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2. A summary of approximately 50 words must accompany the paper, a 300-word synopsis must precede it, and a set of conclusions must end it.

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9. Technical papers intended for publication must be written in the third person.

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March, 1963

Papers

(over)

Note.—Part 2 of this Journal is the 1963-14 Newsletter of the Soil Mechanics Division.

The three preceding issues of this Journal are dated October 1962, December 1962, and February 1963.

KEY WORDS: piles; load; bearing capacity; testing; steel; soil mechanics; foundations

ABSTRACT: Pile load tests on instrumented piles reveal that load transfer to surrounding soil is a significant factor in the capacity of point bearing piles. The piles tested were 14BP89 and 14BP117 steel H-piles driven approximately 40 ft through sand and gravel. The piles were point bearing in shale. Such piles are relatively short and stiff, but the tests revealed that they were of sufficient flexibility to transfer approximately one-third of the applied load to the surrounding soil. A theoretical calculation of load transfer is presented and is in good agreement with the observed results. The mechanism of pile failure, and significance of load transfer in the failure mechanism, is described and examined in terms of significance in design.

REFERENCE: "Load Transfer in End-Bearing Steel H-Piles," by E. D'Appolonia and J. P. Romualdi, Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 89, No. SM2, Proc. Paper 3450, March, 1963, pp. 1-25.

KEY WORDS: compaction; soil; shear; effective stress; density; water

ABSTRACT: The application of a compaction pressure to an unsaturated soil results in the development of shearing stresses at the points of contact between soil particles until the contacts fail. The particles then slide over one another with an increase in density. The compaction characteristics of soil are thus controlled by the shearing resistance of these contacts. The shearing strength is a function of the effective normal stress. When the compaction pressure is applied to a dry soil, the effective stress is approximately equal to the total stress, because the χ-coefficient is small. The addition of water increases the degree of saturation and increases the pore pressures, thus weakening the soil. Hence, the soil deforms more under stress, and higher densities result. The density must increase during compaction until the total stress, the pore-air pressure, and the pore-water pressure combine to give the required effective stresses. The theory is qualitative because little is known of the dynamic stress-strain properties of unsaturated soils.

REFERENCE: "Effective Stress Theory of Soil Compaction," by Roy E. Olson, Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 89, No. SM2, Proc. Paper 3457, March, 1963, pp. 27-45.

KEY WORDS: dams; earthfill; clay; compaction; settlement; plasticity; strains; testing

ABSTRACT: An approximate theory is formulated to calculate the critical tensile strains in an earth dam that result from differential settlements along the axis of the dam. Laboratory apparatus and procedures are developed to estimate the limiting tensile strain at which compacted clay will crack. Comparisons between predicted and observed behavior of five dams indicate that the theory and laboratory tests can be used to predict cracking potential with an accuracy that is satisfactory for practical purposes. The ratio of tensile strain at cracking to compressive strain at failure is a small fraction (approximately 0.01 to 0.1) that shows no evidence of any consistent pattern. Accordingly, it is hazardous to assess the flexibility of compacted earth dams on the basis of stress-strain relations obtained from compression tests. The effects of molding water content and compactive effort on the flexibility of clay was investigated. No consistent relationship between flexibility and plasticity characteristics of the clay was found.

REFERENCE: "Flexibility of Clay and Cracking of Earth Dams," by G. A. Leonards and J. Narain, Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 89, No. SM2, Proc. Paper 3460, March, 1963, pp. 47-98.

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Proceedings of the American Society of Civil Engineers

FLEXIBILITY OF CLAY AND CRACKING OF EARTH DAMS

By G. A. Leonards,[1] F. ASCE, and J. Narain,[2] M. ASCE

SYNOPSIS

An approximate theory was formulated to calculate the critical tensile strains in an earth dam as a result of differential settlements (at the base) along the axis of the dam. Laboratory apparatus and procedures were developed in order to estimate the limiting tensile strain at which a compacted clay will crack.

Comparisons between predicted and observed behavior of four dams and one test embankment indicate that the theory and laboratory tests can be used to predict the cracking potential of an earth dam (if the settlement pattern is known) with an accuracy that is satisfactory for practical purposes.

The tensile strain at cracking is a small fraction of the compressive strain at failure. Their ratio varies from approximately 0.01 to 0.1, with no evidence of any consistent pattern. Accordingly, it would be hazardous to assess the potential flexibility of a compacted earth dam on the basis of stress-strain relations obtained from compression tests.

An increase in molding water content from 2% to 3% dry of optimum to nearly optimum will substantially increase the flexibility of compacted clay; increases in molding water contents (to 3% wet of optimum) may result in little improvement in flexibility, particularly if settlements will occur rapidly. At comparable moisture contents, with respect to optimum, an increase in compactive effort substantially decreases the flexibility.

Note.—Discussion open until August 1, 1963. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Soil Mechanics and Foundations Division, Proceedings of the American Society of Civil Engineers, Vol. 89, No. SM2, March, 1963.

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In general, highly plastic clays are more flexible than clays of low plasticity, but the relative flexibility of the latter soils cannot be distinguished on the basis of plasticity characteristics. Moreover, the effect of time on the tensile strain at cracking cannot be assessed, even approximately, on this basis.

Rapid settlements, such as those which result from collapse of soil structure on wetting, are more conducive to the development of transverse cracks in earth dams than are long-term settlements as a result of consolidation.

Discontinuities at abutments, closure sections, and similar points, often result in sharp changes in slope of the settlement curve, and large tensile strains may develop in the dam; however, no direct relationship exists between these latter two factors. Theoretical analysis can serve as a valuable tool to assess the beneficial effects of proposed changes in the profile of the dam, and provide a framework on the basis of which more meaningful field measurements can be undertaken.

Research is needed on two problems: Prediction of the longitudinal settlement pattern of earth embankments, and the erodability of soils in V-shaped cracks.

INTRODUCTION

Earth dams may be classed among the earliest known civil engineering structures. They were built for a variety of purposes, including irrigation, water supply, and flood protection. Records are available of earth embankments 15 ft to 30 ft high, constructed as early as 500 B.C. (1).[3] An earth dam 40 ft to 90 ft high and 9 miles long, containing 4,500,000 cu yd of soil, was completed in Ceylon circa 1200 A.D. (1).

Between 500 A.D. and 1500 A.D., numerous earth dams were constructed in India. The largest of these, Madduk-Masur Dam, was completed circa 1500 A.D. to a height of 108 ft (2). Because of the lack of a spillway, the dam was overtopped and breached. Other structures over 1,000 years old are still in use (3).

In 1789, Estrecho de Rientes Dam was completed in Spain. It had an unprecedented height of 150 ft and required 34 yr to build (4). Three years later, shortly after the reservoir had been filled, the dam was breached. The Rientes Dam failure served as a strong deterrent to progress in earth dam design for more than a century. Although Druid Lake Dam (Maryland) was completed to a height of 119 ft in 1871 (5), and Barden Dam was built in England to a height of 125 ft in 1876 (6), as late as 1901 (basing their opinion on observations of saturation lines in existing dams), a board of consultants to the New York Aqueduct Commission was considering plans for the New Croton Dam and came to the following conclusion (7):

[3] Numerals in parentheses refer to corresponding items in Appendix I.

> ". . . the maximum height to which an earth embankment, with its top 20 feet above the water line and with outside slope of two to one, can be built with safety, is 70 feet."

Thus even during the 18th and 19th centuries, when the foundations of modern mathematics and mechanics were laid and important developments in other aspects of civil engineering occurred, progress in earth dam construction was scarcely perceptible.

At the turn of the 20th century, largely as a result of expanded irrigation requirements in the western United States, earth dams of increasing size were successfully constructed. Notable among these are Terrace Dam (Colorado), completed to a height of 180 ft in 1909, and Tieton Dam (Washington), completed to a height of 230 ft in 1925. From 1910 to 1940, a number of important developments occurred of which the following are among the most significant: a) Recognition of the principles of soil compaction; b) manufacture of large and efficient earthmoving and compaction equipment; c) understanding of the principles relating to the shear strength and consolidation of clays; and d) development of analytical tools for estimating seepage forces, slope and foundation stability, and spillway requirements. As a result, Winsor Dam (Massachusetts) was completed, in 1940, to a height of 295 ft, Anderson Ranch Dam (Idaho) was completed to a height of 455 ft in 1950, and Trinity Dam (California) was completed to a height of 537 ft in 1960. Oroville Dam (California) is under construction with a height of 730 ft, and Nurek Dam (U.S.S.R.) is being built to a reported height of 990 ft.

In view of the rapidity of these developments, it is remarkable that they were achieved with a minimum of disastrous failures because many critical design problems must still be solved by approximate, semi-empirical methods. The pore pressure distribution during drawdown in a zoned earth dam or the conditions controlling true piping failures (as opposed to heave) are not subject to rational analysis, and despite recent advances, the theoretical basis for slope stability analysis has not yet been established (8). Furthermore, the construction of well-compacted, relatively brittle earth fills on compressible foundations has resulted in an increasing number of cases in which transverse cracks have developed in the dam as a result of differential settlements along the longitudinal axis (9, 10, 11, 12, 13). Such cracks are always potentially dangerous because progressive erosion may lead to serious failures before the danger is recognized and remedial measures are effected. These difficulties were investigated by A. Casagrande (10), F. ASCE, in 1950, who recognized the desirability of using more plastic clays in the core in order to minimize the possibility of cracking and subsequent erosion, as well as the increased flexibility that may be achieved by compacting clay wet of optimum. The first intensive study of the problem was completed by J. L. Sherard (9), M. ASCE, in 1953. Sherard isolated three potentially dangerous conditions: a) steep abutments, b) low compaction moisture contents, and c) construction materials consisting of silts, clayey sands and silts, and silty clays. Sherard also classified the relative danger involved because of the latter factor in terms of index properties (Table 1). E. Tamez and G. Springall (11), and R. J. Marsal (12), F. ASCE, came to essentially the same conclusions, although they distinguished between the effects of settlements caused by volume changes in the embankment material from those caused by compaction or consolidation of the foundation. From such qualitative con-

TABLE 1.—CLASSIFICATION OF EMBANKMENT MATERIALS

Group number	Soil type	A casagrande's airfield classification system symbols	Approximate Ranges of Soil Properties				Number of dams in each soil group	Piping[a]		Cracking[b]		Relative Importance of Moisture-Density Control	
			Median grain-size D_{50} (mm)	Plasticity index[c]	Liquid limit	Per-cent clay sizes (0.005 mm)		Degree of resistance (1) greatest to (6) least	Piping resistance	Degree of susceptibility (1) greatest to (6) least	Susceptibility to cracking when compacted dry	Degree of importance of control (1) greatest to (6) least	Consequence of inadequate moisture control
I	Sands and gravels with plastic fines	SC SF GC GF	0.15-5.0	8-15	20-50	5-30	6	(3)	Intermediate resistance. Heavier compaction and higher plasticity index increase resistance.	(3)	Intermediate susceptibility may crack only under extreme combinations of conditions.	(5)	May fail by cracking or piping only under severe combination of detrimental conditions.
II	Sands and gravels with non-plastic fines	GF SF	0.15-5.0	0-8	10-30	0-15	6	(5)	Low to intermediate resistance. Heavier compaction and higher plasticity index increase the resistance.	(4)	Intermediate susceptibility.	(3)	Most likely to fail by piping. May possibly fail by cracking.
III	Inorganic silts of low compressibility and fine silty sands	ML ML-CL ML-SC ML-SF	0.03-0.15	0-10	10-45	0-25	12	(6)	Uniform sand with P.I. < 6 has lowest resistance. Well-graded material with P.I. > 6 has intermediate resistance.	(2)	High susceptibility. The finer and more uniform the soil, the greater the susceptibility.	(1)	High probability of failure by piping and cracking.
IV	Inorganic silts and clays of low medium plasticity	CL ML	0.10	10-25	20-50	10-40	30	(4)	P.I. < 15--intermediate resistance.	(1)	Material with $D_{50} > 0.02$ mm and P.I. < 15 has highest susceptibility.	(2)	Most likely to fail by cracking. May fail by piping.
								(2)	P.I. > 15--high resistance.	(5)	Material with $D_{50} < 0.02$ mm and P.I. > 20 has high post-construction settlement but sufficient deformability to follow without cracking.	(4)	May fail by cracking or piping only under severe combinations of detrimental conditions.
V	Inorganic clays of high plasticity	CH CL-CH	0.02	25-40	40	30	6	(1)	High piping. Resistance not severely lowered by very poor compaction.	(6)	Unlikely to crack. High post-construction settlement but high deformability.	(6)	Least likely to fail by either piping or cracking.

[a] In general, the coarser the soil and the less the plasticity, the greater the increase in piping resistance due to increased compactive effort.
[b] Susceptibility to cracking was not observed to be decreased appreciably by increase in compactive effort. Rapidly disintegrating residual soils may be especially susceptible to cracking.
[c] No dams constructed of soils with plasticity index greater than 40 were included in the investigation. (After Sherard)

siderations, certain desirable construction practices have evolved, as follows:

1. Whenever possible, avoid the use of fine silty sand, clayey silts, and silty clays in homogeneous dams or in cores of zoned embankments;
2. minimize discontinuities in slopes of abutments, closure sections, and so forth,
3. remove or pre-wet and, if feasible, preload foundation material subject to collapse of structure on wetting;
4. if the foundation material is compressible, compact wet of optimum; and
5. if conditions favorable to cracking cannot be avoided, construct substantial filter zones of well-graded sands and gravels on both sides of the impervious core. The filters should be designed to trap the eroded material and to seal the core.

Although these measures are a valuable guide to the designer, their qualitative nature leaves a wide margin for interpretation. A rational design procedure, while still requiring the exercise of judgment, could narrow this margin materially as the selection of construction materials, compaction specifications, and grading requirements could be tailored to fit the specific job conditions more closely.

Notation.—The letter symbols adopted for use herein are defined where they first appear and are arranged alphabetically in Appendix II.

FORMULATION AND SCOPE OF PROBLEM

The differential movements that cause transverse cracking of earth embankments may be caused by a number of factors:

1. Volume changes in the foundation or embankment material because of gravity forces or collapse of structure when wetted;
2. volume changes caused by shrinkage;
3. shear strains caused by discontinuities in the compressibility of core and shell materials (14).

Item one is by far the most prevalent source of difficulty. The transverse cracks result from excessive tensile strains that develop near the crest as a result of differential settlements along the longitudinal axis of the dam. A typical situation is shown schematically in Fig. 1. A close-up photograph of the crack is shown in Fig. 2. Near the abutments, the loads are light and the compressible stratum is usually thin; accordingly, the settlements are small. As the valley floor is approached, the settlements increase abruptly. A region near the abutment and crest of the dam is subjected to tensile strains the magnitude of which may be sufficient to cause cracking. The cracks are generally widest at the crest and often extend to considerable depths. Although cracking may be caused by compaction or consolidation of the embankment material itself, it frequently results from volume changes that occur either in the foundation or near the base of the embankment. The analysis to follow will be concerned solely with the calculation of maximum ten-

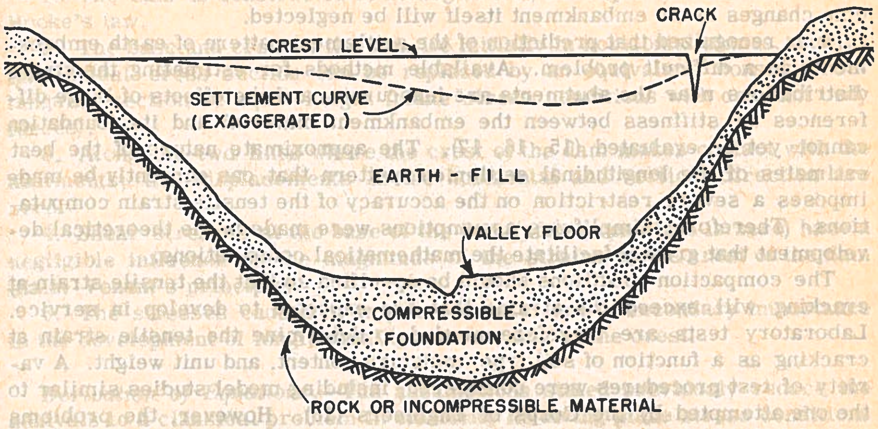


FIG. 1.—SCHEMATIC DIAGRAM ILLUSTRATING TYPICAL SETTLEMENT PATTERN AND ASSOCIATED CRACKING

FIG. 2.—WOODCREST DAM: CLOSE-UP OF TRANSVERSE CRACK (JANUARY, 1960)

sile strain produced by differential settlements at the base of the dam; volume changes in the embankment itself will be neglected.

It is recognized that prediction of the settlement pattern of earth embankments is a difficult problem. Available methods for estimating the stress distributions near the abutments are inaccurate, and the effects of large differences in stiffness between the embankment material and its foundation cannot yet be evaluated (15, 16, 17). The approximate nature of the best estimates of the longitudinal settlement pattern that can currently be made imposes a severe restriction on the accuracy of the tensile strain computations. Therefore, simplifying assumptions were made in the theoretical development that greatly facilitate the mathematical computations.

The compaction conditions should be specified so that the tensile strain at cracking will exceed the strains that are expected to develop in service. Laboratory tests are therefore needed to determine the tensile strain at cracking as a function of soil type, moisture content, and unit weight. A variety of test procedures were considered, including model studies similar to the one attempted by the Corps of Engineers (18). However, the problems associated with achieving similitude are difficult. Moreover, the need for studying a variety of soils over a range in moisture-density relations made the use of models too expensive and time consuming. Accordingly, it was decided to select a laboratory test that simulates the strain conditions in the dam. For this purpose, a simple bending test proved to be the most convenient and effective test procedure.

In order to establish the validity of the approach used, an analysis of five case histories will be presented. Four of these dams cracked in service. The fifth showed no evidence of cracking, even though the maximum settlement exceeded 15% of the embankment height. To conserve space, only the highlights of these case studies will be presented. More complete data and analyses are available (19).

Cracking is not in itself dangerous. If progressive erosion does not develop too rapidly, the crack may seal itself. Alternately, the resulting leakage may be noticed, and the crack may be repaired before any serious damage is done. In fact, cases may arise in which the most economical design would provide for cracking to develop with plans for appropriate remedial measures. However, if the material is highly susceptible to erosion (or is hidden from view by a shell) this practice would be hazardous unless suitable filter zones are provided. The erodability of soils in V-shaped cracks is virtually an unknown entity at present. The qualitative comparisons made by Sherard (9) and the limited laboratory tests conducted by Dunn (20) and by Anderson (21) fall short of permitting an adequate assessment of the danger involved. Research on this problem is needed.

THEORY

For reasons presented in the previous section, the model selected for analysis consists of a beam having a known vertical boundary deflection. Because this model is only an approximate representation of actual conditions, it was deemed appropriate to make the following additional assumptions (their validity is treated in the "Summary of Results") which greatly simplify the mathematical relations:

1. The dam is constructed of homogeneous, isotropic material that obeys Hooke's law.
2. The base and crest of the dam lie initially in horizontal planes, and the trapezoidal cross section can be replaced by an equivalent, constant rectangular section of equal height (plane stress conditions with constant body forces).
3. Along the two lines where the crest of the dam makes contact with the abutments, the displacements in the horizontal and vertical direction are zero.
4. Shear stresses at the base of the dam (whose resultant is zero) have a negligible influence on the magnitude of the strains at the crest of the dam (Saint-Venant's principle).
5. The stresses caused by impounded water are of secondary importance in the development of longitudinal tensile strains at the crest.

Derivation of Equations.—The assumptions stated previously reduce the analysis to a classical problem in elasticity involving plane stress conditions with the exception that one of the boundary conditions is to be specified by the observed (or estimated) settlement pattern. For two-dimensional plane stress conditions with constant body forces, the basic equations are (22):

A. Equilibrium:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0 \quad \text{. . . . . (1)}$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = 0 \quad \text{. . . . . (2)}$$

B. Strains:

$$\epsilon_x = \frac{\partial u}{\partial x} \quad \text{. . . . . (3)}$$

$$\epsilon_y = \frac{\partial v}{\partial y} \quad \text{. . . . . (4)}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \quad \text{. . . . . (5)}$$

C. Hooke's Law:

$$\epsilon_x = \frac{1}{E}\left(\sigma_x - \mu \sigma_y\right) \quad \text{. . . . . (6)}$$

$$\epsilon_y = \frac{1}{E}\left(\sigma_y - \mu \sigma_x\right) \quad \text{. . . . . (7)}$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy} \quad \text{. . . . . (8)}$$

D. Compatibility of strains:

$$\frac{\partial^2 \epsilon_x}{\partial y^2} + \frac{\partial^2 \epsilon_y}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} \quad \ldots\ldots\ldots\ldots\ldots \quad (9)$$

Combining equilibrium and compatibility requirements (Eqs. 1, 2, and 4) and introducing the Airy stress function ϕ, in which

$$\sigma_x = \frac{\partial^2 \phi}{\partial y^2} \quad \ldots\ldots\ldots\ldots\ldots \quad (10)$$

$$\sigma_y = \frac{\partial^2 \phi}{\partial x^2} \quad \ldots\ldots\ldots\ldots\ldots \quad (11)$$

$$\tau_{xy} = - \frac{\partial^2 \phi}{\partial x \partial y} \quad \ldots\ldots\ldots\ldots\ldots \quad (12)$$

gives Maxwell's biharmonic Eq. 22,

$$\nabla^4 \phi = \frac{\partial^4 \phi}{\partial x^4} + 2 \frac{\partial^4 \phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \phi}{\partial y^4} = 0 \quad \ldots\ldots\ldots \quad (13)$$

A solution to Eq. 13 is required to satisfy the following boundary conditions:

$$\sigma_y\Big]_{y=0} = 0 \quad \ldots\ldots\ldots\ldots\ldots \quad (14)$$

$$\tau_{xy}\Big]_{y=0,H} = 0 \quad \ldots\ldots\ldots\ldots\ldots \quad (15)$$

$$v\Big]_{\begin{Bmatrix} x = 0,2L \\ y = 0 \end{Bmatrix}} = u\Big]_{\begin{Bmatrix} x = 0,2L \\ y = 0 \end{Bmatrix}} = 0 \quad \ldots\ldots\ldots \quad (16)$$

$$v\Big]_{y=0} = f(x), \text{ the known crest settlement curve} \quad \ldots\ldots \quad (17)$$

A convenient way to represent the odd-shaped settlement curve mathematically is to find, by harmonic analysis (23), the Fourier coefficients of the series (in the interval 0 to 2L)

$$v\Big]_{y=0} = f(x) = \frac{a_o}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \alpha x + b_n \sin \alpha x\right) \quad \ldots\ldots \quad (18)$$

in which

$$\alpha = \frac{n \pi}{L}$$

Using this procedure, the interval 0 to 2L is divided into a finite number of increments (36 to 60 increments were needed to represent the settlement curves in the case history studies) and the coefficients a_o, a_1, a_2 . . . a_n and b_1, b_2, . . . b_n are evaluated by a standard procedure that can readily be programmed on a computer. Thus, the boundary condition expressed by Eq. 18 can be established from the known settlement pattern. To use this boundary condition, a solution to Eq. 13 was sought in the general form of

$$\phi = \sum_{n=1}^{\infty} \left(A_n \cos \alpha x + B_n \sin \alpha x\right) F(y) \quad \ldots\ldots\ldots \quad (19)$$

Substituting Eq. 19 into Eq. 13, and introducing the boundary conditions (Eqs. 14, 15, 16, and 18), the stress function

$$\phi = \frac{E}{2} \sum_{n=1}^{\infty} \left(a_n \cos \alpha x + b_n \sin \alpha x\right)\left(- \frac{1}{\alpha} \sinh \alpha y + y \cosh \alpha y - \frac{1}{\beta} y \sinh \alpha y\right) \quad \ldots\ldots\ldots \quad (20a)$$

in which

$$\beta = \frac{\sinh \alpha H + \alpha H \cosh \alpha H}{\alpha H \sinh \alpha H} \quad (20b)$$

is obtained, in which the Fourier coefficients a_n and b_n are now known, having been evaluated from the settlement curve. From the definition of the Airy stress function (Eqs. 10, 11, 12) and the relations given by Eqs. 3 through 8, Eq. 20a represents a complete solution for the stresses, strains, and displacements. In particular, the longitudinal strain is given by

$$\epsilon_x = \sum_{n=1}^{\infty} \left(a_n \cos \alpha x + b_n \sin \alpha x\right) \left\{ - \frac{\alpha}{\beta} \cosh \alpha y + \frac{(1 - \mu)}{2} \alpha \sinh \alpha y + \frac{(1 + \mu)}{2} \alpha^2 y \cosh \alpha y - \frac{(1 + \mu)}{2} \frac{\alpha^2}{\beta} y \sinh \alpha y \right\} \quad \ldots\ldots \quad (21)$$

As the tensile strains are a maximum at the crest of the dam (y = 0),

$$\epsilon_x\Big]_{y=0} = -H \sum_{n=1}^{\infty} \left\{\frac{\alpha^2 \sinh \alpha H}{\sinh \alpha H + \alpha H \cosh \alpha H}\right\} \left(a_n \cos \alpha x + b_n \sin \alpha x\right) \quad \ldots\ldots\ldots\ldots\ldots \quad (22)$$

This is the desired expression for the longitudinal strains at the crest of the dam. Eq. 22 can readily be arranged for computer solution.

Simplified Method.—If the length of the dam is long in comparison to the height, the conditions can be approximated by pure bending (plane sections

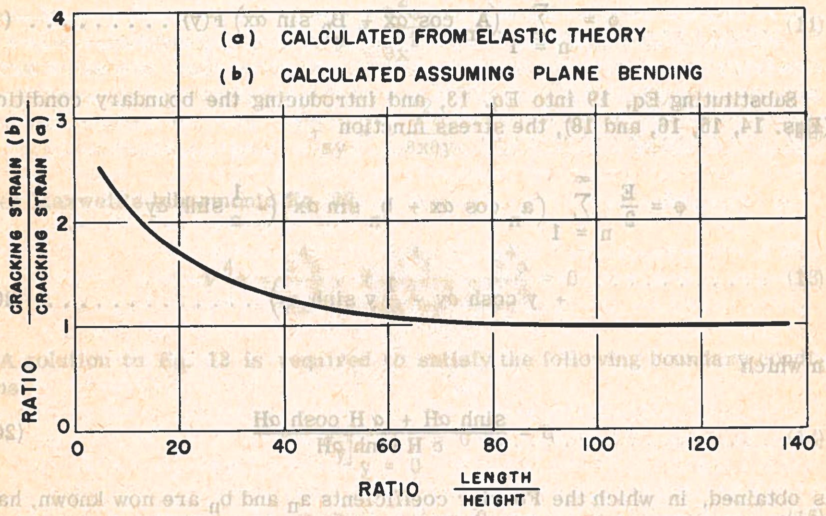


FIG. 3.—INFLUENCE OF LENGTH-HEIGHT RATIO ON CRACKING STRAIN COMPUTED FROM ELASTIC THEORY AND PLANE BEDDING

remain plane after bending). Using the boundary condition of Eq. 18 as the deflection curve, its second derivative with respect to x is

$$f_2(x) = \sum_{n=1}^{\infty} -\alpha^2 \left(a_n \cos \alpha x + b_n \sin \alpha x\right) \quad \ldots\ldots\ldots \quad (23)$$

The radius of curvature, $R \approx \dfrac{1}{f_2(x)}$

$$\epsilon_x = \frac{\text{distance from neutral axis}}{\text{radius of curvature}}$$

$$\epsilon_x\Big]_{y=0} \approx \frac{H}{2} f_2(x)$$

$$\epsilon_x\Big]_{y=0} \approx -H \sum_{n=1}^{\infty} \frac{\alpha^2}{2} \left(a_n \cos \alpha x + b_n \sin \alpha x\right) \quad \ldots\ldots \quad (24)$$

Eq. 22 reduces to Eq. 24 if αH is small ($\sinh \alpha H \approx \alpha H$, $\cosh \alpha H \approx 1$), which occurs when L is much larger than H. Examination of Fig. 3 shows that the simplified procedure can be used without significant error if the length to height ratio exceeds 50.

LABORATORY TESTS

Simple bending tests were performed on beams of compacted clay in order to determine the tensile strains at cracking. For a given soil type, relationships were obtained between tensile strain at the initiation of cracking and compactive effort, molding water content, and rate of loading.

TABLE 2.—INDEX PROPERTIES OF SOILS STUDIED

Source	Liquid Limit	Plasticity Index	Specific gravity of Solids	< No. 200 U. S. Sieve %	Standard Proctor: Optimum Moisture content, in %	Standard Proctor: Optimum dry Density, in pcf
Portland Dam Colorado	29	8	2.74	25	16.3	112
Rector Creek Dam California	38	16	2.60	11	19.8	103
Woodorest Dam California	nonplastic		2.64	21	10.2	127
Shell Oil Dam California	nonplastic		2.65	22	11.2	120
Willard Dam Test Embankment, Utah	31	11	2.72	28	16.4	110
Limestone residual Bedford, Indiana	72	45	2.75	82	25.9	96

Six soil types were used for beam tests. Five of these soils were obtained from earth embankments in which the construction conditions and post-construction performance are known. The sixth soil studied is a limestone residual clay from southern Indiana, which has been studied extensively in previous investigations at Purdue University Lafayette, Ind. A summary of index properties for these soils is given in Table 2.

A grid of pins was inserted in the beam and monitored with a cathetometer. Loads were applied in equal increments and equal time intervals so that fracture occurred in two days (short-term tests) or in four weeks (long-term tests). For comparison purposes, one beam was loaded so that cracking developed after 6 months. All samples were maintained in the as-compacted condition for the duration of the test. The initiation of cracking was monitored visually. Rupture of the beam followed immediately after the first crack formed.

Constant water content cylindrical compression tests were performed on specimens from each beam to investigate the relationship between Young's modulus and fracture strain in tension and compression. Similar tests were also conducted on undisturbed samples from the embankments under study in order to ascertain the effects of recompaction on the stress-strain properties at constant water content.

Apparatus and Procedure.—Each 200 lb soil sample (transported from the field in plastic-lined bags) was thoroughly mixed and sieved through a No. 4 U. S. standard sieve. Clods were broken by hand and evaporated moisture was replenished. The soils were then stored and maintained at their approximate natural moisture content until ready for testing.

Sufficient soil for one beam was taken from storage, and water was added (with an electric vibrator sprayer) to obtain a selected water content. The soil was hand-mixed, covered with water-proof plastic, and stored overnight in the humid room. The compaction mold, a steel channel base with removable sides and ends, was lined with used photographic film. After preliminary testing, the size of beam adopted was 3 in. wide, 2 3/4 in. deep, and 22 1/8 in. long, as a result of the following considerations:

1. Span to depth ratio greater than six;
2. easy manipulation by one man without excessive handling stresses; and
3. deflections large enough to be measured precisely with the cathetometer (sensitivity 10^{-4} in.).

The soil was compacted in ten equal (horizontal) layers with an air-actuated vibrator having a 2.5 in. square base plate, weighing 16 lb, the surface of each layer being scarified before the next layer was placed. The time of compaction for each layer was controlled by stop watch after correlations between compactive effort and compaction time (at selected water contents) had been made for each soil. When dismantling the mold, two gauze support straps were placed at positions causing minimum handling stresses, and the beam was coated with a layer of 50% petrowax plus 50% petrolatum oil by means of hand dipping. Tests showed that this coating was effective in maintaining constant moisture content yet offering negligible resistance to bending. Two rows of 11 tungsten pins, 0.04 in. in diameter and 2 in. long, were inserted 1.75 in. into the beam through a steel template (Fig. 4). The ends of the pins were given a thin coat of yellow paint and etched with a fine cross.

Load was applied by dead weights (Fig. 5). Two sets of ball-bearing pulleys (free to move in mutually perpendicular directions) eliminate the possibility of eccentric loading or twisting of the beam. In order to compensate approximately for dead-load stresses, a load was applied at midspan using a spring of low stiffness. To minimize stress concentrations, the load and support straps were lined with rubber foam. Deflections of the pins were monitored with the cathetometer immediately after adding a load increment, immediately before the next increment was placed, and periodically in-

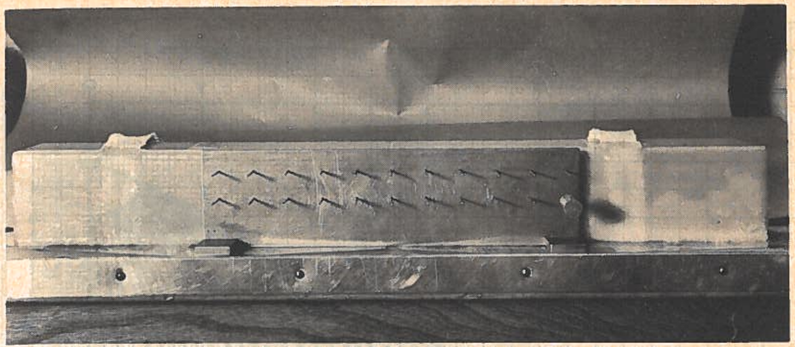


FIG. 4.—TEMPLATE FOR PLACING GRID PINS ON COATED BEAM (NOTE GAUZE HANDLING STRAPS

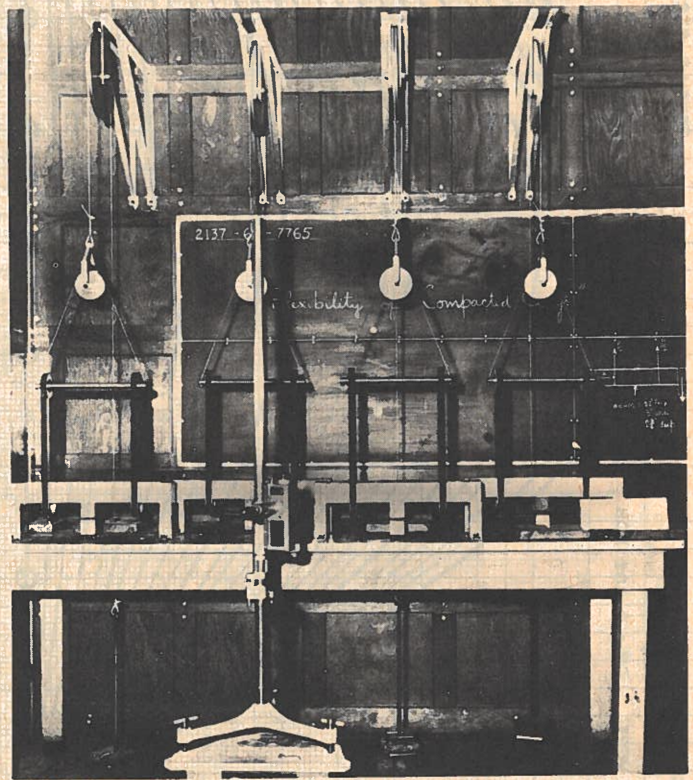


FIG. 5.—APPARATUS FOR FLEXURAL TESTS ON COMPACTED CLAY BEAMS (NOTE ARTICULATED PULLEY SYSTEM)

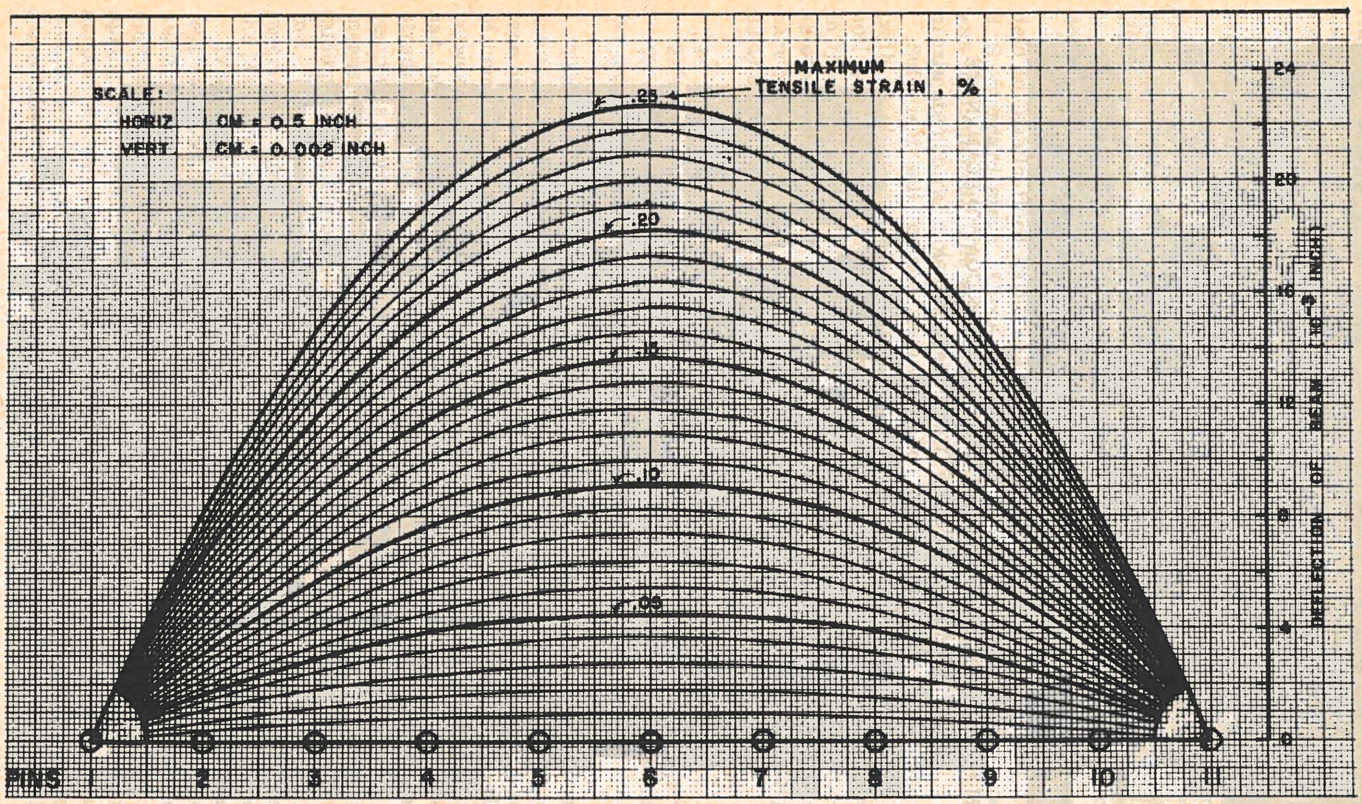


FIG. 6.—RELATIONSHIP BETWEEN THEORETICAL DEFLECTIONS AND TENSILE STRAINS

TABLE 3.—SUMMARY OF TEST RESULTS

Soil				Beam Tests												Compression Tests					
Source		Compaction Condition		Tensile strain at failure, p. c.				Tensile strength at failure, psi		Tangent modulus at failure in tension, in 1000 psi				Compressive strain at failure, p. c.		Compressive strength at failure, psi		Initial tangent modulus in compression, x1000 psi	
		Dry density, in pcf	Water content, %	Short term (a)	Short term (b)	Long term (a)	Long term (b)	Short term	Long term	Short term (a)	Short term (b)	Long term (a)	Long term (b)	Short term (b)	Long term (b)	Short term	Long term	Short term (b)	Long term (b)
Portland Dam	Std. Proctor As Placed	112	16.3	0.14	0.17	0.15	0.19	8.0	8.0	2.1	1.8	3.4	2.6	1.8	2.2	97	81	4.9	4.7
		111	13.5	0.08	0.12	0.13	0.16	12.5	10.5	6.2	5.3	3.2	3.0	2.0	1.9	112	83	6.5	6.1
Rector Creek Dam	Std. Proctor As Found	103	19.8	0.12	0.16	0.13	0.17	9.3	7.4	5.0	3.8	4.2	3.1	2.0	2.7	38	22	4.1	2.6
		96	18.8	0.11	0.12	0.11	0.13	6.7	6.2	2.7	2.5	3.0	2.5	2.0	2.8	60	40	3.3	2.3
Woodcrest Dam	Std. Proctor As Found	127	10.2		0.18		0.24	11.5	12.1		4.9		3.9	2.1	2.5	57	71	5.8	4.3
		124	7.2		0.17		0.20	7.5	6.8		1.7		1.3	1.8	3.2	53	63	4.6	2.8
Shell Oil Co. Dam	Std. Proctor As Found	120	11.2	0.06	0.07			5.7		4.7	4.0			3.7	3.8	34	30	1.9	1.5
		114	12.3	0.05	0.06			4.3		5.8	4.8			4.4	4.7	24	23	2.1	0.8
Willard Dam Test Embankment	Std. Proctor As Found	110	16.4	0.18	0.20	0.22	0.24	6.5	5.5	2.5	2.5	1.7	1.6	3.1	3.7	63	26	3.6	2.2
		105	12.5	0.15	0.18	0.21	0.24	10.2	9.6	4.3	3.8	3.5	3.1	1.3	2.6	51	40	5.7	3.1
Limestone Clay	Std. Proctor Optimum	96	25.9	0.24	0.26	0.24	0.29	27.5	23.6	8.1	7.6	5.2	4.4	4.9	3.0	65	50	5.0	3.3
						0.29*	0.33*		23.5*			4.5*	3.8*						
	Std. Proctor 2% Dry Opt.	93	23.9	0.15	0.17			23.8		7.0	7.0			1.5	2.8	68	63	6.1	5.7
	Std. Proctor 2% Wet Opt.	93	27.9	0.20	0.24	0.28	0.32	15.3	16.5	5.4	4.4	5.1	4.4	5.1	3.0	44	40	3.8	2.4
	Std. Proctor 3.6% Wet Opt.	89	29.5	0.24	0.28			11.7		4.0	3.2								
	Mod. AASHO Optimum	110	17.5	0.105	0.12			37.8		23.0	19.4								
	Mod. AASHO 2% Dry Opt.	107	15.5	0.07	0.08			32.9		30.7	26.4								
	Mod. AASHO 2% Wet Opt.	107	19.5	0.09	0.105			25.4		21.2	17.3								
	Mod. AASHO 3.5% Wet Opt.	103	21.0	0.12	0.14			22.9		14.0	11.0								

(a) Based on observations at start of loading periods.
(b) Based on observations at end of loading periods.
* Very long term test (6 months).

between depending on the test duration. The deflection curve of the pins was plotted on transparent paper (using the end pins as a reference) and compared with the theoretical curves for plane bending to obtain the tensile strain (Fig. 6). Rupture invariably occurred near midspan.

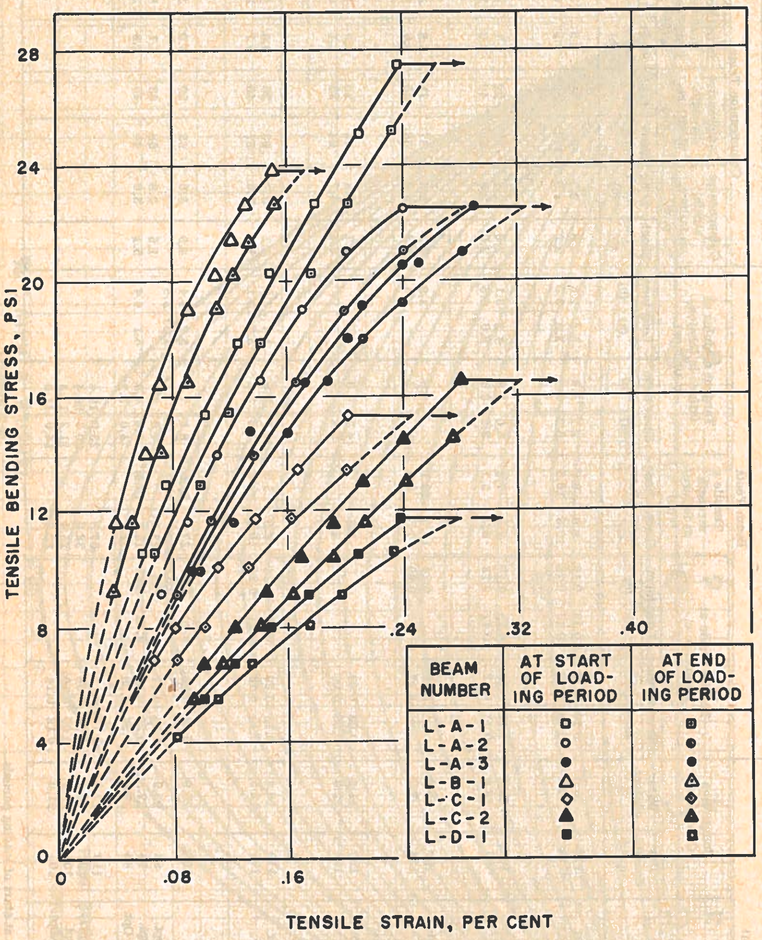


FIG. 7.—TENSILE STRESS-STRAIN CURVES FROM BEAM TESTS ON LIMESTONE CLAY

Constant water content cylindrical compression tests (unconfined and with τ_3 = 30 psi) were performed on samples from each compacted beam and on undisturbed samples from each of the dams studied. Constant-stress increments were applied over time intervals comparable to the short-term and long-term flexure tests. Temperature variations were minimized ($\pm$ 1° to 2°C) for all tests.

Test Results.—A summary of the most pertinent test results is given in Table 3. Typical tensile stress-strain curves are shown in Figs. 7 and 8; typical results for cylindrical compression tests are shown in Figs. 9 and

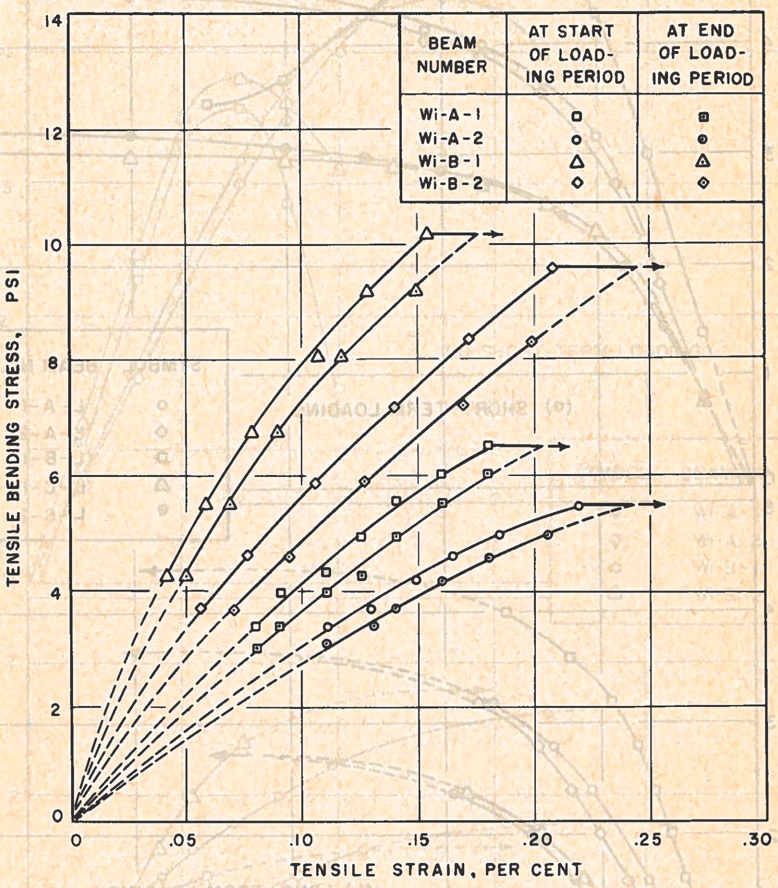


FIG. 8.—TENSILE STRESS-STRAIN CURVES FROM BEAM TESTS ON SOIL FROM WILLARD DAM TEST EMBANKMENT

10. The test conditions corresponding to the beam designations are given in Table 4. Attention is directed to the following points:

1. Tensile stress-strain curves are approximately linear almost to fracture.
2. Tensile strain at cracking is at least an order of magnitude smaller than compressive strain at failure. Their ratio varies at random from approximately 0.11 to 0.014.

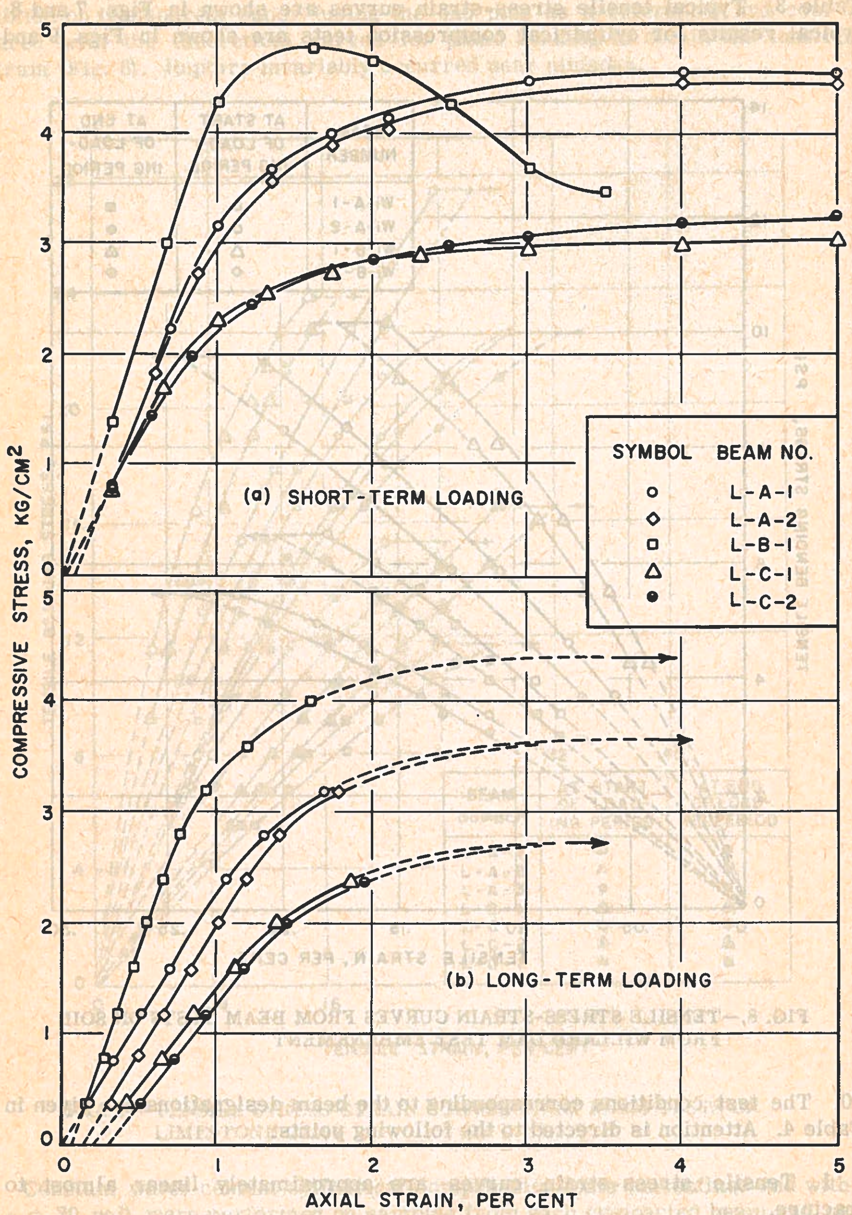


FIG. 9.—STRESS-STRAIN CURVES FROM UNCONFINED COMPRESSION TESTS ON SAMPLES FROM BEAMS OF LIMESTONE CLAY

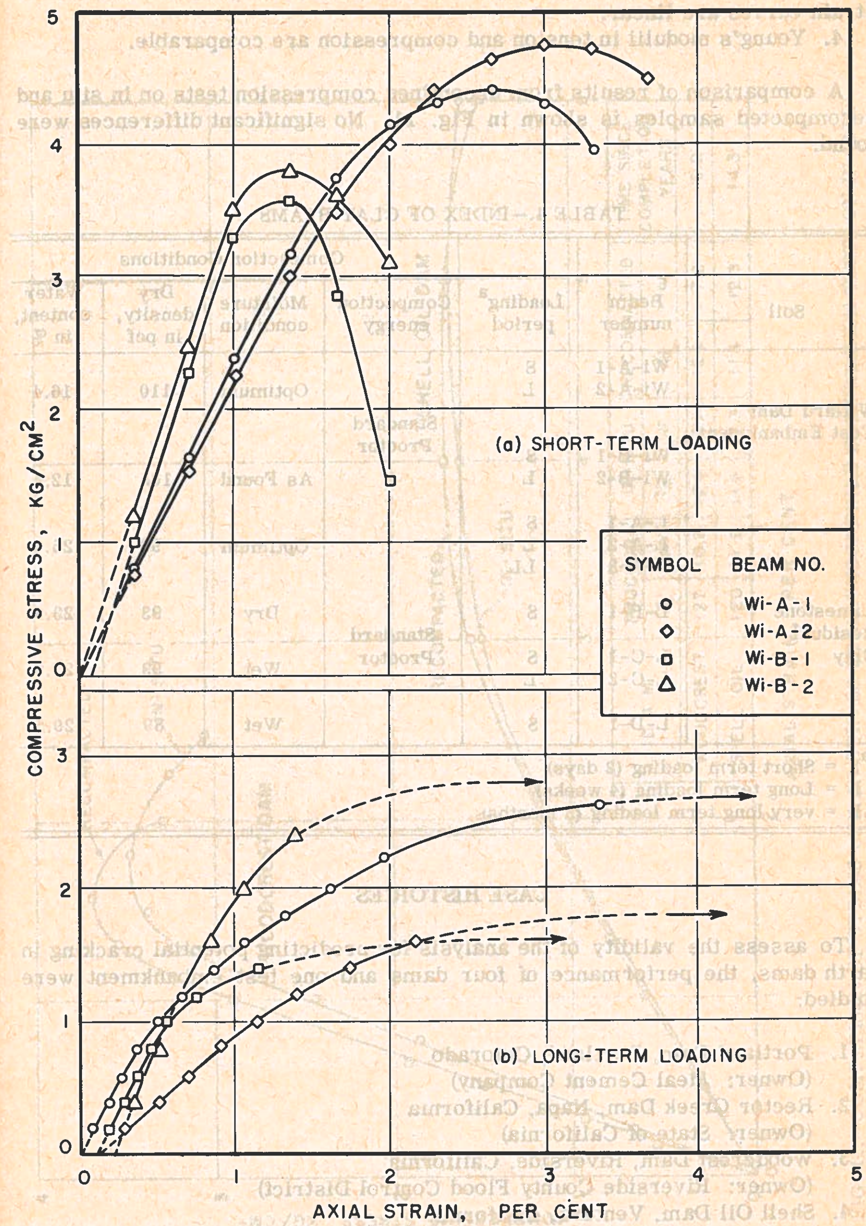


FIG. 10.—STRESS-STRAIN CURVES FROM UNCONFINED COMPRESSION TESTS ON SAMPLES FROM BEAMS OF WILLARD DAM TEST EMBANKMENT

3. At strains comparable to fracture in tension, the compressive stress-strain curves are linear.
4. Young's modulii in tension and compression are comparable.

A comparison of results from unconfined compression tests on in situ and recompacted samples is shown in Fig. 11. No significant differences were found.

TABLE 4.—INDEX OF CLAY BEAMS

Soil	Beam number	Loading[a] period	Compaction Conditions			
			Compaction energy	Moisture condition	Dry density, in pcf	Water content, in %
Willard Dam Test Embankment	Wi-A-1 Wi-A-2	S L	Standard Proctor	Optimum	110	16.4
	Wi-B-1 Wi-B-2	S L		As Found	105	12.4
Limestone Residual Clay	L-A-1 L-A-2 L-A-3	S L LL	Standard Proctor	Optimum	96	25.9
	L-B-1	S		Dry	93	23.9
	L-C-1 L-C-2	S L		Wet	93	27.9
	L-D-1	S		Wet	89	29.5

[a]S = Short term loading (2 days)
L = Long term loading (4 weeks)
LL = very long term loading (6 months)

CASE HISTORIES

To assess the validity of the analysis for predicting potential cracking in earth dams, the performance of four dams and one test embankment were studied:

1. Portland Dam, Portland, Colorado (Owner: Ideal Cement Company)
2. Rector Creek Dam, Napa, California (Owner: State of California)
3. Woodcrest Dam, Riverside, California (Owner: Riverside County Flood Control District)
4. Shell Oil Dam, Ventura, California (Owner: Shell Oil Company)
5. Willard Dam Test Embankment, Willard, Utah (Owner: U. S. Bureau of Reclamation)

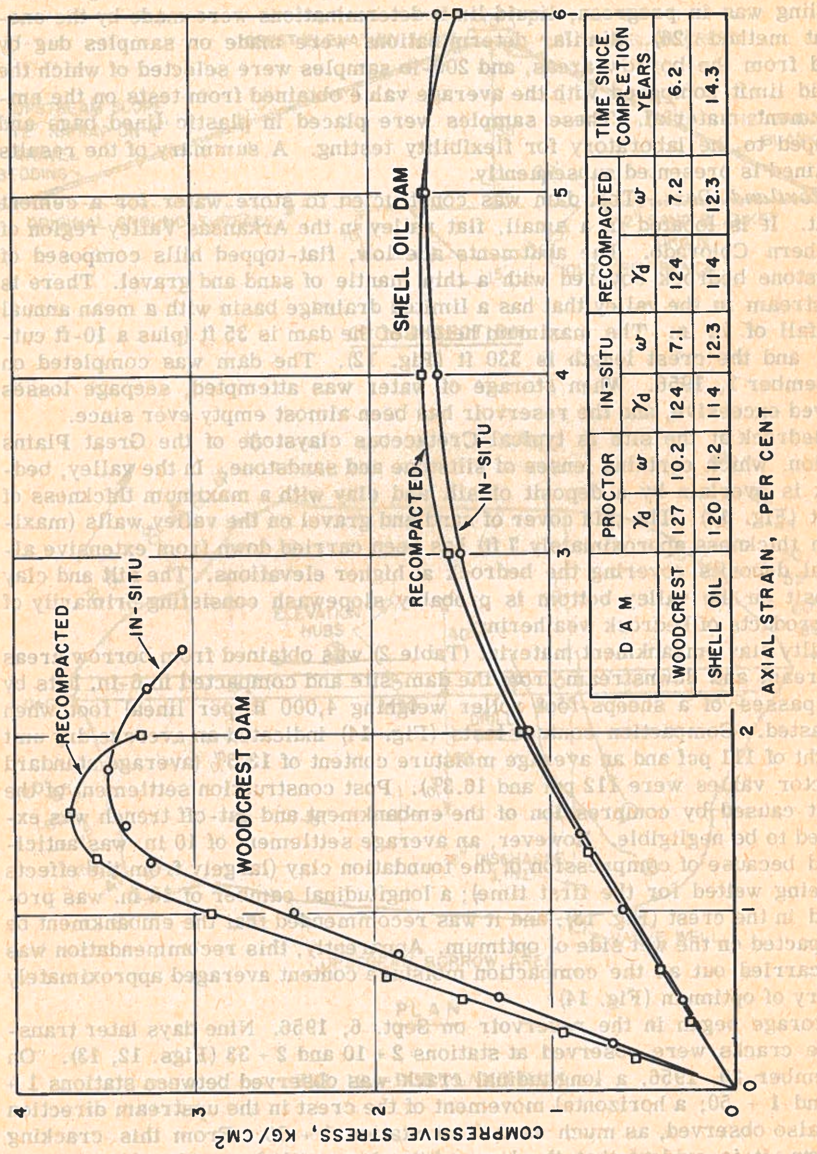


FIG. 11.—COMPARISON OF STRESS-STRAIN CURVES FROM UNCONFINED COMPRESSION TESTS ON IN-SITU AND RECOMPACTED SAMPLES

Each of the dams was visited by Narain in January, 1960, and all available design, construction, and performance data were assembled. Borings were made from the crest of the dam adjacent to the locations of cracks and 2 in. "undisturbed" samples were recovered in copper liners 4 in. long. While drilling was in progress, liquid limit determinations were made by the one-point method (26). Similar determinations were made on samples dug by hand from the borrow areas, and 200-lb samples were selected of which the liquid limit compared with the average value obtained from tests on the embankment material. These samples were placed in plastic lined bags and shipped to the laboratory for flexibility testing. A summary of the results obtained is presented subsequently.

Portland Dam.—The dam was constructed to store water for a cement plant. It is located in a small, flat valley in the Arkansas Valley region of southern Colorado. The abutments are low, flat-topped hills composed of claystone bedrock covered with a thin mantle of sand and gravel. There is no stream in the valley that has a limited drainage basin with a mean annual rainfall of 12 in. The maximum height of the dam is 35 ft (plus a 10-ft cut-off), and the crest length is 330 ft (Fig. 12). The dam was completed on September 1, 1956. When storage of water was attempted, seepage losses proved excessive, and the reservoir has been almost empty ever since.

Bedrock at the site is typical Cretaceous claystone of the Great Plains Region, which contains lenses of siltstone and sandstone. In the valley, bedrock is overlain by a deposit of silt and clay with a maximum thickness of 23 ft (Fig. 13). The thin cover of sand and gravel on the valley walls (maximum thickness approximately 7 ft) has been carried down from extensive alluvial deposits covering the bedrock at higher elevations. The silt and clay deposit on the valley bottom is probably slopewash consisting primarily of the products of bedrock weathering.

Silty clay embankment material (Table 2) was obtained from borrow areas upstream and downstream from the dam-site and compacted in 6-in. lifts by ten passes of a sheeps-foot roller weighing 4,000 lb per lineal foot when ballasted. Compaction control tests (Fig. 14) indicated an average dry unit weight of 111 pcf and an average moisture content of 13.5% (average standard Proctor values were 112 pcf and 16.3%). Post construction settlement of the crest caused by compression of the embankment and cut-off trench was expected to be negligible. However, an average settlement of 10 in. was anticipated because of compression of the foundation clay (largely from the effects of being wetted for the first time); a longitudinal camber of 18 in. was provided in the crest (Fig. 13), and it was recommended that the embankment be compacted on the wet side of optimum. Apparently, this recommendation was not carried out as the compaction moisture content averaged approximately 3% dry of optimum (Fig. 14).

Storage began in the reservoir on Sept. 6, 1956. Nine days later transverse cracks were observed at stations 2 + 10 and 2 + 33 (Figs. 12, 13). On November 29, 1956, a longitudinal crack was observed between stations 1 + 00 and 1 + 50; a horizontal movement of the crest in the upstream direction was also observed, as much as 2 ft at station 1 + 50. From this cracking pattern, it is evident that the base of the dam settled considerably more on the upstream side than on the downstream side and that the settlement was caused by a collapse of structure when the foundation material was first wetted.

Rector Creek Dam.—The dam is situated on Rector Creek in the Napa area of California just north of San Pablo Bay and was constructed to provide a water supply for the Veterans Home in Napa County. The terrain of the

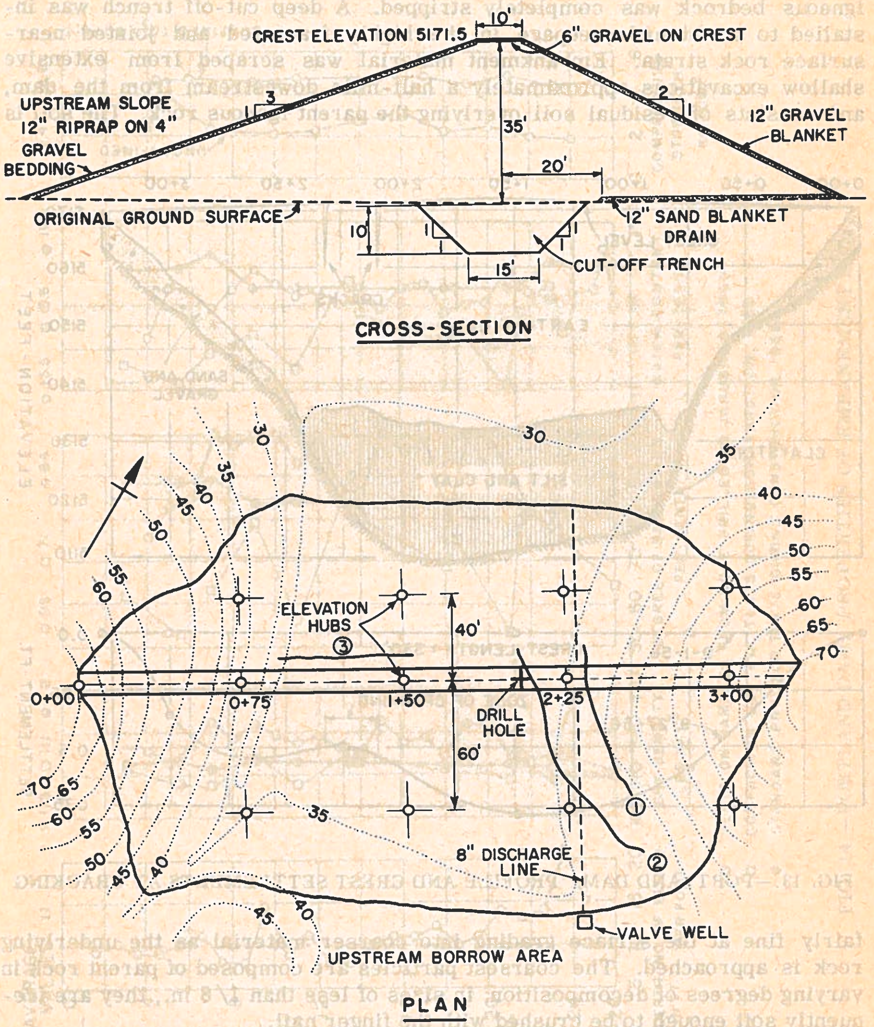


FIG. 12.—PORTLAND DAM

region varies from undulating to mountainous, and the soils consist of alluvium outwashed from basic igneous rocks. The alluvium contains large fractions of coarse materials including boulders to 6 in. in size. The maximum height of the dam is 150 ft (plus a 50 ft cut-off), and the crest length is 900 ft

(Fig. 15). The dam was completed and storage was begun in early January, 1947. Rainfall in the area is sufficient to maintain the reservoir nearly full most of the time (Fig. 16).

Prior to construction of the embankment, the residual soil overlying the igneous bedrock was completely stripped. A deep cut-off trench was installed to avoid underseepage in the highly fractured and jointed near-surface rock strata. Embankment material was scraped from extensive shallow excavations approximately a half-mile downstream from the dam, and consists of residual soil overlying the parent igneous rock. The soil is

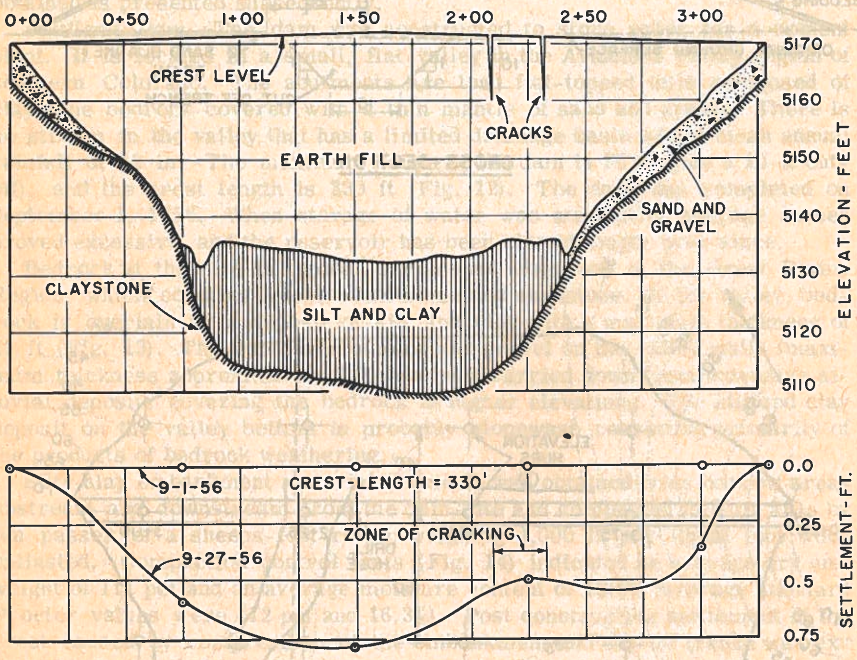


FIG. 13.—PORTLAND DAM: PROFILE AND CREST SETTLEMENTS AT CRACKING

fairly fine at the surface grading into coarser material as the underlying rock is approached. The coarsest particles are composed of parent rock in varying degrees of decomposition; in sizes of less than 1/8 in., they are frequently soft enough to be crushed with the finger nail.

The clayey-gravelly-sand embankment material (Table 2) was spread in 6-in. layers and compacted by ten passes of a heavy sheepsfoot roller. Moisture control consisted of sprinkling the borrow pits. From interpretations of the construction records, it is believed that the embankment was compacted to an average dry unit weight of 111 pcf in zone 1, and 100 pcf in zone 2, at a moisture content approximately 4% below standard Proctor optimum. The condition of the embankment material as found in January, 1960, are shown in Fig. 17.

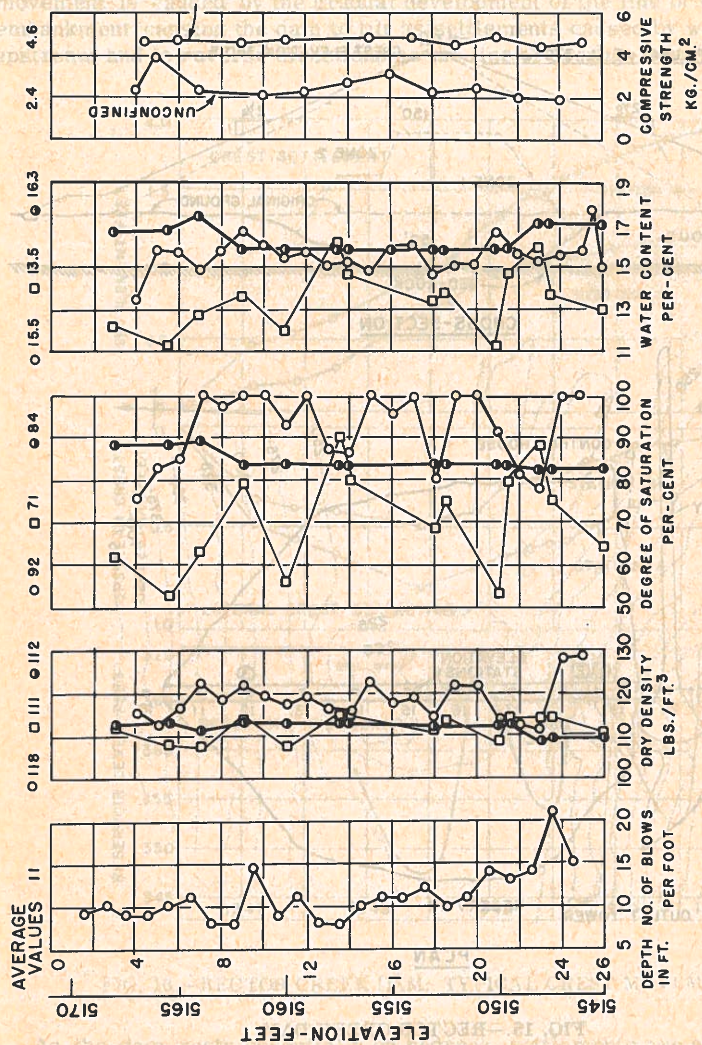


FIG. 14.—PORTLAND DAM: PROPERTIES OF EMBANKMENT SOIL

Immediately after construction was completed hubs were established on the crest 100 ft apart for observations of settlements and of horizontal motions in the longitudinal and transverse directions (Fig. 15, 16, and 18). In February, 1947, one month after storage was begun, a 3/4-in. crack opened across the crest of the dam at station 18 + 65 (Fig. 15). A test pit was sunk

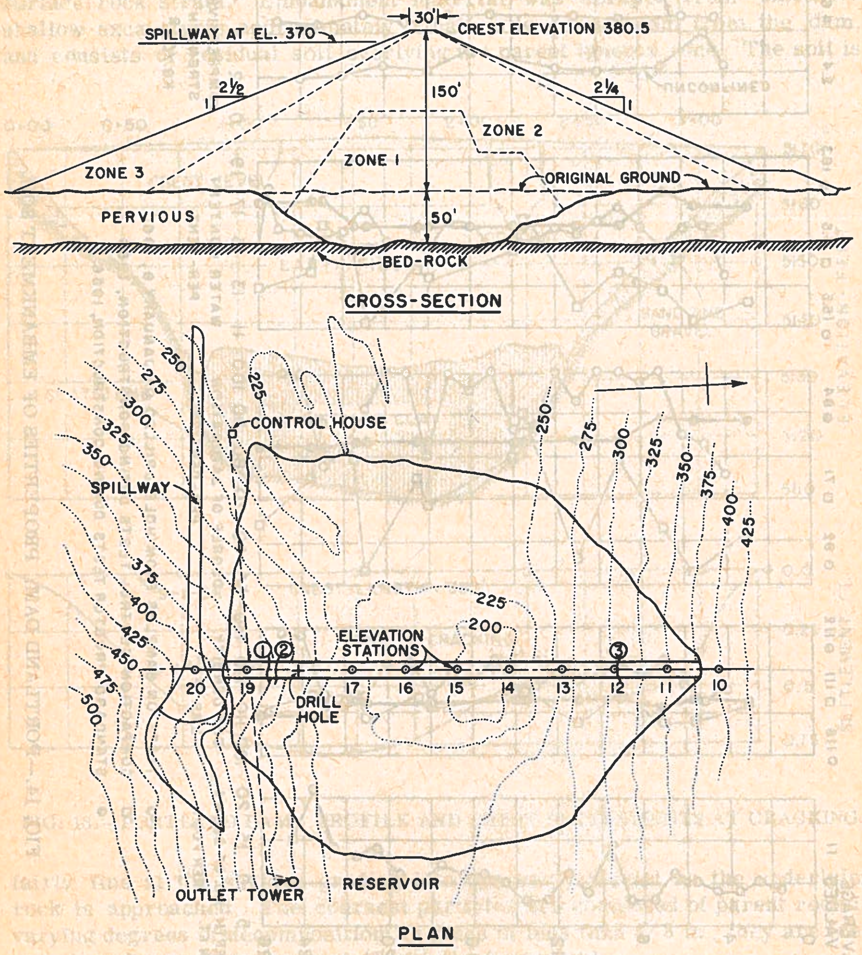


FIG. 15.—RECTOR CREEK DAM

in order to follow the course of the crack. At a depth of 24 ft, the crack still had a width of 1/2 in. When subsequent observations for several months showed no apparent increases in width, the crack was grouted with cement. In December, 1949, a crack approximately 1/2 in. wide was observed at station 11 + 95. Periodically, additional cracks were detected. For example, in

May, 1953, a crack 1 in. wide was discovered at station 18 + 50 in a slab of cement probably wasted during the 1947 grouting operation.

Fig. 16 shows that two years after storage was begun, the crest moved upstream a maximum distance of 0.8 ft. The movement then changed direction; at present (1960), the crest is almost back to its initial position. This movement is caused by the gradual development of the line of seepage in the embankment causing the dam to tilt as settlements caused by wetting develop upstream and to reverse directions as the line of seepage advances (27).

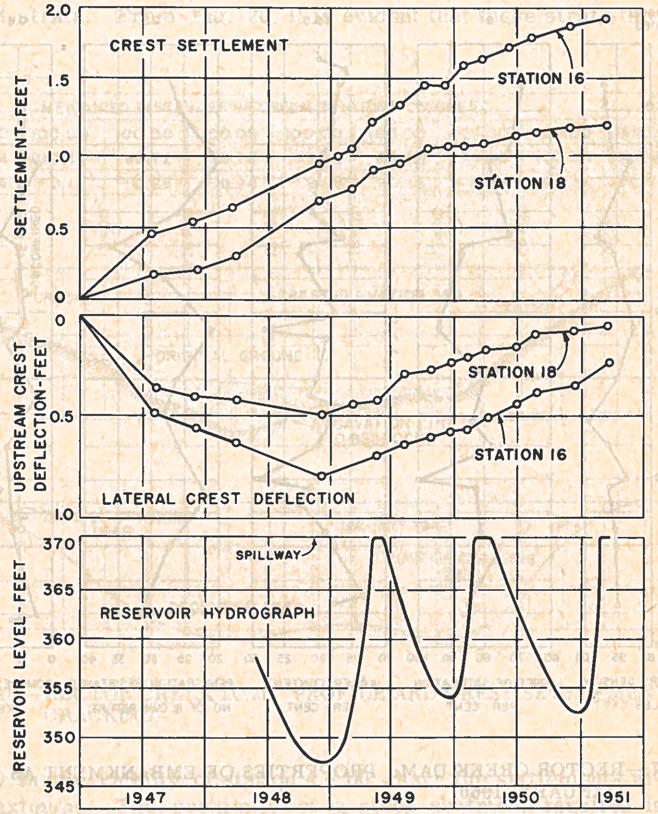


FIG. 16.—RECTOR CREEK DAM: TYPICAL CREST MOVEMENTS

As the dam rests essentially on bedrock, settlements are almost entirely caused by collapse in structure of the embankment material on wetting. Note the correspondence between the zones of cracking and the sharp curvature of the settlement curve.

Woodcrest Dam.—The dam is located in Riverside County in the suburbs of Los Angeles, Calif., and was built for the purpose of flood control. As the drainage area is only 5.6 sq miles, the reservoir has little storage for the greater part of the year. The maximum height of the dam is 42.5 ft, and the crest length is 1,125 ft (Fig. 19). The dam was completed in November, 1953.

Subsurface soils in the valley consist of approximately 5 ft of coarse sand underlain by silt, fine sand, coarse sand, and silty sand strata (Fig. 20). A shallow cut-off trench penetrated the upper pervious stratum of coarse sand. Granite bedrock was found at El. 1054 at station 6 + 55 but was not encountered in the other holes. The silt and silty sand strata are compressible.

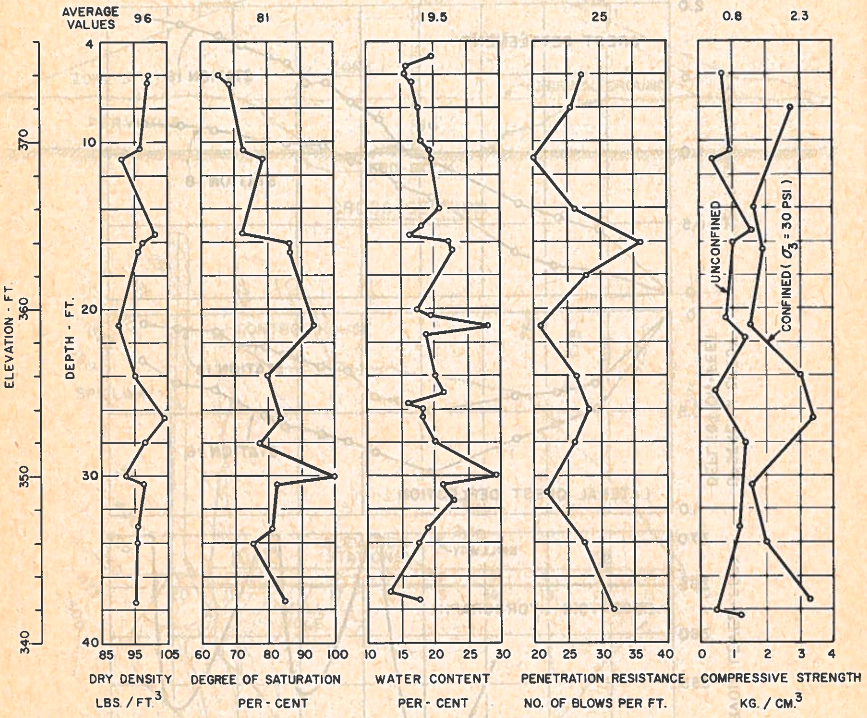


FIG. 17.—RECTOR CREEK DAM: PROPERTIES OF EMBANKMENT AS FOUND (JANUARY, 1960)

Embankment material was obtained from shallow borrow pits upstream from the dam and consisted of an essentially nonplastic mixture of fine sand and silt (Table 2). The dry unit weights from control tests during construction ranged between 118 pcf and 126 pcf (average, 120 pcf), and the compaction moisture content was approximately 8.5%; standard Proctor optimum values are 127 pcf and 10.2%, respectively. Compaction specifications called for a dry unit weight of 124 pcf and a (Proctor needle) penetration resistance of 200 psi. The condition of the embankment material, as found in January 1960, is shown in Fig. 21.

In October, 1954, a transverse crack was discovered at station 2 + 54 (Fig. 19); the settlement curve at this time is shown in Fig. 22. The dam was inspected periodically and each year minor cracks (1/8 in. to 1/16 in. wide) were noted between stations 2 + 50 and 3 + 00 until January, 1959, at which time no new cracks were found. In January, 1960, a 1.5 in. wide crack was discovered at station 2 + 70. The crack was roughly perpendicular to the axis of the dam and could be traced on both slopes for almost the full height of the embankment. The reservoir was empty at the time.

Settlements are partly caused by compression of embankment material but were caused primarily by consolidation of the underlying silt and fine silty sand strata. From Fig. 20, it is evident that these strata thicken considerably in the vicinity of station 4 + 00 where the settlements are found to be a maximum. The proximity of a steep abutment resulted in a sharp change in the settlement pattern and a concomittant development of large tensile strains at the crest (Fig. 22). It is to be noticed that in January, 1960, the curvature of the settlement curve is sharper at station 5 + 00 than in the vicinity of cracking, yet no cracks developed near this station. Examination of the average tensile strains measured between stations on the crest (Fig. 22) indicates that a tensile strain of at least 0.09% was required to initiate cracking (station 2 + 50).

MEASURED DISTANCES BETWEEN STATIONS ON CREST								DATE
100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	JANUARY 1947
100.24	100.11	99.72	99.76	99.92	99.84	99.56	100.36	DECEMBER 1949
+0.24	+0.11	-0.28	-0.24	-0.08	-0.16	-0.44	+0.36	AVG. STRAIN, %

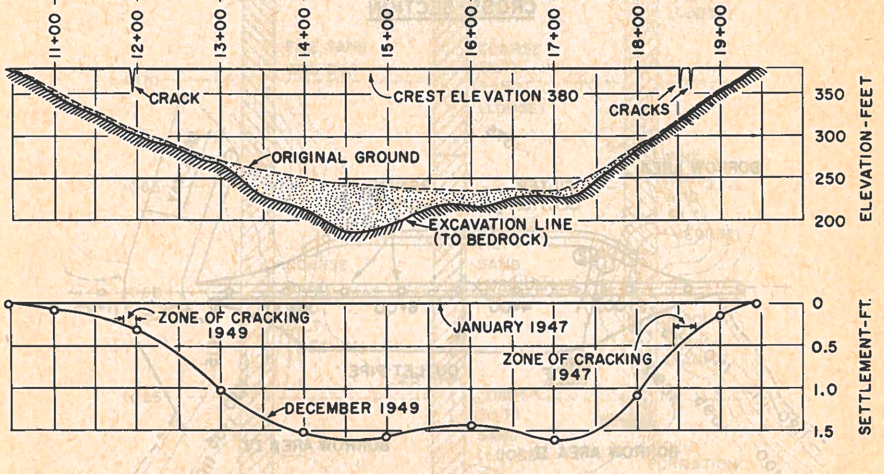


FIG. 18.—RECTOR CREEK DAM: PROFILE AND CREST SETTLEMENTS AT CRACKING

Shell Oil Dam.—The dam is situated on a drainage depression tributary to Ventura River (Rancho Canada de Miguelito) and was constructed to store waste drilling fluids on the Taylor lease at the Coastal Division of Shell Oil

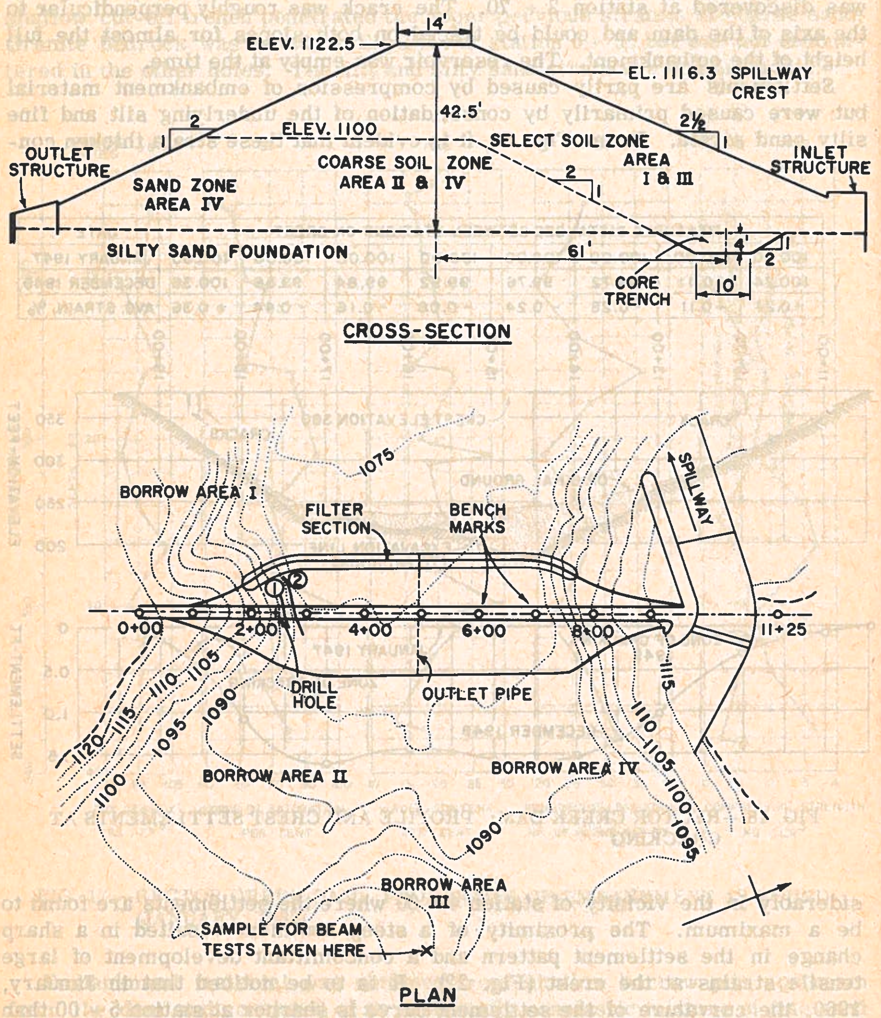


FIG. 19.—WOODCREST DAM

Company, Ventura, Calif. The catchment area is 52 acres. The dam has a maximum height of 77 ft and a crest length of 350 ft (Fig. 23). The valley is V-shaped with side-slopes averaging 2.25 to 1. Surface soils are unconsolidated water-laid deposits exhibiting moderately good drainage characteristics and little resistance to erosion. Post-holes to 8 ft deep were dug in the reservoir area. The soil samples obtained consisted mainly of fine silty

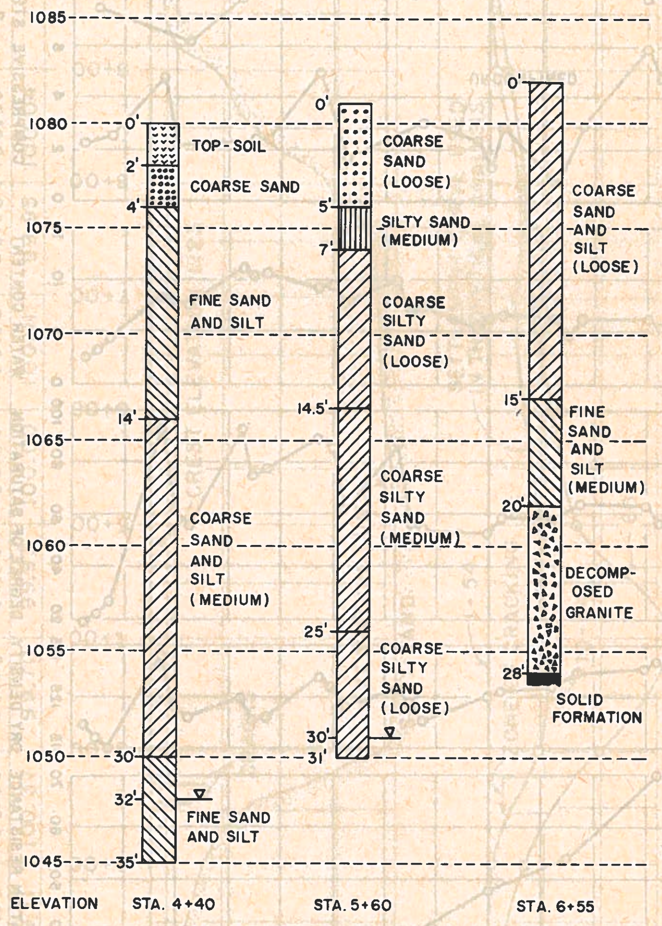


FIG. 20.—WOODCREST DAM: LOGS OF FOUNDATION TEST HOLES

sand having an average moisture content of 7%. No additional information on the foundation conditions is available. All of the soil used in the embankment was taken from the reservoir area and is essentially a nonplastic silty sand (Table 2). Specifications required compaction to 95% of modified AASHO (average optimum modified AASHO values are 122.5 pcf and 11.4%, respectively). Fill material was spread in 6 in. layers and compacted with a

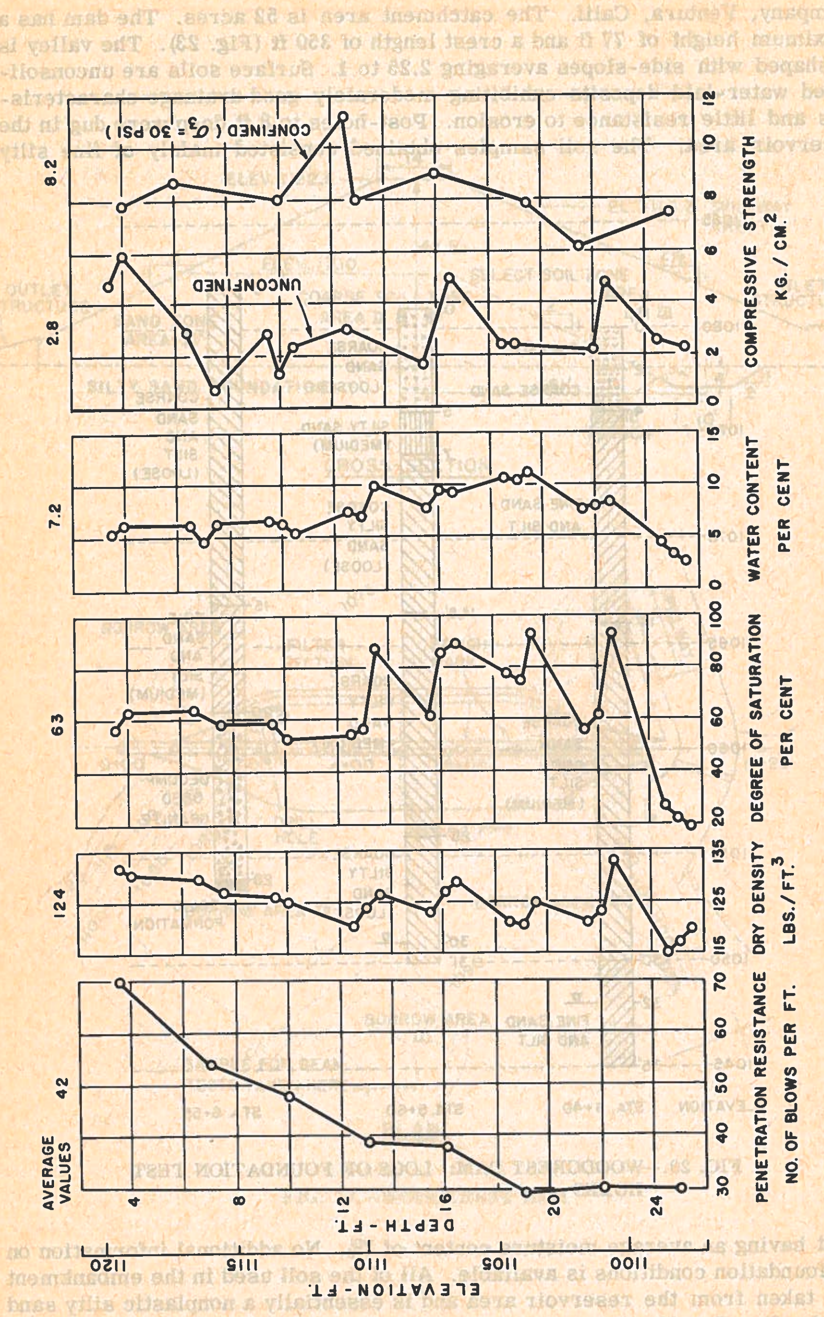
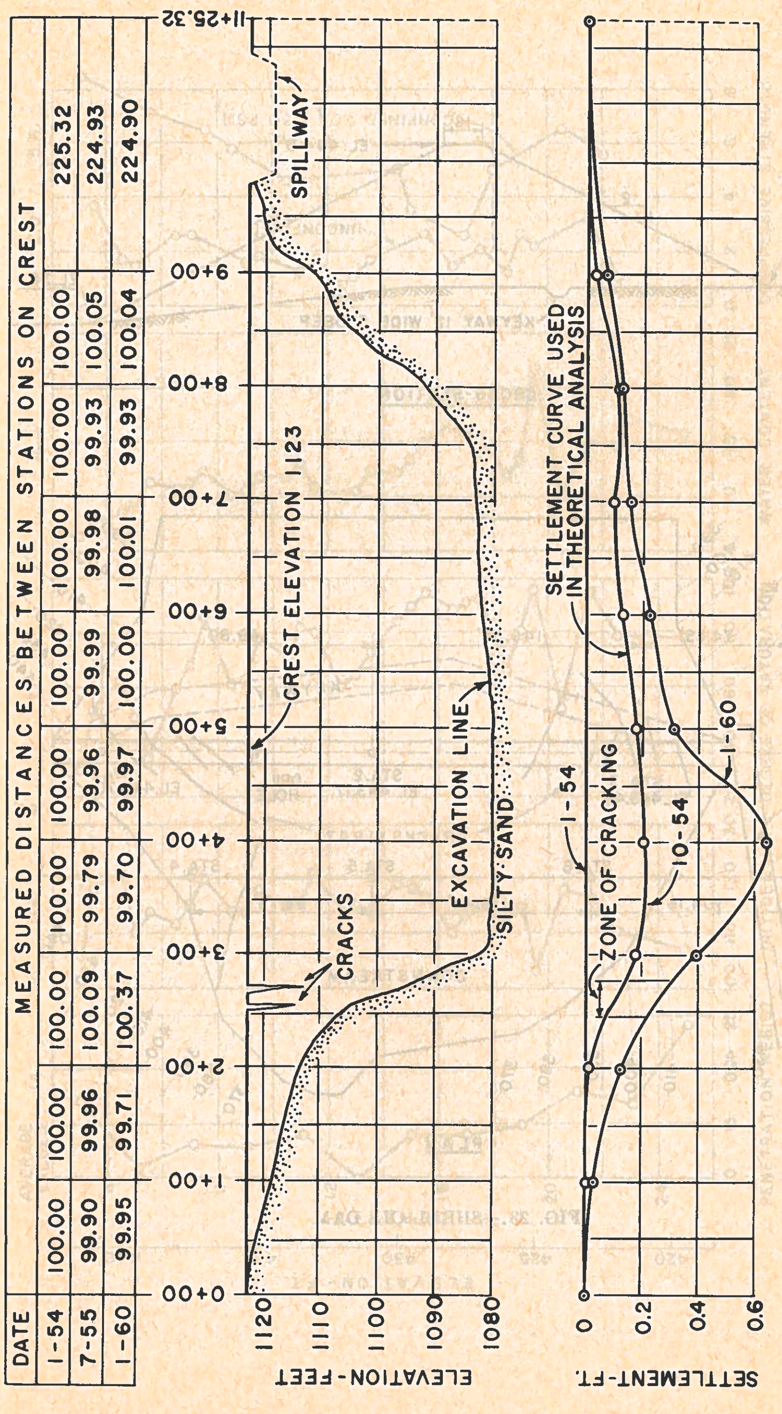


FIG. 21.—WOODCREST DAM: PROPERTIES OF EMBANKMENT AS FOUND (JANUARY, 1960)



DATE										
1-54	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	225.32
7-55	99.90	99.96	100.09	99.79	99.96	99.99	99.98	99.93	100.05	224.93
1-60	99.95	99.71	100.37	99.70	99.97	100.00	100.01	99.93	100.04	224.90

FIG. 22.—WOODCREST DAM: PROFILE AND CREST SETTLEMENTS AT CRACKING

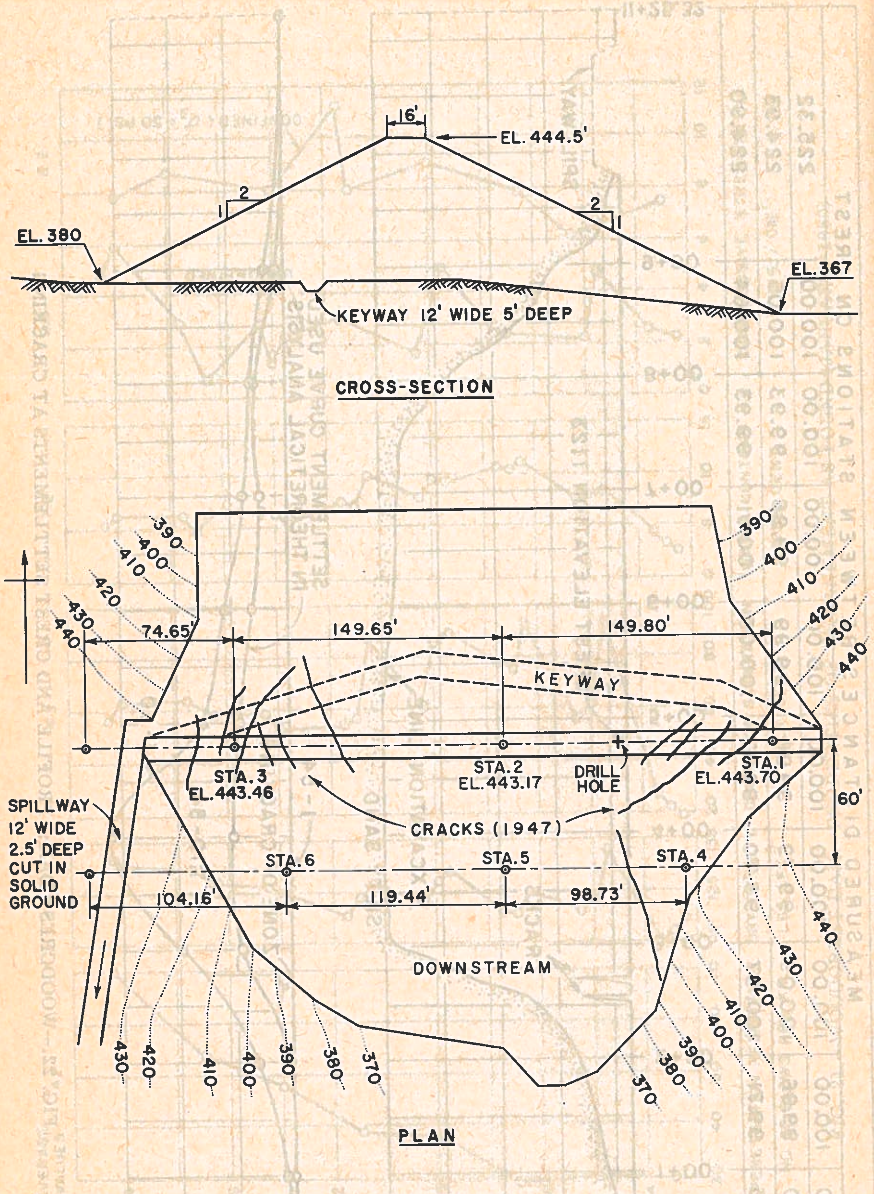


FIG. 23.—SHELL OIL DAM

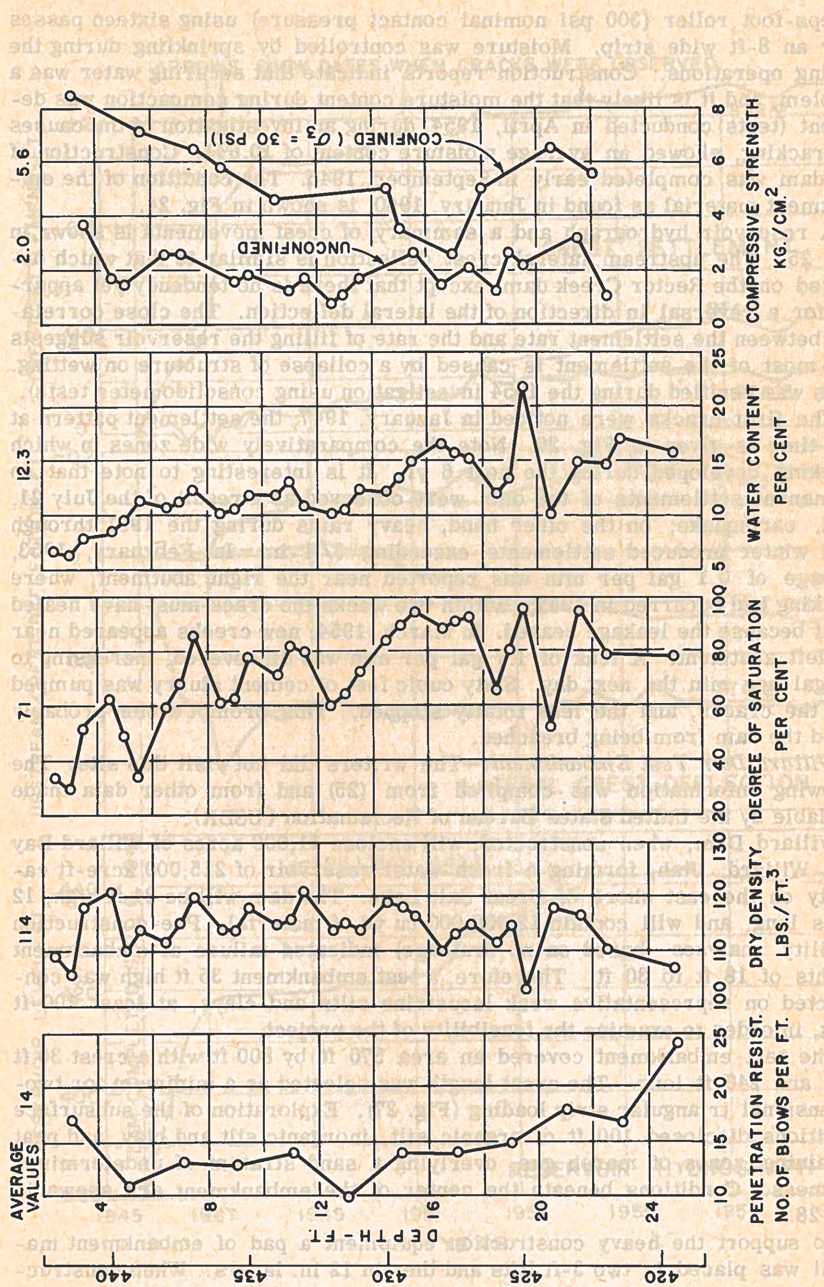


FIG. 24.—SHELL OIL DAM: PROPERTIES OF EMBANKMENT AS FOUND (JANUARY, 1960)

sheeps-foot roller (300 psi nominal contact pressure) using sixteen passes over an 8-ft wide strip. Moisture was controlled by sprinkling during the rolling operations. Construction reports indicate that securing water was a problem, and it is likely that the moisture content during compaction was deficient (tests conducted in April, 1954, during an investigation of the causes of cracking, showed an average moisture content of 10.6%). Construction of the dam was completed early in September, 1945. The condition of the embankment material as found in January, 1960, is shown in Fig. 24.

A reservoir hydrograph and a summary of crest movements is shown in Fig. 25. The upstream lateral crest deflection is similar to that which occurred on the Rector Creek dam, except that there is no tendency yet apparent for a reversal in direction of the lateral deflection. The close correlation between the settlement rate and the rate of filling the reservoir suggests that most of the settlement is caused by a collapse of structure on wetting. (This was verified during the 1954 investigation using consolidometer tests).

The first cracks were noticed in January, 1947; the settlement pattern at this time is given in Fig. 26. Note the comparatively wide zones in which cracking developed during the next 6 yr. It is interesting to note that no permanent settlements of the dam were observed as a result of the July 21, 1952, earthquake; on the other hand, heavy rains during the 1951 through 1952 winter produced settlements exceeding 3/4 in. In February, 1953, seepage of 0.1 gal per min was reported near the right abutment, where cracking had occurred in 1947. Within two weeks the crack must have healed itself because the leakage ceased. In March, 1954, new creeks appeared near the left abutment. A leak of 1.7 gal per min was discovered, increasing to 2.6 gal per min the next day. Sixty cubic feet of cement slurry was pumped into the cracks, and the leak totally stopped. This prompt action probably saved the dam from being breached.

Willard Dam Test Embankment.—The writers did not visit this site. The following information was compiled from (25) and from other data made available by the United States Bureau of Reclamation (USBR).

Willard Dam, when constructed, will enclose 11,000 acres of Willard Bay near Willard, Utah, forming a fresh water reservoir of 215,000 acre-ft capacity on the east shore of Great Salt Lake. The dam will be 31 ft high, 12 miles long, and will contain 12,000,000 cu yd of material. Pre-construction stability analyses (based on no drainage) indicated failure at embankment heights of 18 ft to 30 ft. Therefore, a test embankment 35 ft high was constructed on representative weak lacustrine silts and clays, at least 100 ft thick, in order to examine the feasibility of the project.

The test embankment covered an area 570 ft by 800 ft with a crest 30 ft wide and 240 ft long. The crest length was selected as a minimum for two-dimensional triangular strip loading (Fig. 27). Exploration of the subsurface conditions disclosed 100 ft of organic silt, inorganic silt and clay, and peat containing zones of marsh gas, overlying a sand stratum of undetermined thickness. Conditions beneath the center of the embankment are shown in Fig. 28.

To support the heavy construction equipment a pad of embankment material was placed in two 3-ft lifts and then in 12 in. layers. When construction was begun on June 30, 1955, large capacity earth-moving equipment mired in the wet subsoil of the borrow areas. Haul roads were built and lighter equipment brought in. Fill was placed slowly until July 27, 1955, and

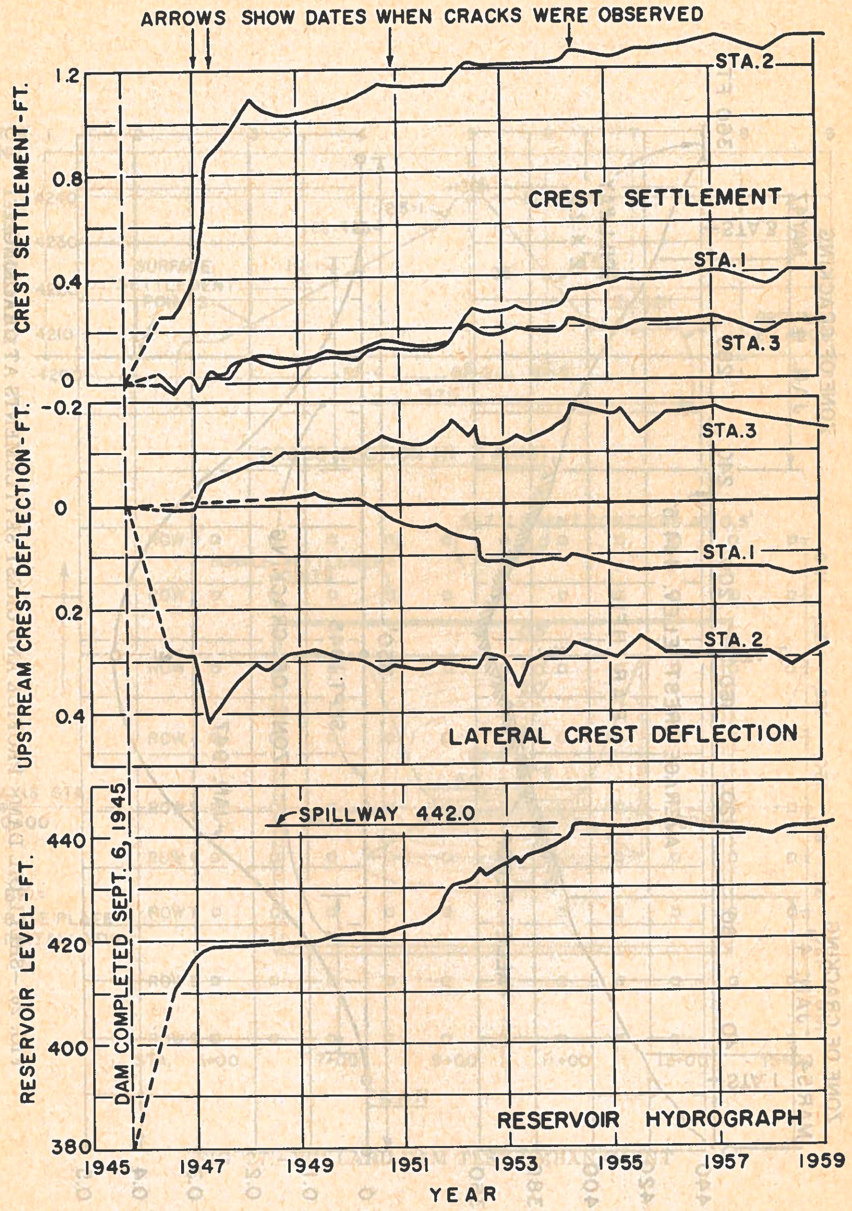


FIG. 25.—SHELL OIL DAM: CREST MOVEMENTS

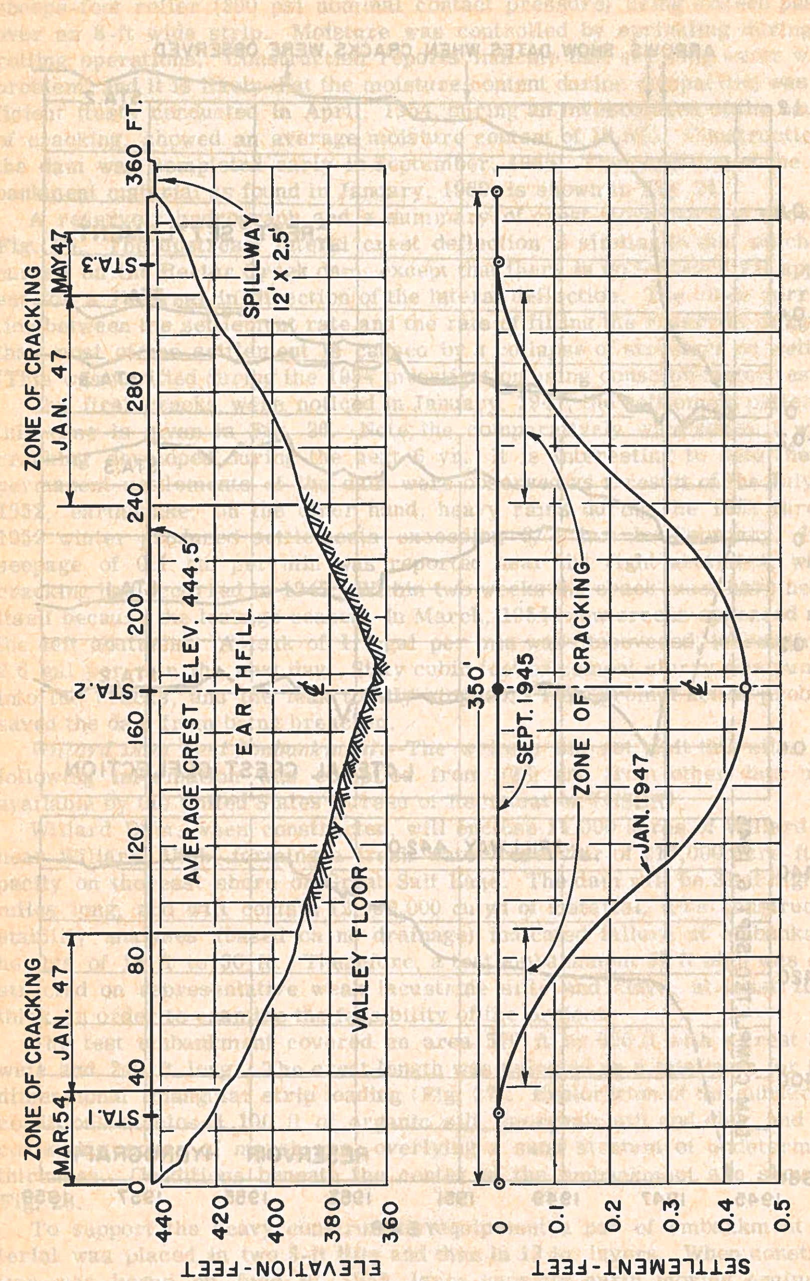


FIG. 26.—SHELL OIL DAM: PROFILE AND CREST SETTLEMENTS AT CRACKING

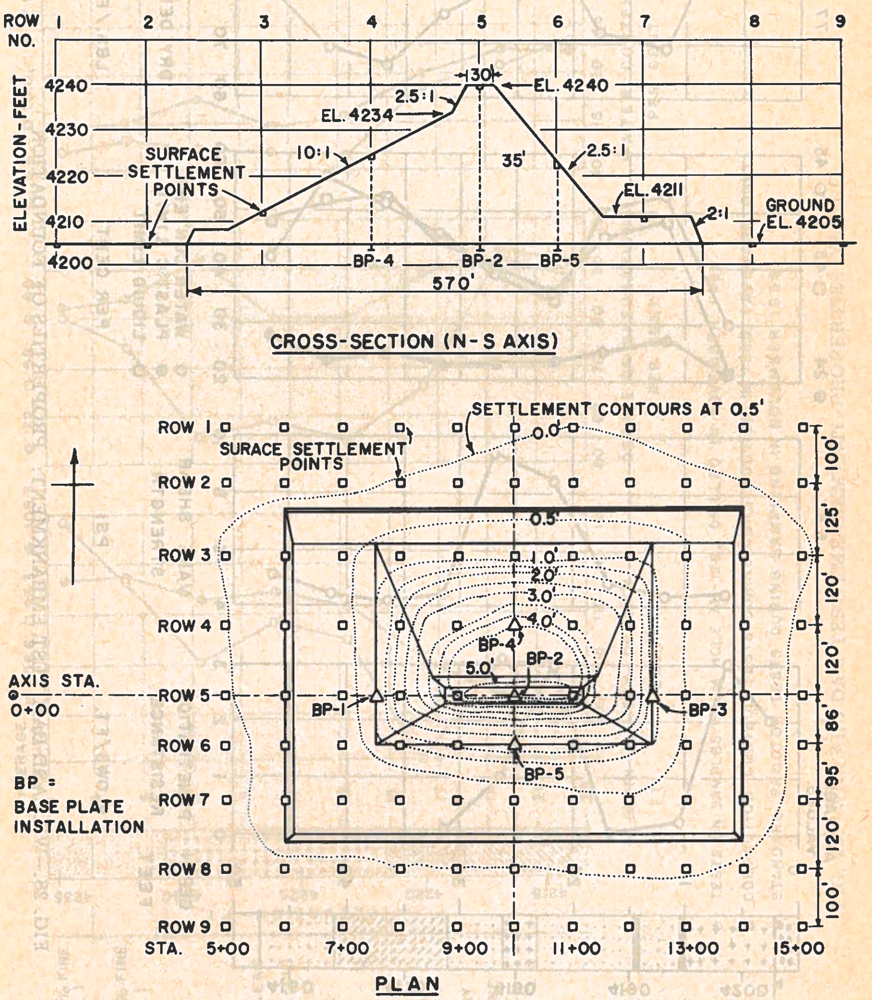


FIG. 27.—WILLARD DAM TEST EMBANKMENT

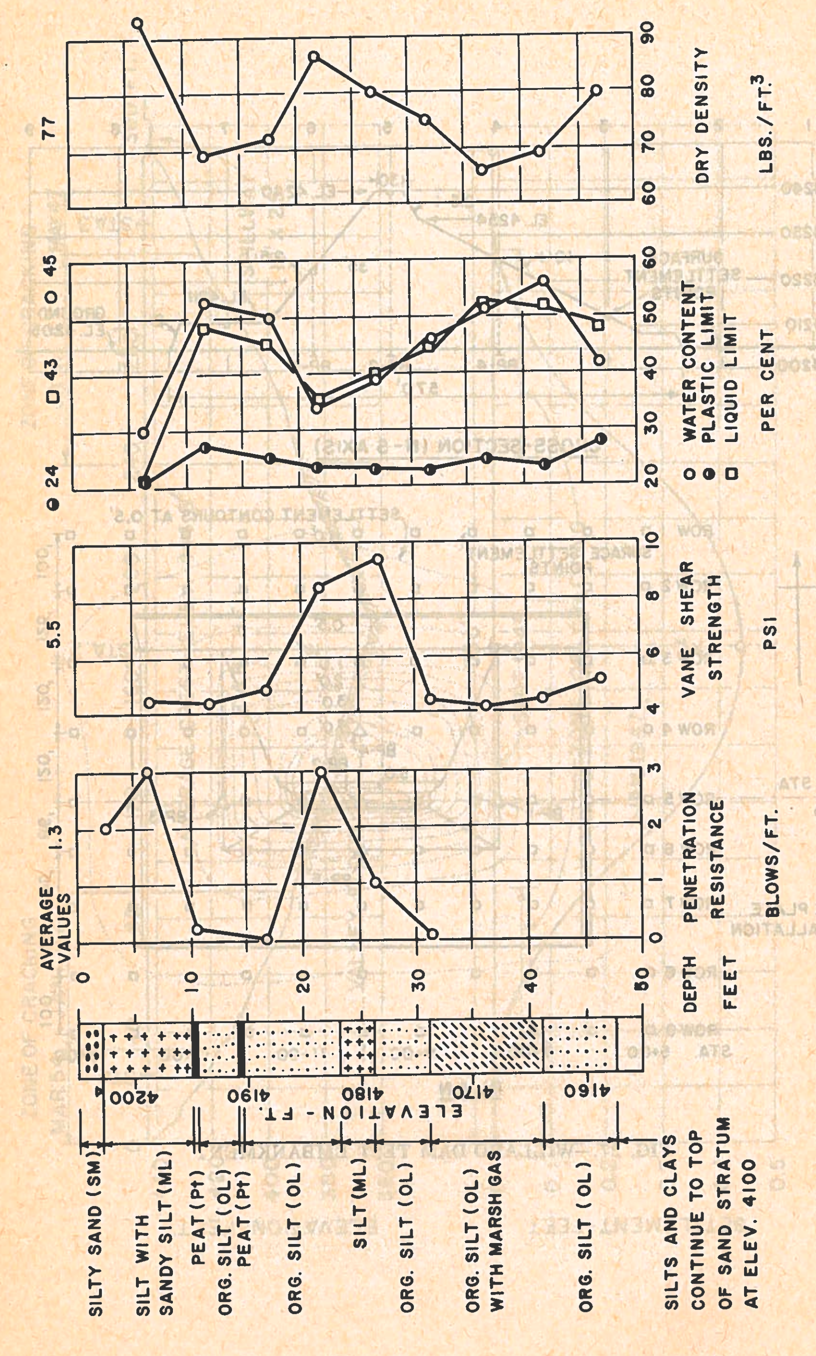


FIG. 28.—WILLARD DAM TEST EMBANKMENT: PROPERTIES OF FOUNDATION

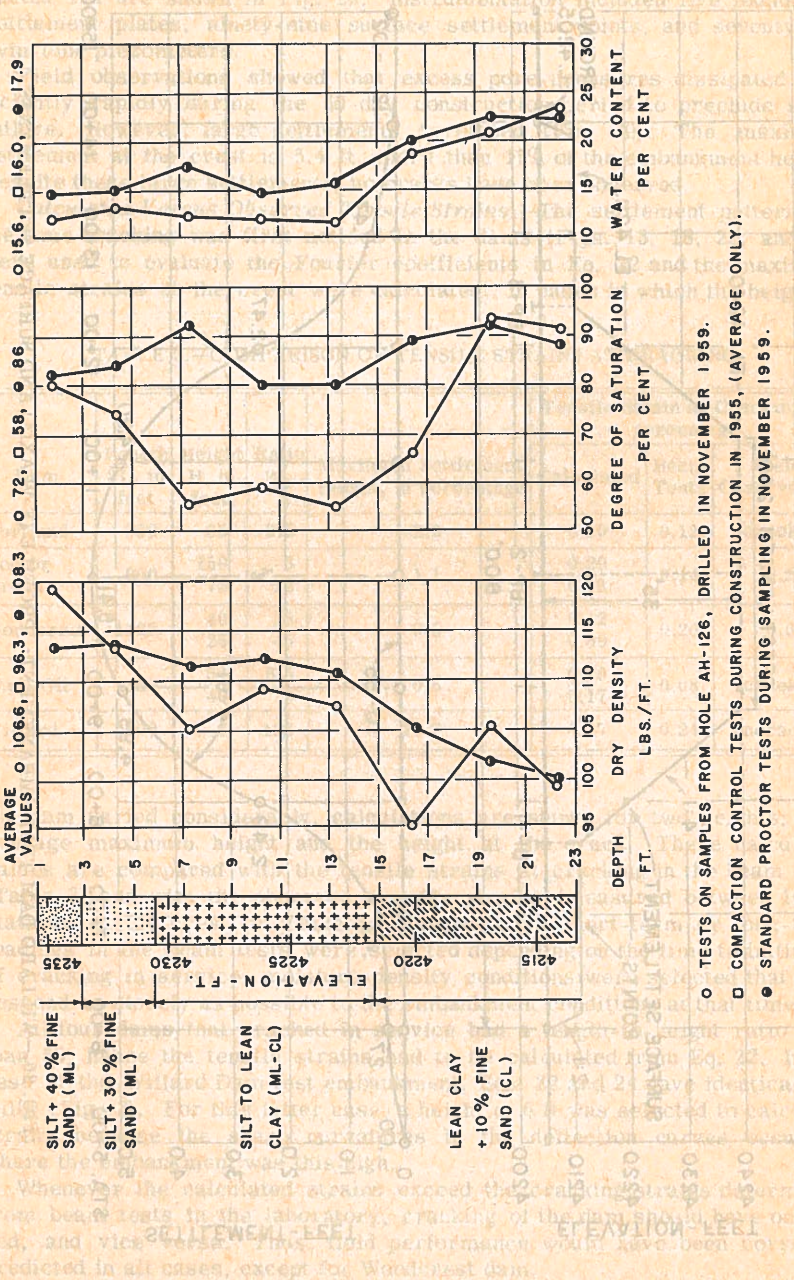


FIG. 29.—WILLARD DAM TEST EMBANKMENT: PROPERTIES OF FILL

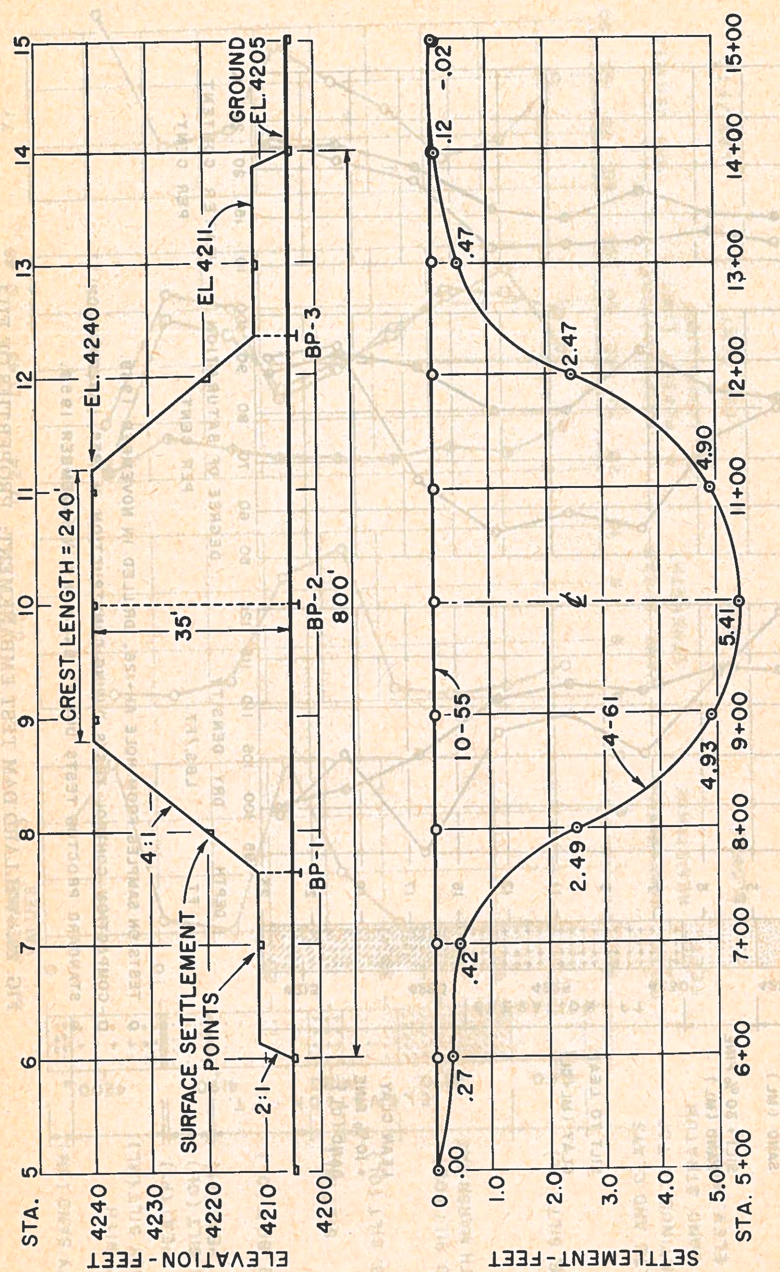


FIG. 30.—WILLARD DAM TEST EMBANKMENT: PROFILE (E-W AXIA) AND SURFACE SETTLEMENTS

then continued rapidly to completion on September 24, 1955. The major part of the embankment was placed in 60 days. Typical conditions of the compacted fill are shown in Fig. 29. Instrumentation included five foundation settlement plates, ninety-nine surface settlement points, and seventy-two twin-tube piezometers.

Field observations showed that excess pore pressures dissipated sufficiently rapidly during the 60-day construction period to preclude slope failure. However, large settlements developed (Fig. 30). The maximum settlement at the crest is 5.4 ft, more than 15% of the embankment height. Despite these large settlements, no cracks have been observed.

Calculated Versus Observed Tensile Strains.—The settlement patterns at the time cracking was first noticed in the dams (Figs. 13, 18, 22, and 26) were used to evaluate the Fourier coefficients in Eq. 22 and the maximum tensile strains at the crest were calculated. In cases in which the height of

TABLE 5.—COMPARISON OF TENSILE STRAINS AT CRACKING

Dam	Length 2L, in feet	Height H, in feet	Ratio $\frac{2L}{H}$	Maximum Settlement Height, in percentage	Tensile Strain at Cracking, in percentage		
					Calculated	Beam Tests	Field Observations
Portland	330	35	9.5	2.2	0.30	0.12	cracked
Rector Creek	900	150 75	6 12	1.1	0.29 0.23	0.14	0.24
Woodcrest	1125	40 25	28 45	0.5	0.12 0.09	0.20	0.09
Shell Oil	350	54 40	6.5 9	0.6	0.20 0.17	0.08	cracked
Willard	800	6	133	15.7	0.17	0.24	uncracked

the dam varied considerably, calculations are shown for two heights: The average maximum height and the height at the crack. These calculated values are compared with the tensile strains at cracking in the beam tests (Table 3) and with the observed average strains (measured between 100-ft stations at the crest) in Table 5. Results from short-term or long-term loadings in the beam tests were selected depending on the time to initiation of cracking in service; moisture-density conditions were selected that correspond as closely as possible to the embankment conditions at that time.

All four dams that cracked in service had a length-to-height ratio less than 50, hence the tensile strains had to be calculated from Eq. 22. In the case of the Willard Dam test embankment, Eqs. 22 and 24 gave identical results (Fig. 3). For this latter case, a height of 6 ft was selected to calculate strains because the sharp curvatures in the deflection curves occurred where the embankment was this high.

Whenever the calculated strains exceed the cracking strains determined from beam tests in the laboratory, cracking of the dam should have occurred, and vice versa. Thus, field performance would have been correctly predicted in all cases, except for Woodcrest dam.

EXAMINATION OF RESULTS

Of the five assumptions made previously, in the section entitled "Theory," numbers 4 and 5 are of little consequence. The observed settlement curves (Figs. 13, 18, 22, 26, and 30) attest to the validity of assumption 3. An earth dam that is compacted properly with sheepsfoot rollers will be as homogeneous and isotropic as any natural deposit is likely to be. (A core can be treated as a separate entity.) Figs. 7 and 8 show that Hooke's law is approximately valid to the point of cracking in tension; also, Young's modulii in tension and compression are approximately equal (Table 3). Accordingly, assumption 1 is considered to be in tolerable agreement with reality.

The validity of assumption 2 cannot be evaluated directly, particularly the effect of replacing a trapezoidal section of variable height by a rectangular section of constant height. In cases in which the span to height ratio is less than 40, substantial variations in the value of the height used has only a secondary influence on the calculated maximum tensile strains (Eq. 22 and Table 5). When this ratio is greater than 60, the calculated strains are approximately proportional to the height (Eq. 24). For long spans and appreciable height variations, a more sophisticated theory would be useful.

The tensile strains at failure, whether calculated from the measured deflection curves, determined from laboratory beam tests, or measured directly between 100-ft stations on the crest, are comparatively small. The maximum range encountered in this study was 0.05% to 0.33%. In all cases, brittle fracture was observed. The ratio of the compressive strain at failure to the tensile strain at cracking ranged from approximately 8 to 80 and showed no consistent pattern in relation to the index properties of the soils studied. Accordingly, the potential flexibility of a compacted clay--from the standpoint of its tensile failure strain--cannot be assessed on the basis of either soil characteristics or compressive stress-strain relations.

The influence of compaction conditions (water content and compactive effort) on the tensile strains at cracking is illustrated in Fig. 31. At comparable water contents, increasing the compactive effort from standard Proctor to modified AASHO approximately halved the flexibility characteristics of the limestone residual clay. Compaction 2% dry of optimum moisture content reduces the flexibility approximately 40%, and compaction 2% wet of optimum caused a slight reduction in flexibility for short-term tests. Thus, if settlement of the dam will occur rapidly (due to collapse of structure on wetting) no improvement in flexibility would be achieved by compacting the limestone clay 2% to 3% wet of optimum.

The effect of time on the tensile strain at cracking is illustrated in Fig. 32 for compaction conditions corresponding to standard Proctor optimum. The importance of avoiding rapid settlements is evident. Again, the plasticity characteristics prove to be unreliable indicators of potential flexibility.

Examination of Table 5 indicates that, despite the simplifications used in the analysis and laboratory tests, performance of the embankments would have been correctly predicted on a quantitative basis in four of the five cases studied. In the case of Woodcrest dam, it is likely that the moisture content in the beam tests did not correspond to the moisture content of the embankment at the time of cracking; also, the number of settlement points is hardly sufficient to define the settlement curve reliably. It is felt that the results demonstrate that the proposed analytical and laboratory techniques are ap-

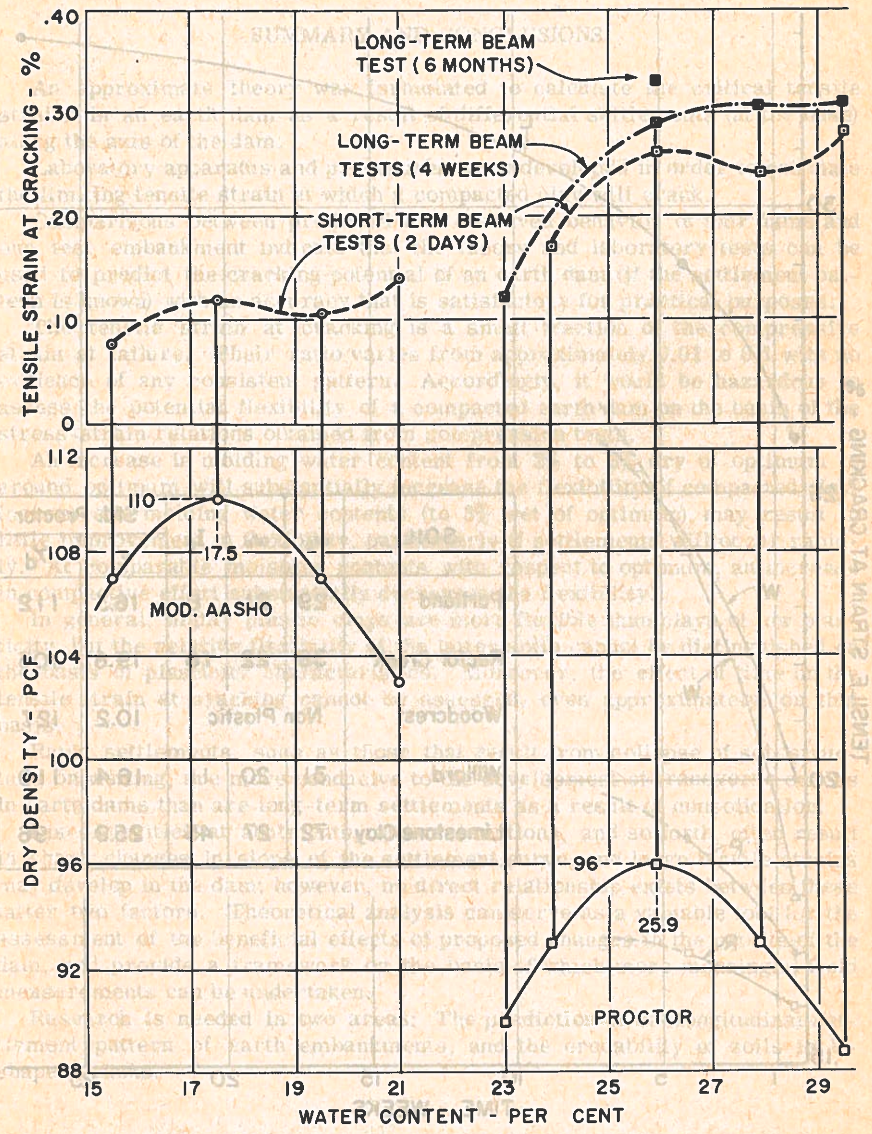
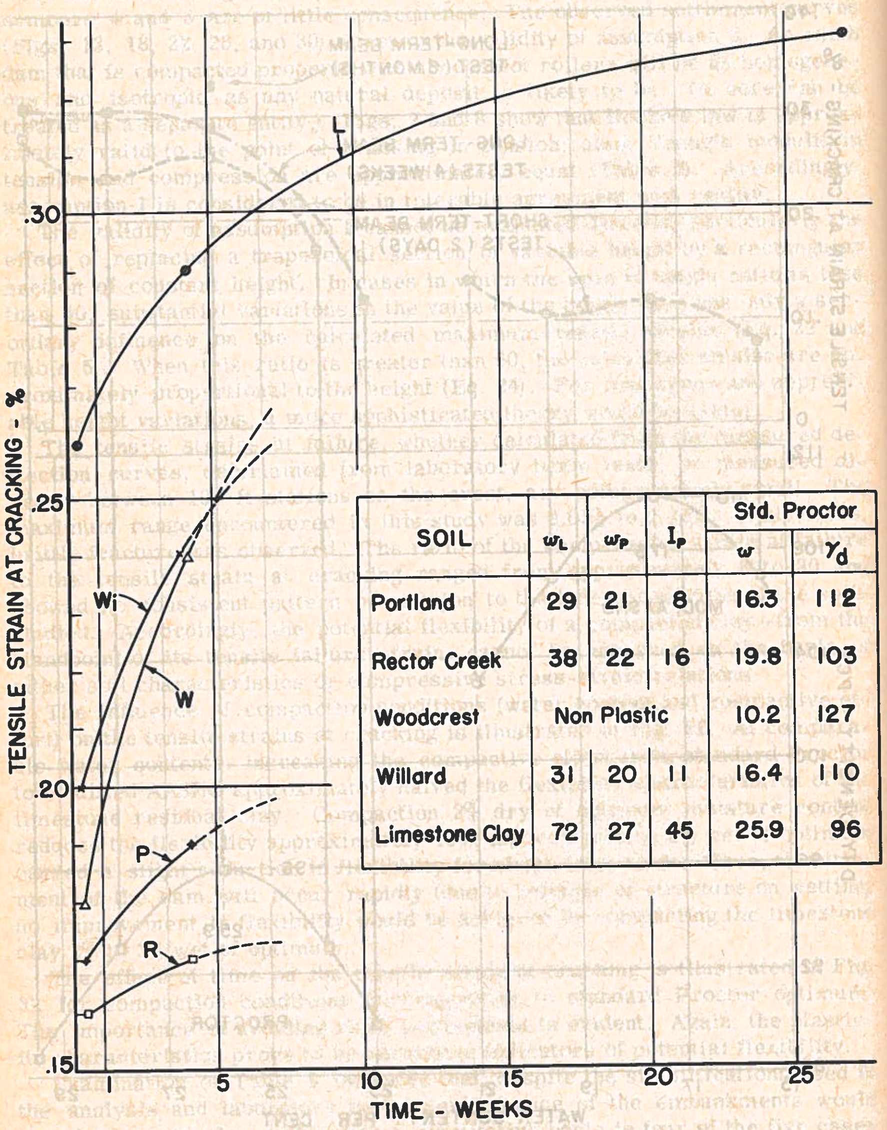


FIG. 31.—EFFECT OF COMPACTION CONDITIONS ON TENSILE STRAIN AT CRACKING: LIMESTONE CLAY



SOIL	ω_L	ω_P	I_P	Std. Proctor ω	Std. Proctor γ_d
Portland	29	21	8	16.3	112
Rector Creek	38	22	16	19.8	103
Woodcrest	Non Plastic			10.2	127
Willard	31	20	11	16.4	110
Limestone Clay	72	27	45	25.9	96

FIG. 32.—EFFECT OF TIME ON TENSILE STRAIN AT CRACKING

proximately valid and can serve as a working tool to assess potential transverse cracking in earth dams as a result of settlements at the base.

SUMMARY AND CONCLUSIONS

An approximate theory was formulated to calculate the critical tensile strains in an earth dam as a result of differential settlements (at the base) along the axis of the dam.

Laboratory apparatus and procedures were developed in order to estimate the limiting tensile strain at which a compacted clay will crack.

Comparisons between predicted and observed behavior of four dams and one test embankment indicate that the theory and laboratory tests can be used to predict the cracking potential of an earth dam (if the settlement pattern is known) with an accuracy that is satisfactory for practical purposes.

The tensile strain at cracking is a small fraction of the compressive strain at failure. Their ratio varies from approximately 0.01 to 0.1 with no evidence of any consistent pattern. Accordingly, it would be hazardous to assess the potential flexibility of a compacted earth dam on the basis of the stress-strain relations obtained from compression tests.

An increase in molding water content from 2% to 3% dry of optimum to around optimum will substantially increase the flexibility of compacted clay; increases in molding water contents (to 3% wet of optimum) may result in little improvement in flexibility, particularly if settlements will occur rapidly. At comparable moisture contents with respect to optimum, an increase in compactive effort substantially decreases the flexibility.

In general, highly plastic clays are more flexible than clays of low plasticity, but the relative flexibility of the latter soils cannot be distinguished on the basis of plasticity characteristics. Moreover, the effect of time on the tensile strain at cracking cannot be assessed, even approximately, on this basis.

Rapid settlements, such as those that result from collapse of soil structure on wetting, are more conducive to the development of transverse cracks in earth dams than are long-term settlements as a result of consolidation.

Discontinuities at abutments, closure sections, and so forth, often result in sharp changes in slope of the settlement curve, and large tensile strains may develop in the dam; however, no direct relationship exists between these latter two factors. Theoretical analysis can serve as a valuable tool for the assessment of the beneficial effects of proposed changes in the profile of the dam, and provide a framework on the basis of which more meaningful field measurements can be undertaken.

Research is needed in two areas: The prediction of the longitudinal settlement pattern of earth embankments, and the erodability of soils in V-shaped cracks.

ACKNOWLEDGMENTS

The writers are indebted to the owners of the dams for permission to make the study and publish the results. Design and performance data for the Willard Dam test embankment are available (2); the performance data were

brought up to date, and 200-lb soil samples were furnished to the writers by the Bureau of Reclamation, U. S. Department of the Interior. This assistance is greatly acknowledged.

The drill rig used and the operators employed for drilling and sampling at the dam sites were furnished by the firm of Woodward-Clyde-Sherard and Associates. For this courtesy, the writers express their sincerest thanks.

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APPENDIX II.--NOTATION

The following symbols have been adopted for use in this paper:

A_n, a_n = coefficients of cosine terms in Fourier series;

B_n, b_n = coefficients of sine terms in Fourier series;

E = Young's modulus;

$F(y)$ = function of y;

$F_n(y)$ = n^{th} derivative of F(y) with respect to y;

$f(x)$ = function of x, representing crest-settlement curve of dam;

$f_n(x)$ = n^{th} derivative of f(x) with respect to x;

G = modulus of rigidity (shear modulus);

H = height of dam;

R = radius of curvature;

u = displacement in x direction;

v = displacement in y direction;

$2L$ = length of dam;

$\alpha = \frac{n\pi}{L}$, in which n represents 1, 2, 3, and so forth;

$\beta = \frac{\sinh \alpha H + \alpha H \cosh \alpha H}{\alpha H \sinh \alpha H}$;

γ_{xy} = shearing strain on x-y plane;

ϵ_x, ϵ_y = normal strains in x and y directions;

μ = Poisson's ratio;

σ_x, σ_y = normal stresses on planes perpendicular to x and y axes;

τ_{xy} = shear stress on x-y plane; and

ϕ = Airy stress function.

Journal of the
SOIL MECHANICS AND FOUNDATIONS DIVISION
Proceedings of the American Society of Civil Engineers

OAHE DAM: GEOLOGY, EMBANKMENT, AND CUT SLOPES[a]

By Donald K. Knight,[1] F. ASCE

FOREWORD

This symposium is composed of three papers originally presented before the Society's first Water Resources Conference in May 1962 in Omaha, Nebr. The paper "An Unusual Spillway," by H. S. Kidd, E. R. Bloomquist, and C. E. Johnson is currently under review by the Soil Mechanics and Foundations Division. The paper "Influence of Shale on Oahe Power Structures Design," by E. A. Johns, R. G. Burnett, and C. L. Craig was published as Proc. Paper 3423 in the February 1963 Journal of the Soil Mechanics and Foundations Division.

SYNOPSIS

General data on the geology of the project are reviewed and the major soils problems of cut slope and embankment stability are examined. The principal

Note.—Discussion open until August 1, 1963. Separate discussions should be submitted for the individual papers in this symposium. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Soil Mechanics and Foundations Division, Proceedings of the American Society of Civil Engineers, Vol. 89, No. SM2, March, 1963.

[a] Presented at the May 1962 ASCE Conference in Omaha, Nebr.

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