shear surface ($l = 76$ ft) is shown in Fig. 8. A symmetric failure, with a value of the factor of safety of 0.944, on both sides of the loaded section is calculated by this analysis. On the other hand, knowing the actual failure length on both sides of the loaded section ($2L = 1{,}030$ ft), the factor of safety was computed in each case and was found to be 1.01 on one side and 1.06 on the other. Therefore, it can be seen that while the failure length was substantially underestimated by the present analysis, the end effects on the factor of safety is in the range of 19% (the ratio of F to F^o is $0.944/0.794 = 1.19$) to 34% (the ratio of F, corresponding to actual failure length, to F^o is $1.06/0.794 = 1.34$).

Finally, a sensitivity analysis performed subsequently by perturbing different parameters showed that: (1) The ratio $(F/F^o)_{min}$ is not very sensitive to small changes in material properties, slope geometry, and shear surface geometry; and (2) the length of failure, $2L$, is quite sensitive to small changes in the parameters defining the problem. For example, a slight increase in the fill resistance will increase the predicted length of failure substantially, thus approaching the observed failure. The same result is achieved if a slightly shallower shear surface is considered, i.e., when R_{max} is decreased.

Conclusions

The end effects on the stability of cohesive slopes were investigated by extending the circular arc method to three-dimensional problems. The results obtained by means of the computer program STAB3D show that:

1. The end effects increase the factor of safety obtained by means of a

conventional two-dimensional solution. When the geometry of the slope and the soil properties do not change along the slope axis, this increase depends on the ratio between the length of failure, $2L$, and the depth of failure, ΔR. Figs. 4 and 5 show that for long shallow failures where l_c/H or $L/\Delta R$ exceeds four, the increase in the factor of safety is less than 5% and can, therefore, be neglected. On the other hand, for short deep slope failures where $L/\Delta R$ is less than one to two, the increase in the factor of safety can exceed 20%–30%, and the use of a three-dimensional analysis is thus recommended.

2. Slopes having a variable cross section or having variable soil properties along their axis are subjected to more pronounced end effects than uniform slopes. A three-dimensional analysis in such cases is needed for a wider range of $L/\Delta R$ than mentioned previously.

3. Even though the end effects on the factor of safety can be predicted with reasonable accuracy, the length of failure is very sensitive to the parameters defining the problem and is thus difficult to predict.

4. The slope angle, β, has little influence on the increase in the factor of safety due to end effects. For a value of $L/\Delta R$ equal to three, the increase in the factor of safety due to end effects is about 10% for $\beta = 10°$, whereas the corresponding increase for $\beta = 90°$ is about 12% (Fig. 5).

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Appendix II.—Notation

The following symbols are used in this paper:

a = moment arm of W about cylinder axis (z-axis);
F = factor of safety based on three-dimensional analysis;
F^o = factor of safety against plane strain failure;
g = mathematical function;
H = height of vertical cut;
L = total length of failure surface in plan;
l = length of failure beyond the cylindrical failure surface;
l_c = length of cylinder;
l_e = semiaxis of ellipsoid;
l_n = cone height;
M_d = driving moment;
M_r = resisting moment;
M_d^o = driving moment based on plane-strain analysis;
M_r^o = resisting moment based on plane-strain analysis;
R = radius of cylinder;
r = radius of cone or ellipse in three-dimensional analysis;
S_u = undrained shear strength;
s = arc length along failure surface in longitudinal direction;
T = integral of shearing stresses acting along circular arc;
W = weight of soil above circular arc;
z = z coordinate;
β = slope angle;
γ = unit weight of soil;
τ = magnitude of shearing resistance; and
ϕ = angle of internal friction.