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Proceedings of the American Society of Civil Engineers

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CONTENTS

March, 1964

Papers

Page
Prediction of Pile Capacity by the Wave Equation
by P. W. Forehand and J. L. Reese, Jr. .......................... 1
Lateral Resistance of Piles in Cohesive Soils
by Bengt B. Broms ........................................... 27
Rebound in Redesign of Oahe Dam Hydraulic Structures
by Lloyd B. Underwood, Stanley T. Thorfinnson,
and William T. Black ........................................ 65
Mechanics of Inclined Filters in Earth Dams
by V. J. Patel, A. V. Gopala Krishnayya,
and K. L. Arora ............................................. 87

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Note.—Part 2 of this Journal is the 1964-13 Newsletter of the Soil Mechanics Division.

Unwatering Akosombo Cofferdams
by Donald J. Bleifuss ......................... 111

Soil Lime Research at Iowa State University
by Manuel Mateos .............................. 127

---

DISCUSSION

---

Analysis of the Engineering News Pile Formulas,
by Hans A. Agerschou. (October, 1962).
Prior discussion: December, 1962, March,

errata ........................................... 157

Soil-Cement as Slope Protection for Earth Dams,

by W. G. Holtz and F. C. Walker (closure) ........... 159

Basic Experiment into Soil-Structure Interaction,
by Robert V. Whitman and Ulrich Lascher.
(December, 1962. Prior discussion: May,
September, 1963. Discussion closed.

by Robert V. Whitman and
Ulrich Lascher (closure) ....................... 163

Study of Failure Envelope of Soils, by T. H. Wu,
Discussion closed.

by T. H. Wu, A. K. Loh,
and L. E. Malvern (closure) ................. 165

Shear Strength Properties of a Sodium Iillite,
by Roy E. Olson. (February, 1963. Prior discussion:
September, 1963. Discussion closed.

by Roy E. Olson (closure) .................... 167

Earthquake Resistance of Sloping Core Dams,

by H. Bolton Seed and R. W. Clough (closure) .......... 169

Effective Stress Theory of Soil Compaction,
by Roy E. Olson. (March, 1963. Prior discussion:

by Roy E. Olson (closure) ................... 171

Seepage Under Dams, by Hammad Y. Hammad.
(July, 1963. Prior discussion: None.
Discussion closed.

errata ........................................... 185

Foundation Modulus Tests for Karaj Arch Dam,
by W. A. Waldorf, J. A. Veltrop, and J. J. Curtis.
Discussion closed.

by K. V. Swaminathan ............................ 187
by E. W. Stropponi and G. H. Kruse ................. 191

Laminar and Turbulent Flow of Water through Sand,
by M. Anandakrishnan and G. H. Varadaraju. 
Discussion closed.

by Ralph R. Rimer ................................ 205

(September, 1963. Prior discussion: None.
Discussion closed.

by A. D. M. Penman ............................. 209

Boundary Value Problems of Soil Mechanics,
by W. D. L. Finn. (September, 1963. Prior discussion:
None. Discussion closed.

by T. H. Wu ................................... 211
CONCLUSIONS

The history of a foundation problem in the Pierre Shale at the Ohio project has been followed. The geology of the area has been examined, including the effect of formation and the development of the river trends. Differential settlement of the foundation area has been experienced and accommodated further movements have been outlined. Two points appear to be of major significance: first, about major differential settlement or movement can occur in the foundation under conditions and second, that the structures to be placed therein appear to be virtually immune to the effects of the structures to be placed therein and can be brought to the problem.

ACKNOWLEDGMENTS

The authors acknowledge the contribution of the dam design and construction, the stilling basin structures were accomplished by the Ohio District Corps of Engineers, and the construction work was supervised by the Ohio Branch of the American Society of Civil Engineers. The work was accomplished under the direct supervision of John Short, Jr., Area Engineer, Ohio District.

SOIL MECHANICS AND FOUNDATIONS DIVISION

M.ECHANICS OF EMBANKED FILTERS IN EARTH DAMS

By V. J. Pian, A. V. Gopal, Krishnapet, 2 and K. L. Amin

ABSTRACT

The mechanics of the upstream and downstream slopes of earth dams are discussed and a method of calculating the pressure on the filter is presented. The pressure on the filter is calculated using the principles of soil mechanics and the characteristics of the upstream and downstream slopes. The method is based on the assumption that the filter is placed near the base of the dam. The location of the critical slip circle for the given slope and position of the dam is determined, and the method of calculating the pressure on the filter is presented. The average slope of the critical slip circle for the given location of the dam is determined and the method of calculating the pressure on the filter is presented.
slope is practically the same, leading to the conclusion that if an inclined filter with this slope is provided in the dam, the design will become scientific and economical. Based on the calculations, a stability chart is given which can be used for designing earth dams.

INTRODUCTION

The main topic of this paper is the validity (or non-validity) of rapid drawdown flow nets in the case of materials that are not freely draining. This uncertainty still prevents the designer from adopting bold designs because the factor of safety depends on the factor of ignorance. Trollope, the general reporter on earth dams, slopes, and even excavation at the Fifth International Conference on Soil Mechanics and Foundation Engineering wrote, concerning the paper of Patel and Maheshwari,4 that "the authors have made most interesting and valuable study of the influence of the location of an upstream sloping filter on the flow net and hence on the stability of earth dams. Their work is based on the model experiments in which steady state conditions were developed. Consequently care must be taken in extrapolating their findings in the case of complete rapid drawdown to materials that are not freely draining."5 Anonymous commenting on the same work6 wrote, "It is necessary to emphasize that the results presented in the report and calculations may be used only in the case of rigid incompressible embankments. Pore pressures in an earth fill embankment due to consolidation of fill after drawdown should be investigated, applying the theory of consolidation or by introducing the triaxial test constants A and B in the well-known Skempton's equation. Therefore, the pore pressure due to consolidation of the fill is to be added to the pore pressure of the water head of non-steady filtration. It is not clear at the moment what the effect of the excessive pore pressure is to the filtration flow net."5 Doubts have also been expressed as to whether the same critical slip circle would be valid for various positions of the filter.6 Thus, further investigation is necessary to discover: (1) Whether the flow net of rapid drawdown can be used for the stability analysis of earth dams with materials that are not freely draining; and (2) whether only one critical slip circle exists in earth dams independent of the position of the filter.

Notation.—The symbols adopted for use in this paper are defined where they first appear and are arranged alphabetically in the Appendix.

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CRITERIA FOR STABILITY ANALYSIS

The stability analysis is generally carried out for the different conditions possible such as construction period, full reservoir, partial drawdown, slow drawdown, and complete rapid drawdown of the reservoir water level. From all these possibilities, generally speaking, the complete rapid drawdown condition provides the minimum factor of safety. The factor of safety for earth dams on pervious foundations is higher than for dams on impervious foundations. Similarly, slow drawdown gives a higher factor of safety than rapid drawdown.7 Thus, in general, sudden drawdown for an earth dam on an impervious foundation represents the worst condition and hence is accepted as the design criterion herein.

EQUATION STABILITY ANALYSIS

Adopting the Fellnian method of slices for the cylindrical slip surface and assuming homogeneous isotropic material, Eq. 1a can be derived for the two-dimensional case.

\[ F_s = \frac{\Sigma(\sigma - u) \tan \phi - c \Sigma d_l}{\Sigma d_r} \]  

in which

\[ F = \text{Factor of safety} = \frac{\text{total restoring force}}{\text{total force acting the slice}} \]  

\( \sigma \) is the normal component of the weight of the slice acting on an area, dl, of the slip surface; dl denotes the length of the slip surface of the slice; du represents the neutral force due to pore pressure acting normal to the slip surface over an area, dl, during the case of rapid drawdown; c refers to the cohesive strength of the soil per unit area on the slip surface; \( \phi \) stands for the tangential component of the weight of the slice acting on the area, dl; \( \phi \) equals the angle of internal friction of soil; and \( \Sigma \) indicates summation.

Eq. 1a can be written after summation as

\[ F_s = \frac{(\sigma - u) \tan \phi + c l}{\sigma} \]  

It is possible to write

\[ \sigma = F_s \frac{H^2}{\gamma_s} \]  

---

the total normal force;
\[ U = F_u H^2 \gamma_w \]  
(3a)

the total neutral force;
\[ I = b'H \]  
(2c)

the total length of the arc of the slip circle, \( b' \) being a constant; and
\[ \tau = F_{\tau} H^2 \gamma_{sat} \]  
(3d)

the total tangential force in which \( H \) is depth of water retained on the upstream face; \( \gamma_{sat} \) corresponds to the saturated density of the soil; \( \gamma_w \) is the density of water; \( b' \) equals the coefficient of slip circle as defined by Eq. 3c; and \( F_u \) and \( F_{\tau} \) are the coefficients of normal, tangential and neutral forces, respectively, as defined by Eqs. 3a, 3b and 3d.

Substituting for the various terms from Eqs. 3a to 3d in Eq. 2 the factor of safety can be expressed as
\[ F_s = \left( \frac{F_u H^2 \gamma_{sat} - F_{\tau} H^2 \gamma_w}{F_{\tau} H^2 \gamma_{sat}} \right) \tan \phi + c b'H \]  
(4a)

or
\[ F_s = \left[ \frac{F_u}{F_{\tau}} \cdot \frac{F_u \gamma_w}{F_{\tau} \gamma_{sat}} \right] \tan \phi + \frac{c b'}{\gamma_{sat}} \]  
(4b)

For a given failure surface and pore pressure distribution
\[ F_s = \left( B_1 - B_2 \right) \frac{\gamma_w}{\gamma_{sat}} \tan \phi + \frac{c B_2}{H \gamma_{sat}} \]  
(5)

in which \( B_1 \) and \( B_2 \) are constants equal to \( F_u / F_{\tau} \), \( F_u / F_{\tau} \), and \( b' / F_{\tau} \), respectively.

METHODS OF COMPUTING PORE WATER PRESSURE FOR RAPID DRAWDOWN

If the material in the dam is assumed to be able to drain the water freely after complete rapid drawdown, the pore pressures can be computed from the rapid drawdown flow nets.

If the material in the dam is of impervious clay, free drainage is questionable. The pore pressure in this case can be computed using Skempton's pore pressure coefficients as suggested by Bishop.9 Referring to Fig. 1 the pore pressure, \( u \), at any point on the slip circle is
\[ u = \gamma_w \left[ h_c - h' + (1 - n) h_r \right] \]  
(6)

in which \( n \) refers to the porosity of the pervious material of the casing of the dam; \( h_c \) denotes the depth of the pervious material; \( h' \) represents the difference between the full depth of water, \( H \), and the head corresponding to the potential under full reservoir condition at the point where the pore pressure is required; and \( h_r \) is the vertical intercept between the upstream face of the impervious zone and the point at which the pore pressure is required. If the pervious material upstream of the impervious zone is not included in the stability analysis, Eq. 6 can be written as
\[ u = \gamma_w \left( h_c - h' \right) \]  
(7)

The validity of Eq. 7 is confined to steady state conditions only.

![FIG. 1.—EARTH FILL DAM CROSS SECTION](image)

Another method usually adopted in the absence of flow nets is to consider the pore pressure equal to full buoyancy. According to this method, \( F_u \) is assumed equal to \( F_{\tau} \). Taking \( \gamma_w \) as unity, Eq. 5 can be written as
\[ F_s = B_1 \left( 1 - \frac{1}{\gamma_{sat}} \right) \tan \phi + \frac{c B_2}{H \gamma_{sat}} \]  
(8)

All three methods have been used in calculating the factor of safety herein.

CALCULATION OF THE FACTOR OF SAFETY

Eq. 5 can be written in a more convenient form for a given slip circle as

LOCATION OF CRITICAL SLIP CIRCLES

The stability analysis for complete rapid drawdown was carried out for an earth dam with a vertical filter for different upstream slopes, i.e., 1.0 on 1.1, on 1.25, 1 on 1.5, 1 on 2.1, on 2.5, 1 on 2.75 and 1 on 3 (vertical on 3 horizontal) for filter positions of 0.25 H, 0.5 H, 1.0 H, 1.5 H, 2.0 H and 2.5 H from the axis of the earth dam, and for combinations of the soil characteristics, i.e., tangent of the angle of internal friction, tan \( \phi \), of 0.1, 0.2, 0.3, and 0.4; and saturated soil density, \( \gamma_{sat} \), of 1.8, 2.0 and 2.2. A number of slip circles were taken, all passing through the upstream toe of the dam and a few passing through the dam foundation.

The following assumptions were considered necessary for the simplification of calculations:

1. The material above the phreatic line is assumed fully saturated and without pore pressure. The material in this zone will become saturated due to capillary rise. This assumption will not lead to any appreciable change from the true values. The values of \( c \) and \( \phi \) for this material are assumed to be similar to the fully saturated zone for the simplification of calculations.

   Thus the calculation yields a lower factor of safety than the actual factor of safety, thus allowing the designer to be on the conservative side.

2. The material is homogenous, and isotropic.

3. Whenever part of the critical slip circle passes below the base of the dam, the material in that part of the foundation is considered to be the same as that in the earth dam. This part of the slip circle is rather small and lies near the upstream toe of the dam. The equipotential lines for the rapid drawdown flow net or for Skempton's constant as suggested by Bishop, are vertical.

4. The water level in the reservoir is assumed to be at the dam height.

It can be seen that these assumptions will provide a calculated factor of safety less than the actual factor of safety.

The values of \( c_{sat}/H \) (Eq. 9c) have been computed, employing the variables previously mentioned, for each slip circle, and using the neutral forces obtained by different methods. Tables 1(a) to 1(f) show values of \( c_{sat}/H \) for the upstream slope of 1 on 2.5 for a number of slip circles surrounding the critical circle. The critical circle is defined by the maximum value of \( c_{sat}/H \). The critical circles have been located for other upstream slopes also considering all the variables (Fig. 2). In each case the center of the critical slip circle is surrounded by the centers of the circles having lower values of \( c_{sat}/H \).

EXAMINATION OF RESULTS

For the given soil characteristics, the critical circle remains the same even though the neutral forces are calculated by different methods. Tables 1(a), 1(c), 1(d). A qualitative explanation for these results can be given by referring to Tables 2 and 3. The change in neutral force occurs approximately in the same proportion for all circles when the filter position is changed. From Table 2, it can be seen that the ratio of \( F_u/F_r \) for the vertical filter positions at 2.5 H and 0.25 H for a number of circles is practically equal to 1.3. Similarly, from Table 3 it can be seen that the ratios of \( F_u/F_r \) calculated...
by different methods are practically the same for different slip circles. Thus the critical circle remains the same even though the position of the filter or the method of calculation of the neutral force is changed.

The critical circle is the same for a given value of tan \( \phi_d \) even though the value of \( \gamma_{sat} \) are changed. Tables 1(a), 1(e) and 1(f) show the values of \( c_d/H \)

| Table 2a |  |
|---|---|---|---|---|
| Circle Number | \( F_u/F_T \) at 2.5 H | \( F_u/F_T \) at 0.25 H | \( F_u/F_T \) at 2.5 H | \( F_u/F_T \) at 0.25 H |
| 4 | 3.28 | 2.73 | 1.20 |  |
| 7 | 3.04 | 2.35 | 1.29 |  |
| 8 | 3.06 | 2.28 | 1.34 |  |
| 12 | 3.25 | 2.44 | 1.33 |  |
| 13 | 3.99 | 3.07 | 1.30 |  |
| 15 | 3.49 | 2.71 | 1.28 |  |
| 16 | 3.28 | 2.73 | 1.20 |  |
| 19 | 3.24 | 2.40 | 1.35 |  |

\( u \) is vertical. Pore pressure by Bishop’s Method.

| Table 3a |  |
|---|---|---|---|---|---|---|
| Circle Number | \( F_u/F_T \) | \( F_u/F_T \) | \( F_u/F_T \) | \( F_u/F_T \) | \( F_u/F_T \) | \( F_u/F_T \) |
| 4 | 2.73 | 2.50 | 3.18 | 1.09 | 1.27 | 1.17 |
| 7 | 2.35 | 2.31 | 3.20 | 1.02 | 1.38 | 1.36 |
| 8 | 2.26 | 2.20 | 3.10 | 1.03 | 1.41 | 1.36 |
| 12 | 2.44 | 2.13 | 3.08 | 1.14 | 1.44 | 1.26 |
| 13 | 3.07 | 2.70 | 3.93 | 1.14 | 1.45 | 1.28 |
| 15 | 2.71 | 2.42 | 3.51 | 1.12 | 1.45 | 1.30 |
| 16 | 2.73 | 2.48 | 3.69 | 1.10 | 1.49 | 1.35 |
| 19 | 2.40 | 2.32 | 3.01 | 1.03 | 1.34 | 1.29 |

\( u = 1 \) on 2.5. \( F_u/F_T \) is vertical at 0.25 H

\( \gamma_{sat} \) calculated for \( \gamma_{sat} \) values of 2.0, 1.8 and 2.2. A change in the value of \( \gamma_{sat} \) will equally change the quantity \( (F_u/F_T) \gamma_{sat} \) in Eq. 9c for all slip circles thus yielding the same critical circle.

The critical circle is practically identical, with variations of tan \( \phi_d \). From Table 1(a) it can be seen that for tan \( \phi_d \) values of 0.3 and 0.4, circle No. 12 becomes the critical circle, whereas for lower values of tan \( \phi_d \), circle No. 8 is critical. From Fig. 2(d) it can be seen that these two circles are close to each other lying in a narrow zone. The values of \( c_d/H \) for the two circles are almost identical, indicating that there exists a narrow zone in which any slip circle will yield almost the same factor of safety. Contours of \( c_d/H \) for an upstream slope of 1 on 1.25, vertical ratio at 0.25 H, \( \gamma_{sat} = 2.0 \) tons per cu m and pore pressure from rapid drawdown flow net is plotted in Fig. 3 for different values of tan \( \phi_d \). The provision of a vertical or upstream inclined filter close to the upstream face (Fig. 3) creates a narrow zone in which any slip circle will yield practically the same factor of safety. Table 4 indicates that circle No. 8 gives the minimum factor of safety at the upstream slope of 1 on 1 for all values of tan \( \phi_d \). Similarly it has been observed that the variation of tan \( \phi_d \) has a negligible effect on changing the critical slip cir-

| Table 4a |  |
|---|---|---|---|---|---|---|
| Circle Number | \( F_u/F_T \) | \( F_u/F_T \) | \( F_u/F_T \) | \( F_u/F_T \) | \( F_u/F_T \) | \( F_u/F_T \) |
| 3 | 1.30 | 0.752 | 3.30 | 0.252 | 0.2790 | 0.2505 | 0.2350 | 0.2250 | 0.2090 |
| 4 | 1.30 | 0.755 | 3.32 | 0.216 | 0.2600 | 0.2340 | 0.2140 | 0.1925 | 0.1775 |
| 5 | 1.46 | 0.765 | 3.45 | 0.255 | 0.2610 | 0.2270 | 0.1965 | 0.1785 | 0.1585 |
| 7 | 1.76 | 0.790 | 3.94 | 0.330 | 0.2980 | 0.2530 | 0.2030 | 0.1750 | 0.1430 |
| 8 | 1.18 | 0.766 | 2.69 | 0.357 | 0.3460 | 0.3125 | 0.2820 | 0.2640 | 0.2460 |
| 9 | 1.96 | 0.752 | 3.00 | 0.336 | 0.2530 | 0.2280 | 0.1750 | 0.1575 | 0.1355 |
| 11 | 1.67 | 0.685 | 3.25 | 0.297 | 0.2710 | 0.2450 | 0.2030 | 0.1835 | 0.1600 |
| 12 | 1.75 | 0.644 | 3.07 | 0.318 | 0.2840 | 0.2530 | 0.2150 | 0.1950 | 0.1700 |
| 13 | 1.95 | 0.674 | 3.14 | 0.330 | 0.2740 | 0.2190 | 0.1695 | 0.1370 |  |

\( u = 1 \) on 2.5. \( F_u/F_T \) vertical of 0.25 H. Pore pressure by Bishop’s Method; \( \gamma_{sat} = 2.0 \).
Case 2.—The foundation materials and material comprising the dam are the same. In this case, for slope less than 53°, the critical circle will most probably pass through the toe of the upstream face.

Case 3.—The foundation material is stronger (has a higher shear strength) than the material in the impervious zone. The foundation material may be pervious or impervious. It is already established that the pore pressure decreases considerably and hence the factor of safety increases as the foundation material becomes more pervious, with higher shear strength.1,7,11,12

It is more likely that the failure surface will be “deep” i.e. passing through the foundation material and not through the upstream toe.

FIG. 3.—CONTOUR LINES OF $c_1/H$ FOR SLOPE AND FILTER SHOWN IN FIG. 4.

FIG. 4.

FIG. 5.


of safety is normally specified to two significant figures only. Thus the controversy of the validity (or non validity) of the rapid drawdown flow net hardly affects the factor of safety, leading to the conclusion that the rapid drawdown flow net for freely draining material can also be used for materials that are not freely draining when the filter is placed within 0.25 H of the axis because the curves of $F_0/F_T$ for the freely and non freely draining conditions show a tendency to meet as the filter is moved upstream (Fig. 6).

**LOCATION OF INCLINED FILTER**

A drainage filter should have an inclination and location such that it reduces the pore pressure to the minimum, thereby increasing the stability of the dam.

**TABLE 6.—VALUES OF CONSTANTS B₁ AND B₂ FOR U/S OF 1 ON 2**

<table>
<thead>
<tr>
<th></th>
<th>B₁</th>
<th>B₂</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.14</td>
<td>7.50</td>
<td>Earthquake during full reservoir(^{13,14})</td>
</tr>
<tr>
<td>2</td>
<td>2.77</td>
<td>7.29</td>
<td>Using rapid drawdown flow net for earth dam on previous foundation(^7)</td>
</tr>
<tr>
<td>3</td>
<td>2.49</td>
<td>7.50</td>
<td>Using rapid drawdown flow net for earth dam on impervious foundation(^4)</td>
</tr>
<tr>
<td>4</td>
<td>2.55</td>
<td>7.50</td>
<td>Earth dam on impervious foundation</td>
</tr>
</tbody>
</table>

**TABLE 7**

<table>
<thead>
<tr>
<th>Filter position</th>
<th>Difference in $F_u/F_T$</th>
<th>Difference in $F_0$ for $\gamma_{sat} = 2.0$ and values of $\tan\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25 H</td>
<td>0.50</td>
<td>0.025, 0.05, 0.075</td>
</tr>
<tr>
<td>1.00 H</td>
<td>0.70</td>
<td>0.035, 0.07, 0.105</td>
</tr>
<tr>
<td>2.50 H</td>
<td>1.50</td>
<td>0.75, 0.15, 0.225</td>
</tr>
</tbody>
</table>

This can be achieved by providing an inclined filter in the vicinity of the critical slip circle. The arcs of the critical slip circles located for different upstream slopes pass through a narrow region (Fig. 7), suggesting that the location of the filter for the maximum pore pressure reduction is approximately the same for all upstream slopes. Table 5 lists the values of $\alpha$ and $\beta$ described below.
for different slope value used in Fig. 10. The centers of the critical circles for various upstream slopes lie almost in a straight line meeting the axis at a distance 0.5 H below the base of the dam. An inclined filter (slope 1 on 2) located at -0.6 H effectively reduces the pore pressure on the different slip circles for the various upstream slopes. The criterion for the location of the filter is that it be parallel and close to that part of the arc of the critical slip circle lying between the base and one-half the height of the dam.

The equipotential lines are traced by electrical analogy on "Teledeltos" paper for full reservoir and rapid drawdown conditions for upstream slopes of 1 on 1.25, 1 on 1.5, 1 on 2, 1 on 2.5, 1 on 2.75, and the upstream inclined filter (slope 1 on 2) placed -0.6 H from the axis of the dam. The neutral forces on the critical slip circle have been computed for different upstream slopes for both the freely draining material and the non freely draining material [Fig. 8(a)]. Bishop's method yields smaller neutral forces than the rapid drawdown flow nets. This is due to the narrow zone of impervious material in which the values of \( h_c \) will be less than \( h' \) (Eq. 6) for most of the critical circle thereby resulting in a negative pore pressure. Hence to be on the safe side, the neutral forces obtained from the rapid drawdown flow nets have been used in the calculations [Fig. 8(b)].

**STABILITY CHART.**

As the values of constants \( B_1, B_2 \) and \( B \) for the various upstream slopes and for the inclined slope (1 on 2) and position (-0.6 H) of the filter are evaluated, it is possible to calculate the value of \( c_H \) of Eq. 9c for various upstream slopes and a filter slope of 1 on 2 at -0.6 H from the axis of the dam,
and for an average $\gamma_{\text{sat}}$ value of 2.0 and $\tan \phi_d$ values of 0.0, 0.1, 0.2, 0.3 and 0.4 as shown in Figs. 9 and 10, which can be used for designing earth dams.

Given the height of the earth dam, the soil characteristics, $\tan \phi$ and $c$, and the factor of safety, $F_s$, for rapid drawdown, it is possible to design the upstream slope of the dam with an inclined filter of slope 1 on 2 at $-0.6 H$ from Fig. 8. For the preliminary stage, it is enough to assume an average value of 2.0 for $\gamma_{\text{sat}}$. For final calculations the actual value can be used. An illustrative example follows.

**Example.** It is required to design an earth dam to retain water to a height of 40 m for the following soil characteristics:

$\phi = 15^\circ$, $c = 7.5$ tons per cu m, and $\gamma_{\text{sat}} = 1.8$ tons per cu m

The required factor for safety, $F_s$, is 1.25. For the preliminary design it can be assumed that $\gamma_{\text{sat}} = 2.00$ tons per cu m.

\[
\tan \phi_d = \frac{\tan \phi}{F_s} = \frac{0.2679}{1.25} = 0.214
\]

\[
c_d = \frac{c}{F_s} = \frac{7.5}{1.25} = 6 \text{ tons per sq m}
\]

\[
\frac{c_d}{H} = \frac{6}{40} = 0.15
\]

For $c_d/H = 0.15$ and $\tan \phi_d = 0.214$, the upstream slope of the earth dam from Fig. 8 is 1 on 2. Now the factor of safety can be calculated with $\gamma_{\text{sat}} = 1.8$ from Eq. 9c. For an upstream slope of 1 on 2, from Table 5 $B_1 = F_{u}/F_{r} = 2.55$ and $B_2 = b'/F_r = 7.50$. From Fig. 7(b) $F_{u}/F_r = 0.898$.

Hence substituting these values in Eq. 9c

\[
F_s = \left(2.55 - \frac{0.898}{1.8}\right) \frac{0.2679 + 7.5 \times 7.5}{1.8 \times 40} = 0.5495 + 0.7188 = 1.2683
\]

which is more than minimum $F_s$ of 1.25.

The filter should have a slope of 1 on 2 located $-0.6 H$ from the axis of dam. In order to illustrate the usefulness of an inclined filter, the effect of the horizontal and inclined filters on the neutral forces, allowable height of dam and seepage loss for an earth dam with an upstream slope of 1 on 2 have been compared in Table 6, in which $\frac{U}{s} = 1$ on 2; $B_1 = F_{u}/F_{r} = 2.55$; $B_2 = b'/F_r = 7.5$; $\gamma_{\text{sat}} = 2.0$; $c = 7$ tons per sq m, and $F_s = 1.3$.

1. The reduction in neutral force is a maximum in the case of the inclined filter.
2. The allowable height of the dam is appreciably increased by the inclined filter for values of $\tan \phi$ equal to or more than 0.4. Hence for higher values of $\tan \phi$, it will be advantageous to use an inclined filter.
3. The quantity of seepage loss is not great in the case of an inclined filter when compared to a horizontal filter. It has also been proved that the increase in quantity of seepage by provision of a vertical or upstream inclined filter forms the negligible part of the total water loss.

4. The provision of an upstream inclined filter makes it possible to design upstream and downstream slopes much steeper than in conventional design with a central impervious core. This will result in saving of earth material making the design more economical.

**DESIGN OF THE FILTER**

The filter should be thick enough to discharge the seepage through the dam. The seepage will be a maximum under full reservoir conditions.

**TABLE 8 a**

<table>
<thead>
<tr>
<th>Comparison of various effects</th>
<th>Horizontal filter at - 0.5 H</th>
<th>Inclined filter (1 on 2) at - 0.6 H</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neutral forces, ( \frac{F_u}{F_F} )</td>
<td>1.30</td>
<td>0.898</td>
</tr>
<tr>
<td>Allowable height of dam, in meters, for ( \tan \phi = 0.20 )</td>
<td>28.2</td>
<td>30.1</td>
</tr>
<tr>
<td>( \tan \phi = 0.30 )</td>
<td>36.3</td>
<td>39.5</td>
</tr>
<tr>
<td>( \tan \phi = 0.40 )</td>
<td>45.6</td>
<td>57.6</td>
</tr>
<tr>
<td>Seepage quantity ( q/1000 )</td>
<td>1.23</td>
<td>1.38</td>
</tr>
</tbody>
</table>

\[ q = \frac{k_f}{2L} \left( t_2 - t_3 \right) \]  

(13)

With a factor of safety of 2 and negligible tail water depth

\[ q = \frac{k_f}{4L} t_2 \]  

(14)

the thickness, \( t_2 \), required for the horizontal filter can be calculated from Eq. 14 after computing the quantity of seepage, \( q \), from the flow nets.

**FIG. 11.—FILTER CROSS SECTION**

A minimum thickness of 2 m for the filter is recommended to facilitate construction. A transition zone should be provided around the filter to avoid migration of fine particles.

**CONCLUSIONS AND RECOMMENDATIONS**

1. There exists only one critical slip circle during rapid drawdown, full reservoir or earthquake during full reservoir condition for a given upstream slope irrespective of the height of the earth dam, soil characteristics and methods of calculating the pore pressures.

2. The provision of a vertical or inclined filter creates a narrow zone in which the factor of safety for different slip circles scarcely varies.

3. The critical slip circle will not move upstream with the provision of a filter. Thus the critical slip circle is a function of the upstream slope only.

4. The validity (or non validity) of rapid drawdown flow nets for materials that are not freely draining does not change the factor of safety appreciably when the vertical filter is placed at a distance of 0.25 H or less from the axis of the dam. Therefore, the rapid drawdown flow net can also be used for materials that are not freely draining.

5. The difference in pore pressures and hence the factors of safety calculated using full buoyancy and Skempton’s constants as suggested by Bishop is negligible when the filter is placed at or beyond 2.5 H.

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The critical slip circles for different upstream slopes located from the values of $\alpha$ and $\beta$ (Table 3) pass through a narrow region indicating the same slope and position of the drainage filter for all upstream slopes. An upstream inclined filter (slope 1 on 2) located -0.6 H from the axis of the earth dam effectively reduces the pore pressure on different critical slip circles for the various slopes.

7. The stability charts and nomographs could be constructed for calculating the factor of safety easily for earth dams with a vertical or inclined filter as the constants $B_1$, $B_2$, and $B_3$ in Eq. 9a are determined. The preliminary results thus obtained using Fig. 9 can be further investigated for stability with possible variations such as a pervious casing and saturated density of the soil. While investigating further, the values of $\alpha$ and $\beta$ given in Table 3 can be used for the location of the preliminary critical circle. This will enable the designer to locate the actual critical slip circle in a few trials.

8. The thickness of the filter should be designed to drain the seepage safely during the full reservoir condition (Eqs. 12 and 14). A minimum thickness of two meters should be adopted to facilitate construction.

9. When the downstream slope is dry the preliminary stage can be determined by the use of stability charts given by Taylor. Further investigation would be necessary for possible variations regarding the distribution of the pervious and impervious materials in the earth dam.

10. The thickness of an upstream pervious casing should be such that it will be able to drain the seepage water upstream after complete rapid drawdown. Since a drainage filter is provided close to the upstream slope, a number of flow lines run towards the filter after rapid drawdown thus reducing the quantity of seepage towards the upstream slope. Moreover, when the drainability of the impervious clay core is poor a small quantity of seepage can be expected upstream. It has been found that with a nominal thickness of 3 m will be sufficient for the pervious casing to drain the seepage as well as to protect the clay core from the effect of drying and wetting.

11. A slope protection consisting of riprap should be provided over the upstream pervious casing to protect the slope from wave action.

12. This information can be used in designing earth dams with inclined zones and rock-fill dams if the upstream inclined filter is extended to the top of the dam and material downstream of the filter is the same pervious material as that of the filter.

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APPENDIX.—NOTATION

The following symbols have been adopted for use in this paper.

- $B_1$, $B_2$, and $B_3$ = constants;
- $b'$ = length of the arc of the slip circle in the impervious zone per unit height of the dam;
- $c$ = cohesive strength of the soil;
- $c_d$ = cohesive strength divided by the factor of safety with respect to the strength of soil;
- $d$ = distance of the filter measured from the axis of the dam along the base;
- $d_l$ = length of the slip circle of an elemental slice in the slip circle analysis;
- $d_u$ = neutral force acting normal to the slip surface over an area, $d_l$;
- $d_r$ = normal component of the weight of the slice acting on the area, $d_l$;
- $d_T$ = tangential component of the weight of the slice acting on the area, $d_l$;
- $F_s$ = factor of safety with respect to the strength of the soil;
- $F_u$ = coefficient of neutral force or pore pressure;
- $F_C$ = coefficient of normal force;
- $F_T$ = coefficient of tangential force;
- $H$ = depth of water retained on the upstream face, which will be referred to as the height of dam in this paper;
- $h_c$ = the vertical intercept between the upstream face of the impervious zone and the point at which the pore pressure is required;
- $h_r$ = the depth of the pervious material;
- $h'$ = difference between the full depth of water, H, and head corresponding to the potential under full reservoir condition at the point where the pore pressure is required;
- $k$ = coefficient of permeability;
- $k_f$ = coefficient of permeability of the filter material.

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UNWATERING AKOSombo COFFERDAMS

By Donald J. Bicluza, F. ASCE

SYNOPSIS

The Akosombo Dam is of the rockfill type, with a central vertical core of impervious material. Unwatering of the foundation area required cofferdams both upstream and downstream. Though the cofferdams were high, particularly the downstream one, they had to be built practically entirely under water. It was necessary to allow one season's flood to pass over the uncompleted cofferdams, and they could be sealed only after the flood had passed. The upstream cofferdam was then sealed in the conventional manner, by dumping sealing material on its upstream face from its crest. Because of the characteristics of material available, sealing the downstream cofferdam in similar fashion would have been expensive and time-consuming, and the time available was short. A method of sealing was therefore devised and used that took advantage of the special circumstances existing and required no equipment that was not already available. Artificial silting was induced by dredging clay dumped into the river, and discharging it near and along the downstream face. This method was entirely successful, and resulted in saving the contractor both time and money.

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