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Proceedings of the American Society of Civil Engineers

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DISCUSSION

Journal of the
SOIL MECHANICS AND FOUNDATIONS DIVISION
Proceedings of the American Society of Civil Engineers

STABILITY OF CUTS IN SOFT SOILS

By T. Cameron Kenney,[1] A.M. ASCE

SYNOPSIS

The purpose of this paper is to provide a means by which the stability of slopes, formed as a result of excavating open cuts in normally-consolidated soils, can be quickly estimated. Undrained (rapid-excavation) and long-term conditions of slope stability are considered for both dry and submerged slopes. For the long-term case, different seepage patterns are used, and in addition the overconsolidation effect on the soil, which results from excavation, is studied and found to be generally of negligible importance. The results of these analyses are presented in the form of design curves showing the required slope inclination for different values of s_u/p' for undrained conditions and different values of ϕ_d for long-term conditions. Available field evidence is presented and none of this evidence conflicts with the results of the stability analyses. It is concluded that both the undrained case and the long-term case can provide the most critical conditions for slope stability in open cuts.

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INTRODUCTION

Slope stability has been the subject of much study, and general solutions have been presented previously for the special cases of soils having constant values of undrained shear strength s_u (Taylor[2]) and constant values of the effective stress shear strength parameters c' and ϕ' (Bishop and Morgenstern).[3] The subject of this paper concerns another special case previously studied by Gibson and Morgenstern,[4] that is, the stability of slopes formed as a result of excavating open cuts in normally consolidated clays. Herein, the term "normally consolidated" has been used to describe those soils having the properties and existing under the simplifying conditions listed as follows: (a) The groundwater surface is at the ground surface; (b) the soil has uniform unit weight independent of depth; (c) the ratio of the undrained shear strength of the soil to the effective overburden pressure (s_u/p') is constant and independent of depth; and (d) from (a), (b), and (c); it follows that the effective overburden pressure and the undrained shear strength of the soil increase linearly with depth.

In the subsequent sections, slope-stability calculations will be made for undrained conditions (rapid excavation) and long-term conditions (steady seepage).

Notation.—The letter symbols adopted for use in this paper are defined where they first appear and are arranged alphabetically in the Appendix.

STABILITY ANALYSES

Undrained Conditions.—If excavation is carried out rapidly, the soil will remain in an undrained state during this period. If it is assumed that the soil possesses isotropic strength properties, then the soil may be considered as an isotropic cohesive material and the $\phi = 0$ analysis (Skempton[5]; Bishop and Bjerrum[6]) can be used to estimate the stability of the slopes.

The undrained shear strength s_u of normally-consolidated soils at any depth h_o below the original ground surface can be expressed in terms of the (s_u/p')-ratio and the effective vertical overburden pressure $(\gamma-\gamma_w)h_o$ existing before construction by means of the following equation:

$$s_u = \left(\frac{s_u}{p'}\right) (\gamma-\gamma_w)\ h_o \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

2 "Fundamentals of Soil Mechanics," by D. W. Taylor, John Wiley & Sons, Inc., New York, N. Y., 1948, 700 pp.

3 "Stability Coefficients for Earth Slopes," by A. W. Bishop and N. Morgenstern, Geotechnique, Vol. 10, No. 4, 1960, pp. 129-150.

4 "A Note of the Stability of Cuttings in Normally Consolidated Clays," by R. E. Gibson and N. Morgenstern, Geotechnique, Vol. 12, No. 3, 1962, pp. 212-216.

5 "The ϕ- 0 Analysis of Stability and its Theoretical Basis," by A. W. Skempton, Proceedings, Internatl. Conf. on Soil Mechanics and Foundation Engrg., Vol. 1, Rotterdam, 1948, pp. 72-78.

6 "The Relevance of the Triaxial Test to the Solution of Stability Problems," by A. W. Bishop and L. Bjerrum, Research Conf. on Shear Strength of Cohesive Soils, ASCE, Boulder, Colo., 1960, pp. 437-501.

in which γ denotes the unit weight of soil and γ_w denotes the unit weight of water.

For circular-arc failure surfaces, it can be shown that, for these strength conditions, the critical surface must pass through the toe of the slope or intersect the slope above the toe as for Case 1 in Fig. 1(a), but that the critical surface will not intersect the bottom of the excavations as for Case 2 in Fig.

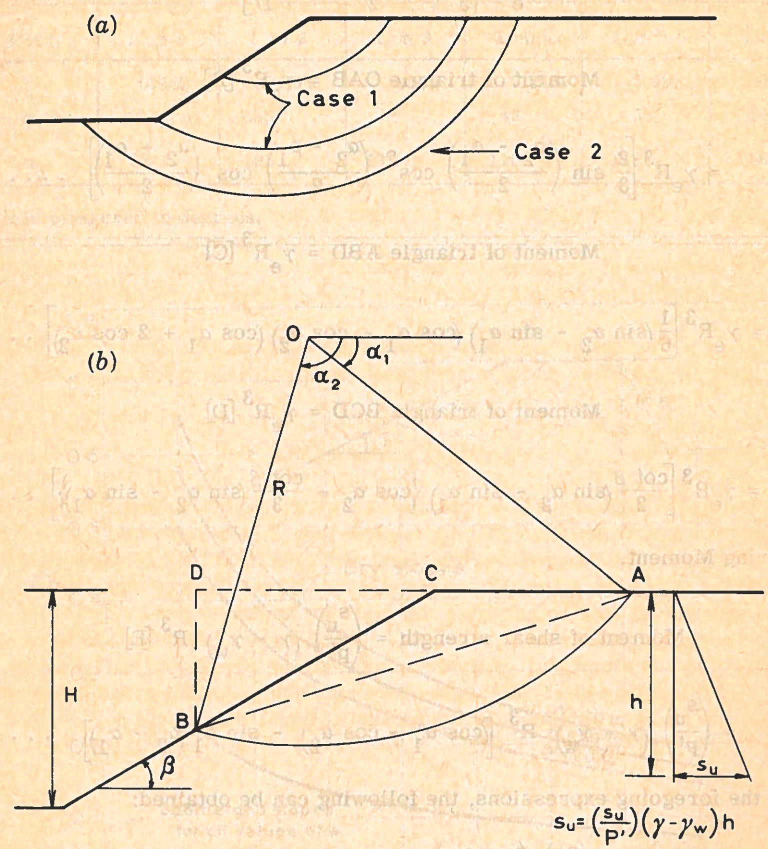

FIG. 1.—STABILITY ANALYSIS FOR UNDRAINED CONDITIONS

1(a). Fig. 1(b) shows a dry slope having an inclination β and being intersected by a circular-arc failure surface. For the purposes of ultimately being able to consider submerged slopes, use will be made of the equivalent unit weight of the soil γ_e for dry-slope conditions. At failure, the moments about point 0 of the disturbing forces and the restoring forces must be equal. These moments can be expressed as follows:

Disturbing Moments.

$$\text{Moment of sector OAB} = \gamma_e R^3 [A]$$

$$= \gamma_e R^3 \left[\frac{1}{3} \left(\sin \alpha_2 - \sin \alpha_1\right)\right] \quad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

$$\text{Moment of triangle OAB} = \gamma_e R^3 [B]$$

$$= \gamma_e R^3 \left[\frac{2}{3} \sin \left(\frac{\alpha_2 - \alpha_1}{2}\right) \cos^2 \left(\frac{\alpha_2 - \alpha_1}{2}\right) \cos \left(\frac{\alpha_2 + \alpha_1}{2}\right)\right] \quad \ldots . \quad (3)$$

$$\text{Moment of triangle ABD} = \gamma_e R^3 [C]$$

$$= \gamma_e R^3 \left[\frac{1}{6} \left(\sin \alpha_2 - \sin \alpha_1\right) \left(\cos \alpha_1 - \cos \alpha_2\right) \left(\cos \alpha_1 + 2 \cos \alpha_2\right)\right] \quad \ldots \quad (4)$$

$$\text{Moment of triangle BCD} = \gamma_e R^3 [D]$$

$$= \gamma_e R^3 \left[\frac{\cot \beta}{2} \left(\sin \alpha_2 - \sin \alpha_1\right) \left\{\cos \alpha_2 + \frac{\cot \beta}{3} \left(\sin \alpha_2 - \sin \alpha_1\right)\right\}\right] \quad \ldots \quad (5)$$

Restoring Moment.

$$\text{Moment of shear strength} = \left(\frac{s_u}{p'}\right) \left(\gamma - \gamma_w\right) R^3 [E]$$

$$= \left(\frac{s_u}{p'}\right) \left(\gamma - \gamma_w\right) R^3 \left[\left(\cos \alpha_1 - \cos \alpha_2\right) - \sin \alpha_1 \left(\alpha_2 - \alpha_1\right)\right] \quad \ldots . \quad (6)$$

From the foregoing expressions, the following can be obtained:

$$\left(\frac{s_u}{p'}\right) \left(\frac{\gamma - \gamma_w}{\gamma_e}\right) = \frac{A - B + C - D}{E} = n \quad \ldots\ldots\ldots\ldots\ldots \quad (7)$$

in which A, B, C, D, and E are the expressions contained within the brackets in the previous expressions. A similar equation was previously derived, evaluated, and examined by Gibson and Morgenstern.[4] Eq. 7 shows that n is dependent only on the inclination of the slope (cot β) and the orientation of the slip surface (α_1 and α_2) and is independent of the depth of the excavation (H) and the depth of the slip surface (R).

The maximum values of n required for the stability of slopes having different inclinations were obtained by trial by evaluating Eq. 7 numerically for

TABLE 1.—RESULTS OF STABILITY CALCULATIONS FOR UNDRAINED CONDITIONS

Cot β	1 on 1	1 on 2	1 on 3	1 on 4	1 on 5.5	1 on 8
n	0.243	0.171	0.135	0.115	0.092	0.071
α_1	31	35	38	42	45	50
α_2	92	105	112	115	118	118

α is measured in degrees.

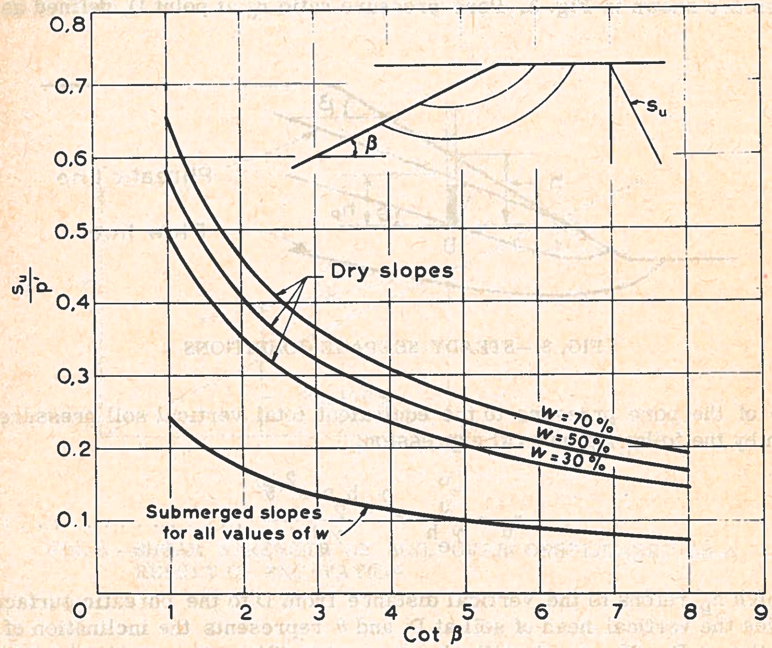

FIG. 2.—RESULTS OF STABILITY CALCULATIONS FOR UNDRAINED CONDITIONS

different values of α_1 and α_2. These maximum values of n, together with the values of α_1 and α_2 for the critical slip surfaces, are listed in Table 1.

For dry slopes where there is no external pressure applied to the slopes $\gamma_e = \gamma$. For this condition, the (s_u/p')-values necessary for slope stability were calculated from the values of n listed in Table 1 using saturated soil unit weights corresponding to water contents of w = 30%, 50%, and 70% and a specific gravity of the soil particles G = 2.70. These results are plotted in Fig. 2. For the case of submergence, it has been shown[2] that the stability conditions of a submerged slope of soil having a unit weight γ are exactly the same as those for a dry slope having an equivalent unit weight $\gamma_e = (\gamma - \gamma_w)$. Thus, from Eq. 7 it is seen that the stability of a submerged slope for undrained conditions is independent of the unit weight of the soil, and the (s_u/p')-values necessary for slope stability are numerically equal to the values of n listed in Table 1. These values are also plotted in Fig. 2.

Long-term Conditions.—The estimation of the long-term stability of slopes in soils requires the consideration of two factors, or pore pressure and shear strength.

Pore Pressure.—The pore pressure for steady-seepage conditions for dry slopes are shown in Fig. 3. Pore-pressure ratio r_u at point D, defined as the

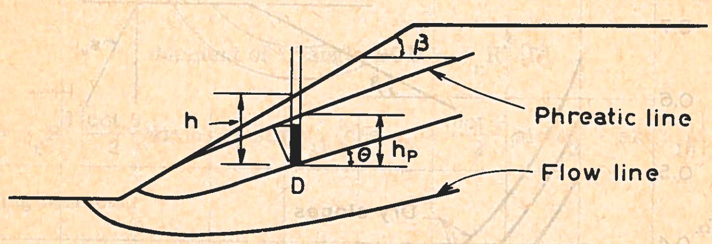

FIG. 3.—STEADY-SEEPAGE CONDITIONS

ratio of the pore pressure to the equivalent total vertical soil pressure, is given by the following general expression:

$$r_u = \frac{u}{\gamma_e h} \approx \frac{\gamma_w h_p \cos^2 \theta}{\gamma_e h} \quad \ldots\ldots\ldots\ldots\ldots (8)$$

in which h_p refers to the vertical distance from D to the phreatic surface, h denotes the vertical head of soil at D, and θ represents the inclination of the flow line at D. The most critical seepage conditions are obtained when the ground-water surface is at the ground surface, that is, $h_p = h$. For this condition, two seepage cases will be considered: flow parallel to the slope and horizontal flow.

(1) Parallel flow, in which $h_p = h$, $\theta = \beta$: $\gamma_e = \gamma$ and

$$r_u = \frac{\gamma_w}{\gamma} \cos^2 \beta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

(2) Horizontal flow, in which $h_p = h$, $\theta = 0$: $\gamma_e = \gamma$ and

$$r_u = \frac{\gamma_w}{\gamma} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

For the case of submerged slopes, the stress conditions are similar to those for dry slopes in which the excess pore pressures are equal to zero ($h_p = 0$)

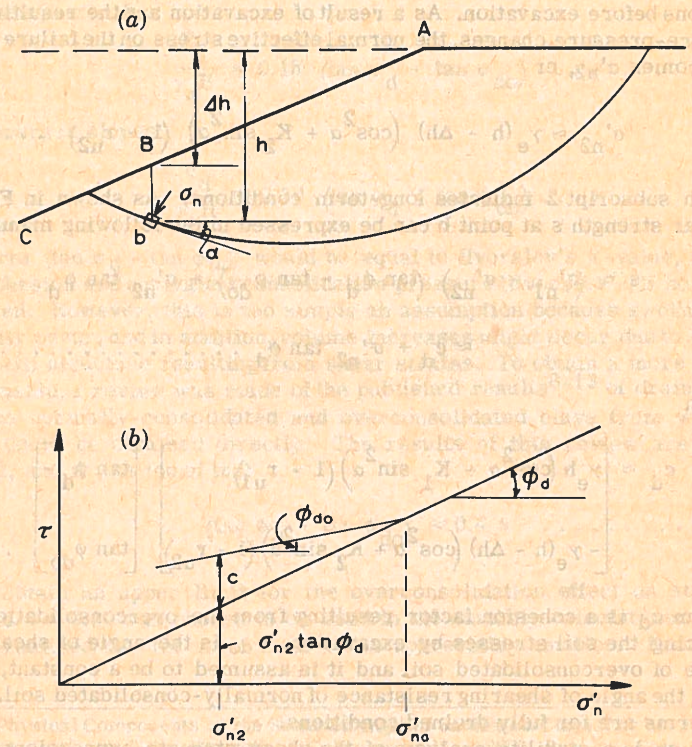

FIG. 4.—SHEAR STRENGTH OF SOIL OVERCONSOLIDATED AS A RESULT OF EXCAVATION

and the equivalent unit weight of the soil is $\gamma_e = (\gamma - \gamma_w)$.

Submergence, in which $h_p = 0$: $\gamma_e = \gamma - \gamma_w$ and

$$r_u = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

Shear Strength.—Fig. 4(a) shows a slope formed as a result of excavation of the soil from above surface ABC. Below and to the right side of A the soil remains normally consolidated, whereas below surface ABC the soil is in a state of overconsolidation as a result of the excavation operations. Consider the element b on some potential failure surface. The consolidation stress normal to this surface before excavation began is σ'_{n1}:

$$\sigma'_{n1} = \gamma_e h \left(\cos^2\alpha + K_1 \sin^2\alpha\right) \left(1 - r_{u1}\right) \quad \dots\dots\dots(12)$$

in which α denotes the slope angle of the failure surface at b, K represents the ratio of horizontal and vertical effective stresses, and subscript 1 indicates conditions before excavation. As a result of excavation and the resulting long-term pore-pressure changes, the normal effective stress on the failure surface at b becomes σ'_{n2}, or

$$\sigma'_{n2} \approx \gamma_e (h - \Delta h) \left(\cos^2\alpha + K_2 \sin^2\alpha\right) \left(1 - r_{u2}\right) \quad \dots\dots(13)$$

in which subscript 2 indicates long-term conditions. As shown in Fig. 4(b), the shear strength s at point b can be expressed in the following manner:

$$s \approx \left(\sigma'_{n1} - \sigma'_{n2}\right) \left(\tan\phi_d - \tan\phi_{do}\right) + \sigma'_{n2} \tan\phi_d$$

$$\approx c_d + \sigma'_{n2} \tan\phi_d \quad \dots\dots\dots\dots\dots\dots\dots(14)$$

in which

$$c_d \approx \begin{bmatrix} \gamma_e h \left(\cos^2\alpha + K_1 \sin^2\alpha\right)\left(1 - r_{u1}\right) \\ - \gamma_e (h - \Delta h) \left(\cos^2\alpha + K_2 \sin^2\alpha\right)\left(1 - r_{u2}\right) \end{bmatrix} \begin{bmatrix} \tan\phi_d - \\ \tan\phi_{do} \end{bmatrix} \quad \dots(15)$$

The term c_d is a cohesion factor resulting from the overconsolidation effect of reducing the soil stresses by excavation, ϕ_{do} is the angle of shearing resistance of overconsolidated soil and it is assumed to be a constant, and ϕ_d denotes the angle of shearing resistance of normally-consolidated soil. These three terms are for fully drained conditions.

The use in a stability analysis of the shear strength expressions in Eqs. 14 and 15 would require a great deal of time. To avoid this, a study was made to determine the average value of c_d which acted along the entire length of the failure surface. This average value is denoted by $\bar{c}_d$, and with its use the shear strength at any point along the failure surface can be approximately expressed as follows:

$$s = \bar{c}_d + \sigma'_n \tan\phi_d \quad \dots\dots\dots\dots\dots(16)$$

Values of $\bar{c}_d$ were obtained numerically by evaluating Eq. 15 along potentially critical slip surfaces for slopes having inclinations between cot β = 2 and 8, for soil unit weights corresponding to water contents from 30% to 70%, and for dry and submerged slopes having the pore-pressure conditions previously described. To evaluate Eq. 15 it was assumed that $K_1 = K_2 = 0.5$ and

this assumption would tend to overestimate $\bar{c}_d$ because actually K_1 for normally-consolidated soil is less than K_2 for overconsolidated soil. It was found that the calculated $\bar{c}_d$ values were essentially independent of the magnitude of the slope inclinations that were studied, and were primarily dependent on the unit weight of the soil and the soil properties ϕ_d and ϕ_{do}. (Further details concerning these calculations will not be given because, as it will be noted subsequently, the total $\bar{c}_d$ effect on the stability of slopes excavated in normally-consolidated soil is practically negligible). From the results of these calculations the term $\bar{c}_d$ can be approximately expressed as follows:

Dry slopes:

$$\frac{\bar{c}_d}{\gamma_e H} \approx 0.15 \left(\tan\phi_d - \tan\phi_{do}\right) \quad \dots\dots\dots\dots(17a)$$

Submerged slopes:

$$\frac{\bar{c}_d}{\gamma_e H} \approx 0.30 \left(\tan\phi_d - \tan\phi_{do}\right) \quad \dots\dots\dots\dots(17b)$$

The term $(\tan\phi_d - \tan\phi_{do})$ would be equal to Hvorslev's κ-value, relating true cohesion and equivalent consolidation pressure (Hvorslev[7]), if no swelling occurred. However, this is too simple an assumption because swelling would certainly occur, and in addition, volume increases might occur due to dilatancy of the soil structure resulting from shear strains. To obtain a more realistic relationship, a review was made of the published results[8-13] of drained shear tests on normally-consolidated and overconsolidated clays from which this factor could be obtained directly. The results of this review are given in Table 2, and it was found that

$$\left(\tan\phi_d - \tan\phi_{do}\right) \approx 0.4\,\kappa \quad \dots\dots\dots\dots\dots(18)$$

To obtain an upper limit for the overconsolidation effect on stability, a value of κ = 0.10 was chosen to be used in the stability calculations, although it was realized that for most soils κ was appreciably smaller. From Eq. 18,

7 "Physical Components of the Shear Strength of Saturated Clays," by M. J. Horslev, Research Conf. on Shear Strength of Cohesive Soils, ASCE, Boulder, Colo., 1960, pp. 169-273.

8 "Theoretical and Experimental Investigations on the Shear Strength of Soils," by L. Bjerrum, Norwegian Geotechnical Institute Publication No. 6, Oslo, Norway, 1954, 113 pp.

9 "Triaxial Compression and Extension Tests on Remoulded Saturated Clay," by R. H. G. Parry, Geotechnique, Vol. 10, No. 4, 1960, pp. 166-180.

10 "The Effect of Overconsolidation on the Shear Strength Characteristics of an Undisturbed Oslo Clay," by N. E. Simons, Research Conf. on Shear Strength of Cohesive Soils, ASCE, Boulder, Colo., 1960, pp. 747-763.

11 "The Strength and Structure of Kaolin," by J. R. Morgan, thesis presented to the University of Melbourne, at Melbourne, Australia, in 1961, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

12 "The Shear Strength Properties of Calcium Illite," by R. E. Olson, Geotechnique, Vol. 12, No. 1, 1962, pp. 23-43.

13 "The Compressive Strength of Remoulded Niagara Clay," by D. J. Bazett and S. W. Smotrych, paper presented at the Annual Meeting of the Amer. Soc. for Testing Materials, 1960.

TABLE 2.—VALUES OF $\tan \phi_d - \tan \phi_{do}$ AND κ FROM RESULTS OF DRAINED SHEAR TESTS

Soil		Test	w_L	w_P	k' (a)	κ	k'/κ	Reference
Allschwyel tile	(R)	D	61	18	0.04	0.10 (b)	0.4	8
Zürich talus	(R)	D	41	14	0.04	0.08 (b)	0.5	8
Vienna V	(R)	D	47	22	0.04	0.10 (b)	0.4	7
Little Belt	(R)	D	127	36	0.05	0.15 (b)	0.3	7
Weald	(R)	T	43	18	0.02	0.05 (c)	0.4	9
Oslo	(U)	T	39	21	0.02	–	—	10
Kaolin	(R)	T	59	31	0.02	0.05	0.4	11
Illite	(R)	T	85	37	0.02	-	-	12
Niagara	(R)	T	41	20	0.05	0.10 (c)	0.5	13

(a) $k' = \tan \phi_d - \tan \phi_{do}$

(b) $\kappa = c_e/\sigma_{1c}$ (anisotropic consolidation stresses)

(c) $\kappa = c_e/\sigma_{3c}$ (isotropic consolidation stresses)

R denotes remoulded

U denotes undisturbed

D denotes direct shear

T denotes triaxial compression

this gives an upper-limit value of $(\tan \phi_d - \tan \phi_{do}) = 0.04$, and from Eqs. 17, the following values of $c_d/\gamma_e H$ were obtained:

Dry slopes:

$$\frac{\bar{c}_d}{\gamma_e H} = 0.006 \qquad (19a)$$

and Submerged slopes:

$$\frac{\bar{c}_d}{\gamma_e H} = 0.012 \qquad (19b)$$

To obtain the lower limit of the overconsolidation effect, a value of $(\tan \phi_d - \tan \phi_{do}) = 0$ was also considered, and this gave the condition for soil having negligible cohesive properties or which had reverted to a pseudo normally-consolidated state, in which

$$\frac{\bar{c}_d}{\gamma_e H} = 0 \qquad (20)$$

Calculations.—For the condition in which the average apparent cohesion of the soil is equal to zero (Eq. 20) the implied failure surface is a plane parallel to the slope, and the value of $\tan \phi_d$ required for stability, can be obtained

directly from the expression (Bishop and Morgenstern[3]):

$$\tan \phi_d = \frac{\tan \beta}{\left(1 - r_u \sec^2 \beta\right)} \qquad (21)$$

Values of ϕ_d were obtained from this equation for different slope inclinations, for the pore-pressure and density conditions given by Eqs. 9, 10, and 11 and for three values of bulk density corresponding to water contents of 30%, 50%, and 70% and G = 2.70; the results are given in Fig. 5(a).

For the condition where the average apparent cohesion of the soil is greater than zero (Eqs. 19), stability analyses were made with the use of the stability coefficients presented by Bishop and Morgenstern.[3] Values of ϕ_d required for stability were obtained for different slope inclinations, for the pore-pressure and density conditions given by Eqs. 9, 10, and 11, and for bulk densities corresponding to water contents equal to 30%, 50%, and 70% and $\rho = 2.70$. For slope inclinations less than 1 on 5 (this is the flattest slope studied by Bishop and Morgenstern), ϕ_d-values were obtained by extrapolation, using as a guide the results obtained from Eq. 21 and shown in Fig. 5(a); the results of these calculations are given in Fig. 5(b). Values of ϕ_d for values of $(\tan \phi_d - \tan \phi_{do})$ between 0 and 0.04 (that is, for soils having κ-values between 0 and 0.10) can be obtained sufficiently accurately by interpolation between the results given in Figs. 5(a) and 5(b).

SLOPE DESIGN

An examination of the data in Figs. 5(a) and 5(b) for slopes having the same inclination and soils having the same values of $(\tan \phi_d - \tan \phi_{do})$ indicates that the necessary ϕ_d-values for horizontal and parallel seepage conditions are similar, the difference being of the order of 1° or 2°. It can also be seen by comparing slopes having the same inclination and pore-pressure conditions that overconsolidation of the soil corresponding to a value of $(\tan \phi_d - \tan \phi_{do}) = 0.04$ causes a reduction of the necessary ϕ_d-value by 2° to 3°. In most cases, $(\tan \phi_d - \tan \phi_{do})$ will be appreciably less than 0.04 and therefore the overconsolidation effect on ϕ_d for most cases will be appreciably less than 2° or 3°. In view of these small differences, and in view of the fact that these differences are of the same order of magnitude as, or smaller than, the variation of ϕ_d in a natural soil deposit, it would appear justifiable for the purpose of rapid calculations to ignore the overconsolidation effect. Also, for the case of dry slopes, it would be justifiable to use the results for only one seepage condition, such as parallel seepage. Thus, for the long-term condition, values of ϕ_d necessary for stability can be obtained with sufficient accuracy directly from Fig. 5(a) or from Eq. 21.

The (s_u/p') versus $\cot \beta$ data for undrained conditions given in Fig. 2 and the ϕ_d versus $\cot \beta$ data for long-term conditions given in Fig. 5(a), have been combined in Fig. 6. From Fig. 6, knowing the average water content, the average (s_u/p')-ratio, and the average ϕ_d-value, the critical value of slope inclination ($\cot \beta$) can be obtained, and also it can be ascertained whether the undrained or the long-term condition provides the most critical stability case.

As an example, consider the case in which a dry slope is to be excavated rapidly in a soil deposit having the following properties: w = 50%, s_u/p'

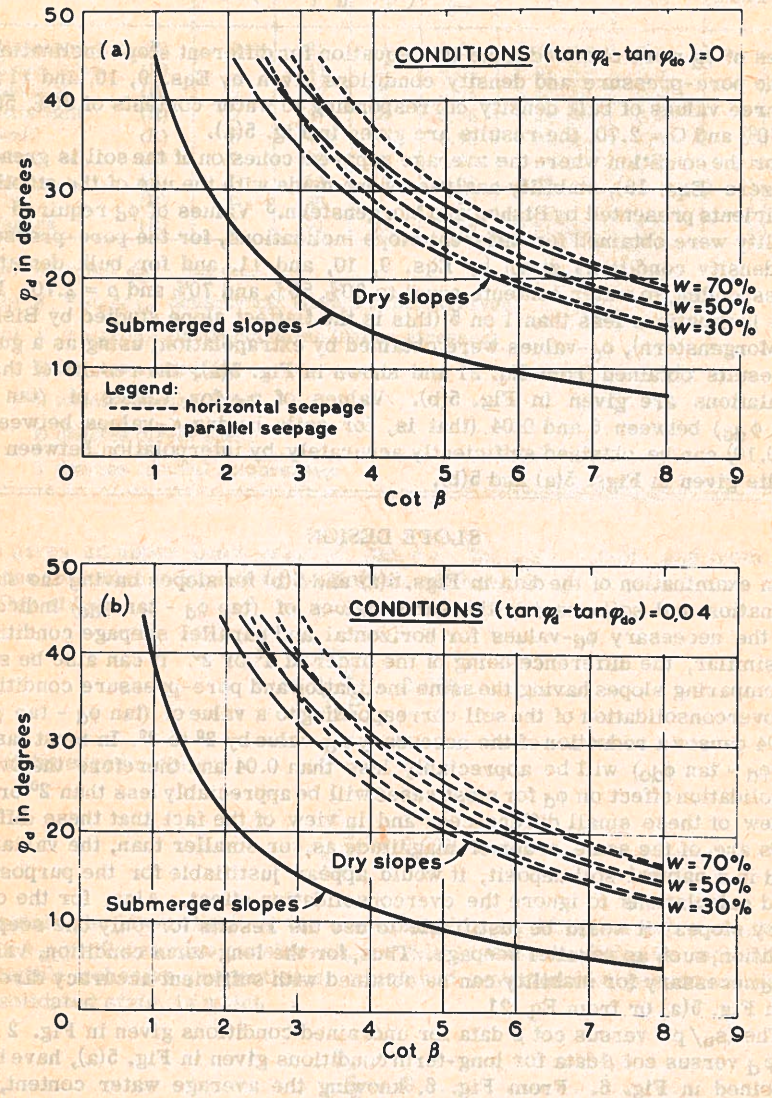

FIG. 5.—RESULTS OF STABILITY CALCULATIONS FOR LONG-TERM CONDITIONS

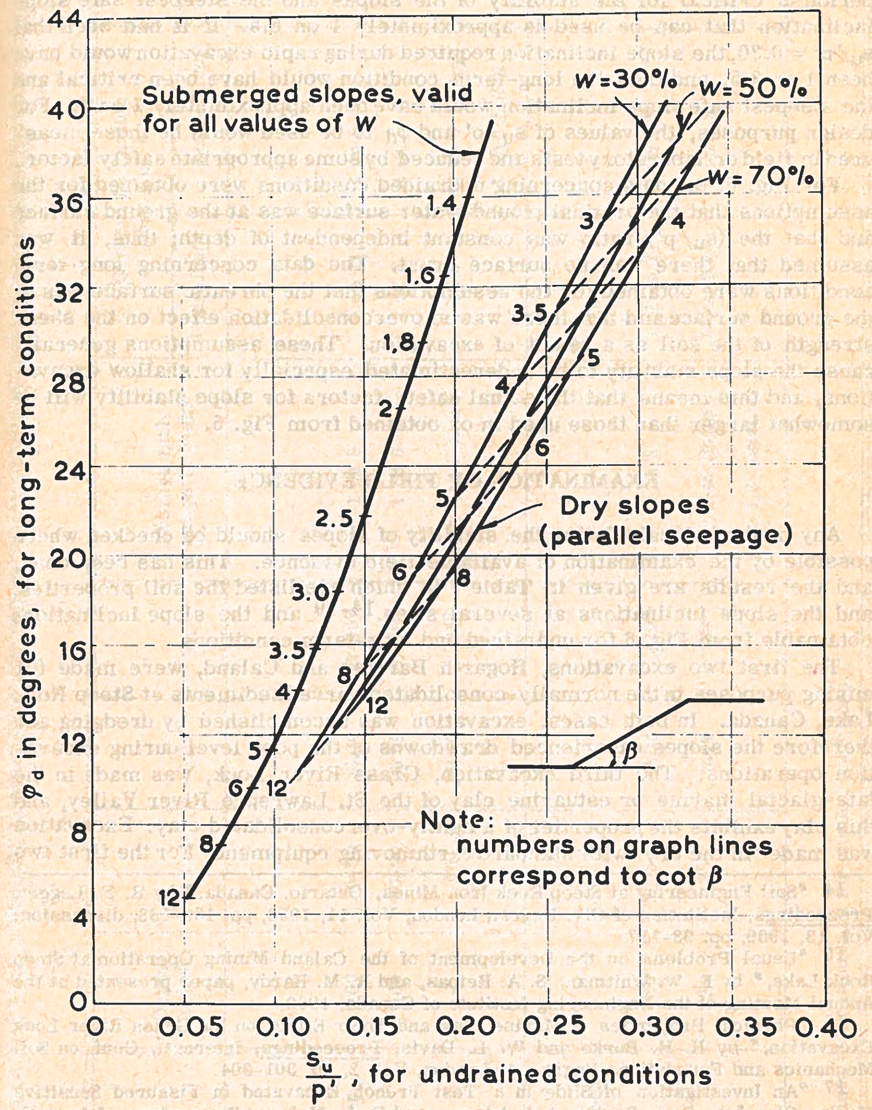

FIG. 6.—CRITICAL VALUES OF COT β FOR VALUES OF s_u/p' AND ϕ_d

$= 0.20$, $\phi_d = 30^0$. From Fig. 6 it can be found that, for undrained conditions and $s_u/p' = 0.20$, an inclination of 1 on 6.5 is required. For long-term stability and $\phi_d = 30^0$, an inclination of 1 on 4.0 is required. Hence, the excavation period is critical for the stability of the slopes and the steepest safe slope inclination that can be used is approximately 1 on 6.5. If it had been that $s_u/p' = 0.30$, the slope inclination required during rapid excavation would have been 1 on 3.5, and thus the long-term condition would have been critical and the steepest safe slope inclination would have been approximately 1 on 4. For design purposes, the values of s_u/p' and ϕ_d to be used would be those measured in field or laboratory tests and reduced by some appropriate safety factor.

For Fig. 6, the data concerning undrained conditions were obtained for the assumptions that the original ground-water surface was at the ground surface and that the (s_u/p')-ratio was constant independent of depth; thus, it was assumed that there was no surface crust. The data concerning long-term conditions were obtained for the assumptions that the phreatic surface was at the ground surface and that there was no overconsolidation effect on the shear strength of the soil as a result of excavation. These assumptions generally cause the slope stability to be underestimated, especially for shallow excavations, and this means that the actual safety factors for slope stability will be somewhat larger than those used in or obtained from Fig. 6.

EXAMINATION OF FIELD EVIDENCE

Any method of calculating the stability of slopes should be checked where possible by the examination of available field evidence. This has been done, and the results are given in Table 3 in which are listed the soil properties, and the slope inclinations at several sites,[14-19] and the slope inclinations obtainable from Fig. 6 for undrained and long-term conditions.

The first two excavations, Hogarth Barrier and Caland, were made for mining purposes in the normally-consolidated varved sediments at Steep Rock Lake, Canada. In both cases, excavation was accomplished by dredging and therefore the slopes experienced drawdowns of the pool level during excavation operations. The third excavation, Grass River Lock, was made in the late-glacial marine or estuarine clay of the St. Lawrence River Valley, and this clay exhibits the properties of a lightly-overconsolidated clay. Excavation was made "in the dry" with standard earthmoving equipment. For the first two

14 "Soil Engineering at Steep Rock Iron Mines, Ontario, Canada," by R. F. Legget, Proceedings, Institution of Civ. Engrs., London, Vol. 11, 1958, pp. 169-188, discussion; Vol. 13, 1959, pp. 93-117.

15 "Usual Problems in the Development of the Caland Mining Operation at Steep Rock Lake," by E. W. Whitman, S. A. Reipas, and R. M. Hardy, paper presented at the Annual Meeting of the Engineering Institute of Canada, 1962.

16 "Physical Properties of Marine Clay and Their Effect on the Grass River Lock Excavation," by H. H. Burke and W. L. Davis, Proceedings, Internatl. Conf. on Soil Mechanics and Foundations Engrg., 4, London, Vol. 2, pp. 301-304.

17 "An Investigation of Slide in a Test Trench Excavated in Fissured Sensitive Marine Clay," by D. J. Bazett, J. I. Adams, and E. L. Matyas, Proceedings, Internatl. Conf. on Soil Mechanics and Foundation Engrg., 5, Vol. 1, Paris, 1961, pp. 431-435.

18 "Pore Pressures Resulting from Driving Piles in Soft Clay," by L. Bjerrum and I. Johannessen, Pore Pressure and Suction in Soils, Conference, London, Butterworths, 1961, pp. 108-111.

19 "Stability Investigations of the North Bank of the Drammen River, by B. Kjaernsli and N. E. Simons, Geotechnique, Vol. 12, No. 2, 1962, pp. 147-167.

TABLE 3.—FIELD EVIDENCE CONCERNING OPEN-CUT EXCAVATIONS IN NORMALLY-CONSOLIDATED SOILS

Excavation	Depth, in feet	w, in %	s_u/p'	ϕ_d	Average Inclination in Field	Critical Inclination from Fig. 6: Undrained	Critical Inclination from Fig. 6: Long-Term	Reference
Dry-Slopes								
Hogarth Barrier	160	30-90	>0.30	25-30°	1 on 8	1 on 4	1 on 5	14
Caland	400	30-90	>0.30	25-30°	1 on 6	1 on 4	1 on 5	15
Grass River	-	60	>0.35	34°	1 on 6 to 1 on 10	1 on 3	1 on 4 (Dry)	16
Lock							1 on 1.5 (Sub.)	17
Submerged Slopes								
Fredrikstad	60	45	0.33	>30°	1 on 2	(>1 on 1)	1 on 1.8	18
Drammen	30	35	0.16	33°	1 on 1.45 (failed) 1 on 1.6 (stable)	(1 on 2.2)	1 on 1.5	19

Note: Underlined inclinations are for critical conditions.

excavations, long-term conditions are critical for slope stability. The slope inclinations obtained from Fig. 6 for long-term conditions are somewhat steeper than the inclinations of the excavated slopes that have to date (1963) remained stable, indicating the existence of a safety factor against the possibility of failure. In the case of the Grass River Lock, no check is possible for the long-term condition because, shortly after construction, the excavation was flooded thus providing safer conditions. It is noted, however, that the inclination of 1 on 10 of the excavated slopes is significantly flatter than the inclination of 1 on 4 obtained from Fig. 6.

The second type of field evidence summarized in Table 3 is for natural slopes formed by river erosion of normally-consolidated sediments. The two cases given are for the Fredrikstad and Drammen Rivers in Norway. In both cases the water level in the river is only several meters below the level of the clay terrace, and thus these can be considered as examples of submerged slopes. These river valleys were eroded slowly and therefore cannot provide a check for the undrained condition; however, they do provide a check for the long-term conditions. In the case of the Fredrikstad River, the critical condition for the river banks is long-term, and from Table 3 it is seen that the critical inclination obtained from Fig. 6 is only slightly steeper than the present-day (1963) inclination of the river bank.

For the Drammen River soil, the critical condition would have been the undrained case. However, because erosion occurred slowly under drained conditions, the soil being allowed to mobilize to a greater degree its effective-stress, shear-strength properties, the critical condition was the long-term case. The critical inclination of 1 on 1.5 found from Fig. 6 compares well with the average inclination of 1 on 1.45 for a part of the river bank that failed, and it also is somewhat steeper than the inclination of 1 on 1.6 for a part of the river bank that is stable at present.

Although the field evidence collected in Table 3 is meager, there is no evidence that conflicts with the use of procedures outlined herein and which are presented in condensed form in Fig. 6.

CHANGE OF SLOPE STABILITY WITH TIME

The question of changes with time of the slope stability of cuttings has been examined by Bishop and Bjerrum,[6] and they concluded that the most critical stability condition was long-term. However, the data given in Fig. 6 imply that either undrained or long-term conditions can be critical for slope stability. Because these views are not consistent, the question will be re-examined.

The line of thought used by Bishop and Bjerrum is as follows: As a result of excavation operations, the vertical stresses are reduced and the pore pressures in the soils are reduced to values that are lower than the ultimate values for long-term conditions (the possible use of deep drainage systems for lowering permanently the ground-water level is neglected). It follows that, with the passage of time after the end of construction, the pore pressures will increase and thus the effective stresses in the soil will decrease. By assuming that changes of shear strength are only dependent on changes of effective stress, Bishop and Bjerrum concluded that if the effective stresses decrease, then the stability of the slopes must also decrease.

However, changes of shear strength are also dependent on the degree of mobilization or degree of development of the effective-stress shear-strength

parameters of the soil, and it is known that, for many soils exhibiting low undrained shear strengths, the effective-stress shear-strength parameters developed at the condition of maximum undrained shear strength can be much smaller than those developed at the conditions of maximum drained shear strength (Kenney;[20] Bjerrum;[21] although questioned by Bjerrum and Simons[22]), that is,

$$c'_u = M_c\, c_d \quad \text{.................(22)}$$

$$\tan \phi'_u = M_\phi \tan \phi_d \quad \text{...............(23)}$$

in which c'_u and ϕ'_u represent values of c' and ϕ' measured at the condition of maximum undrained shear strength, c_d and ϕ_d denote the fully-mobilized values of c and ϕ for drained conditions, and M_c and M_ϕ refer to the degree of mobilization of cohesion and angle of shearing resistance and have values less than unity.

After the construction of the excavation has been completed and if the external loading conditions remain unchanged, the average shear stress τ and the average normal stress σ_n along a potential failure surface remain unchanged with time. Immediately after construction, the shear stress can be expressed in terms of the existing effective stresses and the partially-mobilized shear-strength parameters as follows:

$$\tau = c'_{um} + (\sigma_n - u_u) \tan \phi'_{um} \quad \text{...........(24)}$$

in which u_u denotes the pore pressure for undrained conditions, and c'_{um} and ϕ'_{um} represent partially-mobilized values of c'_u and ϕ'_u. Now c'_{um} and $\tan \phi'_{um}$ can be expressed as follows:

$$c'_{um} = \frac{c'_u}{F_u} \quad \text{..................(25)}$$

and

$$\tan \phi'_{um} = \frac{\tan \phi'_u}{F_u} \quad \text{................(26)}$$

20 Discussion by T. Cameron Kenney of "The Influence of Rate of Strain on Effective Stresses in Sensitive Clay," by C. B. Crawford, Special Technical Publication No. 254, Amer. Soc. for Testing Materials, 1959, pp. 49-58.

21 "The Effective Shear Strength Parameters of Sensitive Clays," by L. Bjerrum, Proceedings, Internatl. Conf. on Soil Mechanics and Foundation Engrg., 5, Paris, Vol. 1, 1961, pp. 23-28.

22 "Comparison of Shear Strength Characteristics of Normally Consolidated Clays," by L. Bjerrum and N. E. Simons, Research Conf. on Shear Strength of Cohesive Soils, Boulder, Colo., 1960, pp. 711-726.

in which F_u denotes safety factor against failure for undrained conditions. Thus, Eq. 24 can be rewritten as

$$\tau = \frac{c'_u}{F_u} + (\sigma_n - u_u)\frac{\tan \phi'_u}{F_u}$$

$$= \frac{M_c\, c_d}{F_u} + (\sigma_n - u_u)\frac{M_\phi \tan \phi_d}{F_u} \quad \ldots\ldots\ldots\ldots (27)$$

Similarly, an expression for τ can be obtained for long-term or drained conditions,

$$\tau = \frac{c_d}{F_d} + (\sigma_n - u_d)\frac{\tan \phi_d}{F_d} \quad \ldots\ldots\ldots\ldots (28)$$

in which F_d refers to the safety factor against failure for drained conditions. Combining Eqs. 27 and 28 and substituting $u_u + \Delta u = u_d$, in which Δu denotes the change of pore pressure from undrained to drained conditions, yields

$$\frac{F_u}{F_d} = \frac{c_d\, M_c + (\sigma_n - u_d)\tan \phi_d\, M_\phi + \Delta u \tan \phi_d\, M_\phi}{c_d + (\sigma_n - u_d)\tan \phi_d} \quad \ldots\ldots (29)$$

For normally-consolidated and some lightly overconsolidated soils, the cohesion terms are sufficiently small in magnitude to be negligible, and for this case Eq. 29 can be simplified to yield

$$\frac{F_u}{F_d} = M_\phi \left(1 + \frac{\Delta u}{\sigma_n - u_d}\right) \quad \ldots\ldots\ldots\ldots (30)$$

For most excavated slopes the term $\Delta u/(\sigma_n - u_d)$ is greater than 0.2, and for most soils M_ϕ is between the values of 0.9 and 1.0. Thus, the ratio $(F_u/F_d) > 1$, which indicates that for these soils the safety factor for undrained conditions exceeds the safety factor for drained conditions, and therefore drained or long-term conditions are critical. However, soils that have low (s_u/p')-ratios, and these are generally of the sensitive type (although not necessarily), would also exhibit values of $\Delta u/(\sigma_n - u_d)$ greater than 0.2 but would exhibit much lower values of M_ϕ, such as 0.7. In this case, the ratio $(F_u/F_d) > 1$, which indicates that the undrained condition provides the lowest safety factor and therefore is critical for stability.

It must therefore be concluded that critical slope stability conditions can exist either during undrained or long-term conditions, and that soils which have critical stability conditions during undrained conditions are restricted to those having low values of s_u/p'-ratio, probably less than 0.20.

CONCLUSIONS

It was assumed that the undrained shear strength of normally-consolidated soil increased linearly with depth from a value of zero at the ground surface. It was found that, if an open-cut excavation were made in the soil, the critical slip surface would intersect the slope and its location would be independent of the depth of the excavation; thus, the stability of the slope is governed only by the rate of increase of shear strength with depth, the soil density, and the inclination of the slope. Slope-stability analyses were made for undrained conditions and the results are given in Fig. 2.

Slope-stability analyses were also made for long-term conditions, consideration being given to different pore-pressure conditions and to the effects of overconsolidation resulting from the removal of soil from above the final slopes of the excavation. The results of the calculations are given in Fig. 5. Even for soils having a Hvorslev κ-value equal to 0.10 (considered to be the upper limit for most soils), it was found that the effect of overconsolidation decreased the required ϕ_d-value by only approximately 3°. This quantity is within the accuracy range by which the ϕ_d-properties of a soil deposit can be determined, and therefore the overconsolidation effect resulting from stress reduction on excavation can for most cases be neglected.

Fig. 6 has been prepared to combine the results given in Figs. 2 and 5(a). Fig. 6 shows the soil strength properties that are required in order that a slope of given inclination remain stable for undrained and long-term conditions. It is implied from Fig. 6, and concluded in a subsequent section, that both the undrained and the long-term conditions can be critical for the stability of slopes in excavations, and that soils that have critical stability conditions during undrained conditions are restricted to those having low values of s_u/p'-ratio, probably less than 0.20.

The available field evidence concerning unsupported excavations in normally-consolidated or lightly overconsolidated soils is collected in Table 3. Although the evidence is meager, none of it conflicts with the procedures for designing slopes that are presented herein.

APPENDIX.—NOTATION

The following symbols have been adopted for use in this paper:

c_d = apparent cohesion, drained conditions, fully mobilized;

$\bar{c}_d$ = average value of c_d;

c'_u = apparent cohesion, effective stresses, measured at the condition of maximum undrained shear strength;

c'_{um} = partially mobilized value of c'_u;

F_d = safety factor, drained (long-term) conditions;

F_u = safety factor, undrained conditions;

G = specific gravity of mineral particles;

h = depth of soil;

h_o = distance below original ground surface;

h_p = distance below phreatic surface;

H = depth of excavation;

K = ratio of the horizontal and vertical effective stresses;

M_{cu} = degree of mobilization of apparent cohesion, undrained conditions;

M_ϕ = degree of mobilization of angle of shearing resistance, undrained conditions;

p' = vertical effective stress due to weight of overlying soil;

r_u = pore-pressure ratio (Eq. 8);

s = shear strength;

s_u = shear strength, undrained conditions;

u_u = pore-water pressure, undrained conditions;

u_d = pore-water pressure, drained conditions;

w = water content;

α = inclination angle of failure surface;

β = inclination angle of slope;

γ = unit weight of soil (bulk density);

γ_e = equivalent unit weight for dry-slope conditions;

γ_w = unit weight of water;

θ = inclination angle of flow line;

κ = Hvorslev's true-cohesion parameter;

σ_n = normal stress;

σ'_n = normal stress, effective stresses;

τ = shear stress;

ϕ_d = angle of shearing resistance, drained conditions, fully mobilized;

ϕ_{do} = ϕ_d in the overconsolidation stress range;

ϕ'_u = angle of shearing resistance, effective stresses, measured at the condition of maximum undrained shear strength; and

ϕ'_{um} = partially mobilized value of ϕ'_u.