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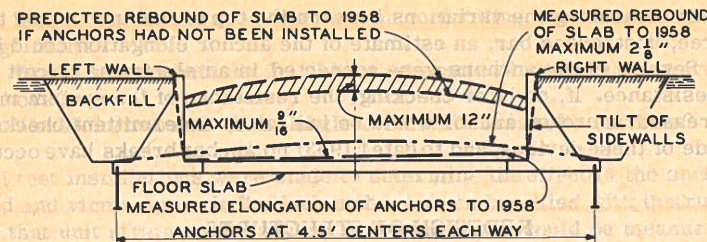


FIG. 13.—SCHEMATIC SECTION OF SECONDARY BASIN SHOWING EFFECT OF ANCHORS

shows the total rise of the concrete surface as measured by differential leveling. The difference between these two represents rebound of the shale below the bottom of the anchors. The top curve shows the predicted rebound range of the slab within two to three years if anchors had not been installed.

#### CONCLUSIONS

The history of a foundation problem in the Pierre shale at the Oahe project has been followed. The geology of the area has been examined, including the effect of glaciation and the development of the river trench. Differential rebound of the Pierre shale following stilling basin excavation has been described, the physical properties of the Pierre shale presented, and probable cause of the movements discussed. The major redesign of the structures to control and accommodate further movements has been outlined. Two points appear to be of major significance; first, abrupt major differential rebound can occur in foundations under certain conditions and, second, once the abrupt movements occur, the quantitative effect on the structures to be placed thereon appears to be virtually impossible to determine. Design must then be based upon qualitative data and the best judgment that can be brought to the problem.

#### ACKNOWLEDGMENTS

Design and construction of the stilling basin structures was accomplished by the Omaha District Corps of Engineers. Design was guided by J. O. Ackerman, F. ASCE, Chief of Engineering Division and construction was accomplished under the direct supervision of John Sibert, Jr., Area Engineer, Oahe Dam.

## Journal of the SOIL MECHANICS AND FOUNDATIONS DIVISION Proceedings of the American Society of Civil Engineers

### MECHANICS OF INCLINED FILTERS IN EARTH DAMS

By V. J. Patel,<sup>1</sup> A. V. Gopala Krishnayya<sup>2</sup> and K. L. Arora<sup>3</sup>

#### SYNOPSIS

The stability of the upstream and downstream slopes of an earth dam increases because of the reduction in pore pressure resulting from the installation of a filter along the base of the dam. A further reduction in pore pressure can be achieved if a vertical or an upstream slanting filter is connected to the downstream by a horizontal filter.

The pore pressure on the slip surface can be calculated from the rapid drawdown flow net constructed under the assumption of steady conditions, free drainage of the material and a rigid incompressible embankment. However, when the material of the dam is impervious clay, which has poor drainage qualities the application of the rapid drawdown flow net is questionable. The validity of the rapid drawdown flow net in such material has been studied in this paper and it is found that, irrespective of the method used for calculating pore pressures, the factor of safety is practically unaffected if the filter is placed near the axis of the dam.

It is shown herein that the position of the critical slip circle for a given upstream slope of an earth dam is practically unaffected by the height of the dam, soil characteristics, method of calculation of the pore pressure and the location of filter. The location of the critical slip circles for various upstream slopes have been determined and a simple method of locating them for a given upstream slope is presented. The average slope of the arc of critical circle lying between the base of the dam and one-half the height for any upstream

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slope is practically the same, leading to the conclusion that if an inclined filter with this slope is provided in the dam, the design will become scientific and economical. Based on the calculations, a stability chart is given which can be used for designing earth dams.

## INTRODUCTION

The main topic of this paper is the validity (or non validity) of rapid drawdown flow nets in the case of materials that are not freely draining. This uncertainty still prevents the designer from adopting bold designs because the factor of safety depends on the factor of ignorance. Trollope, the general reporter on earth dams, slopes and oven excavation at the Fifth International Conference on Soil Mechanics and Foundation Engineering wrote, concerning the paper of Patel and Maheshwari,<sup>4</sup> that "the authors have made most interesting and valuable study of the influence of the location of an upstream sloping filter on the flow net and hence on the stability of earth dams. Their work is based on the model experiments in which steady state conditions were developed. Consequently care must be taken in extrapolating their findings in the case of complete rapid drawdown, to materials that are not freely draining."<sup>5</sup> Anagnosti commenting on the same work<sup>4</sup> wrote, "It is necessary to emphasize that the results presented in the report and calculations may be used only in the case of rigid incompressible embankments. Pore pressures in an earth fill embankment due to consolidation of fill after drawdown should be investigated, applying the theory of consolidation or by introducing the triaxial test constants A and B in the well-known Skempton's equation. Therefore, the pore pressure due to consolidation of the fill is to be added to the pore pressure of the water head of non-steady filtration. It is not clear at the moment what the effect of the excessive pore pressure is to the filtration flow net."<sup>5</sup> Doubts have also been expressed as to whether the same critical slip circle would be valid for various positions of the filter.<sup>6</sup> Thus, further investigation is necessary to discover: (1) Whether the flow net of rapid drawdown can be used for the stability analysis of earth dams with materials that are not freely draining; and (2) whether only one critical slip circle exists in earth dams independent of the position of the filter.

**Notation.**—The symbols adopted for use in this paper are defined where they first appear and are arranged alphabetically in the Appendix.

<sup>4</sup> Patel, V. J., and Maheshwari, B. L., "Pore Pressure in Earth Dams," *Proceedings, 5th Internatl. Conf. on Soil Mechanics and Foundation Engrg.*, Vol. II, Paris, July, 1961, pp. 687-690.

<sup>5</sup> *Proceedings, 5th Internatl. Conf. on Soil Mechanics and Foundation Engrg.*, Paris, July, 1961, Vol. II, pp. 690, Vol. III, pp. 351.

<sup>6</sup> *Proceedings, Symposium on Earth and Rock-fill Dams*, University of Roorkee, Roorkee, India, December, 1961.

## CRITERIA FOR STABILITY ANALYSIS

The stability analysis is generally carried out for the different conditions possible such as construction period, full reservoir, partial drawdown, slow drawdown and complete rapid drawdown of the reservoir water level. From all these possibilities, generally speaking, the complete rapid drawdown condition provides the minimum factor of safety. The factor of safety for earth dams on pervious foundations is higher than for dams on impervious foundations.<sup>7</sup> Similarly, slow drawdown gives a higher factor of safety than rapid drawdown.<sup>8</sup> Thus, in general, sudden drawdown for an earth dam on an impervious foundation represents the worst condition and hence is accepted as the design criterion herein.

## EQUATION STABILITY ANALYSIS

Adopting the Fellenius method of slices for the cylindrical slip surface and assuming homogeneous isotropic material, Eq. 1a can be derived for the two-dimensional case.

$$F_s = \frac{\sum(d\sigma - du) \tan \phi - c \sum dl}{\sum d\tau} \quad (1a)$$

in which

$$F_s = \text{Factor of safety} = \frac{\text{total restoring force}}{\text{total force actuating the slice}} \quad (1b)$$

$d\sigma$  is the normal component of the weight of the slice acting on an area,  $dl$ , of the slip surface;  $dl$  denotes the length of the slip surface of the slice;  $du$  represents the neutral force due to pore pressure acting normal to the slip surface over an area,  $dl$ , during the case of rapid drawdown;  $c$  refers to the cohesive strength of the soil per unit area on the slip surface;  $d\tau$  stands for the tangential component of the weight of the slice acting on the area,  $dl$ ;  $\phi$  equals the angle of internal friction of soil; and  $\sum$  indicates summation.

Eq. 1a can be written after summation as

$$F_s = \frac{(\sigma - u) \tan \phi + cl}{\tau} \quad (2)$$

It is possible to write

$$\sigma = F_\sigma H^2 \gamma_{\text{sat}} \quad (3a)$$

<sup>7</sup> Patel, V. J., and Krishnayya, A. V. G., "Earth Dams with Upstream Slating Filter and Clay Blankets on Pervious Foundation," *Proceedings, Symposium on Earth and Rock-fill Dams*, Roorkee, India, December, 1961.

<sup>8</sup> Patel, V. J., and Arora, K. L., "Effect of Slow Drawdown in Earth Dams with Upstream Slating Filters," *Indian Journal of Power and River Valley Development*, August, 1961, pp. 9-14.



the total normal force;

$$U = F_u H^2 \gamma_w \dots \dots \dots (3b)$$

the total neutral force;

$$1 = b' H \dots \dots \dots (3c)$$

the total length of the arc of the slip circle,  $b'$  being a constant; and

$$\tau = F_\tau H^2 \gamma_{sat} \dots \dots \dots (3d)$$

the total tangential force in which  $H$  is depth of water retained on the upstream face;  $\gamma_{sat}$  corresponds to the saturated density of the soil;  $\gamma_w$  is the density of water;  $b'$  equals the coefficient of slip circle as defined by Eq. 3c; and  $F_\sigma$ ,  $F_\tau$  and  $F_u$  are the coefficients of normal, tangential and neutral forces, respectively, as defined by Eqs. 3a, 3b and 3d.

Substituting for the various terms from Eqs. 3a to 3d in Eq. 2 the factor of safety can be expressed as

$$F_s = \frac{(F_\sigma H^2 \gamma_{sat} - F_u H^2 \gamma_w) \tan \phi + cb' H}{F_\tau H^2 \gamma_{sat}} \dots \dots \dots (4a)$$

or

$$F_s = \left[ \frac{F_\sigma}{F_\tau} - \frac{F_u \gamma_w}{F_\tau \gamma_{sat}} \right] \tan \phi + \frac{cb'}{\gamma_{sat} F_\tau H} \dots \dots \dots (4b)$$

For a given failure surface and pore pressure distribution

$$F_s = \left( B_1 - B \frac{\gamma_w}{\gamma_{sat}} \right) \tan \phi + \frac{cB_2}{H\gamma_{sat}} \dots \dots \dots (5)$$

in which  $B_1$  and  $B_2$  are constants equal to  $F_\sigma/F_\tau$ ,  $F_u/F_\tau$  and  $b'/F_\tau$ , respectively.

#### METHODS OF COMPUTING PORE WATER PRESSURE FOR RAPID DRAWDOWN

If the material in the earth dam is assumed to be able to drain the water freely after complete rapid drawdown, the pore pressures can be computed from the rapid drawdown flow nets.

If the material in the dam is of impervious clay, free drainage is questionable. The pore pressure in this case can be computed using Skempton's pore pressure coefficients as suggested by Bishop.<sup>9</sup> Referring to Fig. 1 the pore pressure,  $u$ , at any point on the slip circle is

$$u = \gamma_w [h_c - h' + (1 - n) h_r] \dots \dots \dots (6)$$

in which  $n$  refers to the porosity of the pervious material of the casing of the dam;  $h_r$  denotes the depth of the pervious material;  $h'$  represents the difference between the full depth of water,  $H$ , and the head corresponding to the potential under full reservoir condition at the point where the pore pressure is required; and  $h_c$  is the vertical intercept between the upstream face of the impervious zone and the point at which the pore pressure is required. If the pervious material upstream of the impervious zone is not included in the stability analysis, Eq. 6 can be written as

$$u = \gamma_w (h_c - h') \dots \dots \dots (7)$$

The validity of Eq. 7 is confined to steady state conditions only.

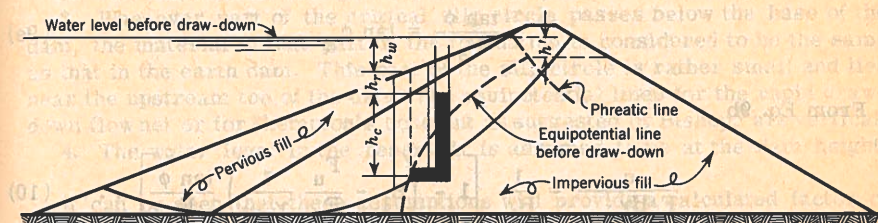


FIG. 1.—EARTH FILL DAM CROSS SECTION

Another method usually adopted in the absence of flow nets is to consider pore pressure equal to full buoyancy. According to this method,  $F_u$  is assumed equal to  $F_\sigma$ . Taking  $\gamma_w$  as unity, Eq. 5 can be written as

$$F_s = B_1 \left( 1 - \frac{1}{\gamma_{sat}} \right) \tan \phi + \frac{cB_2}{H\gamma_{sat}} \dots \dots \dots (8)$$

All three methods have been used in calculating the factor of safety herein.

#### CALCULATION OF THE FACTOR OF SAFETY

Eq. 5 can be written in a more convenient form for a given slip circle as

<sup>9</sup> Bishop, A. W., "The Use of Pore Pressure Coefficients," *Geotechnique*, London, Vol. IV, 1954, pp. 148-152.



$$\left[ B_1 - \frac{F_u}{F_\tau} \frac{1}{\gamma_{\text{sat}}} \right] \frac{\tan \phi}{F_s} + \frac{c B_2}{F_s H \gamma_{\text{sat}}} = 1 \dots \dots \dots (9a)$$

or

$$\frac{c}{F_s H} = \frac{\gamma_{\text{sat}}}{B_2} \left[ 1 - \left( B_1 - \frac{F_u}{F_\tau} \frac{1}{\gamma_{\text{sat}}} \right) \frac{\tan \phi}{F_s} \right] \dots \dots (9b)$$

or

$$\frac{c_d}{H} = \frac{\gamma_{\text{sat}}}{B_2} \left[ 1 - \left( B_1 - \frac{F_u}{F_\tau} \frac{1}{\gamma_{\text{sat}}} \right) \tan \phi_d \right] \dots \dots \dots (9c)$$

in which

$$\frac{c}{F_s} = c_d \dots \dots \dots (9d)$$

and

$$\frac{\tan \phi}{F_s} = \tan \phi_d \dots \dots \dots (9e)$$

From Eq. 9b

$$\frac{c}{F_s H \gamma_{\text{sat}}} = \frac{1}{B_2} \left[ 1 - \left( B_1 - \frac{F_u}{F_\tau} \frac{1}{\gamma_{\text{sat}}} \right) \frac{\tan \phi}{F_s} \right] \dots \dots (10)$$

Using Eq. 9b, values of  $c/F_s H$  for different values of  $\tan \phi / F_s$  can be calculated for different slip circles. The critical circle is the one which has the highest values of  $c/F_s H$  surrounded by lower values of  $c/F_s H$  for a given value of  $\tan \phi / F_s$  and  $\gamma_{\text{sat}}$ . This method of finding the critical circle has the advantage of eliminating two variables, namely  $c$  and  $H$ . If Eq. 5 is used, factors of safety for every slip circle can be calculated for different dam heights, cohesive strengths and angles of internal friction to find the critical slip circle.

The term,  $c/F_s H \gamma_{\text{sat}}$ , of Eq. 10 is similar to Taylor's stability number<sup>10</sup> ( $c/F_c H \gamma_{\text{sat}}$ ) with the difference that the factor of safety is with respect to strength instead of cohesion. Also the term,  $1/\gamma_{\text{sat}}$ , on the right side of Eq. 10 cannot be eliminated as is possible in dry slopes. The lowest and highest possible values of the saturated density of the impervious soil used for earth dam construction may vary between 1.8 tons per cu m and 2.2 tons per cu m. Hence, an average value of 2.0 tons per cu m can be assumed in finding the values of  $c/F_s H$  for various slip circles. It will be shown later that the variation of  $\gamma_{\text{sat}}$  has no effect in changing the critical circle and hence this method of finding the critical circle is advantageous.

<sup>10</sup> Taylor, D. W., "Stability of Earth Slopes," *Journal of the Boston Society of Civil Engineers*, Boston, Mass., July, 1937.

## LOCATION OF CRITICAL SLIP CIRCLES

The stability analysis for complete rapid drawdown was carried out for an earth dam with a vertical filter for different upstream slopes, i.e. 1 on 1, 1 on 1.25, 1 on 1.5, 1 on 2, 1 on 2.5, 1 on 2.75 and 1 on 3 (1 vertical on 3 horizontal) for filter positions of 0.25 H, 0.5 H, 1.0 H, 1.5 H, 2.0 H and 2.5 H from the axis of the earth dam, and for combinations of the soil characteristics, i.e. tangent of the angle of internal friction,  $\tan \phi$ , of 0.1, 0.2, 0.3, and 0.4; and saturated soil density,  $\gamma_{\text{sat}}$ , of 1.8, 2.0 and 2.2. A number of slip circles were taken, all passing through the upstream toe of the dam and a few passing through the dam foundation.

The following assumptions were considered necessary for the simplification of calculations:

1. The material above the phreatic line is assumed fully saturated and without pore pressure. The material in this zone will become saturated due to capillary rise. This assumption will not lead to any appreciable change from the true values. The values of  $c$  and  $\phi$  for this material are also assumed to be similar to the fully saturated zone for the simplification of calculations. Thus the calculation yields a lower factor of safety than the actual factor of safety, thus allowing the designer to be on the conservative side.
2. The material is homogenous, and isotropic.
3. Whenever part of the critical slip circle passes below the base of the dam, the material in that part of the foundation is considered to be the same as that in the earth dam. This part of the slip circle is rather small and lies near the upstream toe of the dam. The equipotential lines for the rapid drawdown flow net or for Skempton's constant as suggested by Bishop,<sup>9</sup> are vertical.
4. The water level in the reservoir is assumed to be at the dam height.

It can be seen that these assumptions will provide a calculated factor of safety less than the actual factor of safety.

The values of  $c_d/H$  (Eq. 9c) have been computed, employing the variables previously mentioned, for each slip circle, and using the neutral forces obtained by different methods. Tables 1(a) to 1(f) show values of  $c_d/H$  for the upstream slope of 1 on 2.5 for a number of slip circles surrounding the critical circle. The critical circle is defined by the maximum value of  $c_d/H$ . The critical circles have been located for other upstream slopes also considering all the variables (Fig. 2). In each case the center of the critical slip circle is surrounded by the centers of the circles having lower values of  $c_d/H$ .

## EXAMINATION OF RESULTS

For the given soil characteristics, the critical circle remains the same even though the neutral forces are calculated by different methods. Tables 1(a), 1(c), 1(d). A qualitative explanation for these results can be given by referring to Tables 2 and 3. The change in neutral force occurs approximately in the same proportion for all circles when the filter position is changed. From Table 2, it can be seen that the ratio of  $F_u/F_\tau$  for the vertical filter positions at 2.5 H and 0.25 H for a number of circles is practically equal to 1.3. Similarly, from Table 3 it can be seen that the ratios of  $F_u/F_\tau$  calculated



TABLE 1

Circle Number	$\frac{F_p}{F_r}$	$\frac{F_u}{F_r}$	$\frac{b}{F_r \gamma_{sat}}$	$F_r$	$\frac{C_d}{H}$ from Eq. 9c			
					$\tan \phi_d = 0.1$	$\tan \phi_d = 0.2$	$\tan \phi_d = 0.3$	$\tan \phi_d = 0.4$
(a) Pore Pressure By Bishop's Method; $\gamma_{sat} = 2.0^a$								
4	3.18	2.73	5.05	0.306	0.1620	0.1260	0.1080	0.0538
7	3.20	2.35	4.52	0.375	0.1765	0.1370	0.0870	0.0489
8	3.10	2.28	3.80	0.465	0.2110	0.1595	0.1083	0.0567
12	3.08	2.44	3.90	0.444	0.2084	0.1610	0.1132	0.0655
13	3.93	3.07	4.66	0.342	0.1625	0.1120	0.0605	0.0094
15	3.51	2.71	4.18	0.426	0.1876	0.1360	0.0840	0.0262
16	3.69	2.73	4.00	0.465	0.1920	0.1350	0.0760	0.0180
19	3.10	2.40	4.05	0.423	0.2001	0.1530	0.1060	0.0539
(b) Pore Pressure By Bishop's Method; $\gamma_{sat} = 2.0^b$								
4	3.18	3.28	5.05	0.306	0.1516	0.1251	0.0887	0.0523
7	3.20	3.04	4.52	0.375	0.1839	0.1478	0.1096	0.0625
8	3.10	3.06	3.80	0.465	0.2217	0.1804	0.1393	0.0978
12	3.08	3.25	3.90	0.444	0.2190	0.1820	0.1446	0.1075
13	3.93	3.99	4.66	0.342	0.1727	0.1314	0.0901	0.0488
15	3.51	3.49	4.18	0.426	0.1969	0.1549	0.1128	0.0707
16	3.69	3.28	4.00	0.465	0.1987	0.1475	0.0962	0.0450
19	3.10	3.24	4.05	0.423	0.2121	0.1753	0.1348	0.1016
(c) Pore Pressure From Rapid Drawdown Flow Nets; $\gamma_{sat} = 2.0^a$								
4	3.18	2.50	5.05	0.306	0.1598	0.1216	0.0833	0.0451
7	3.20	2.81	4.52	0.375	0.1814	0.1419	0.1023	0.0528
8	3.10	2.20	3.80	0.465	0.2104	0.1578	0.1052	0.0626
12	3.08	2.13	3.90	0.444	0.2045	0.1531	0.1016	0.0602
13	3.93	2.70	4.66	0.342	0.1588	0.1036	0.0468	-0.0068
15	3.51	2.42	4.18	0.426	0.1906	0.1411	0.0741	0.0191
16	3.69	2.48	4.00	0.465	0.1887	0.1275	0.0662	0.0050
19	3.10	2.32	4.05	0.423	0.2007	0.1524	0.1041	0.0558
(d) Pore Pressure By Full Buoyancy; $\gamma_{sat} = 2.0^a$								
4	3.18	3.18	5.05	0.306	0.1665	0.1350	0.1035	0.07211
7	3.20	3.20	4.52	0.375	0.1856	0.1503	0.1145	0.0796
8	3.10	3.10	3.80	0.465	0.2222	0.1815	0.1407	0.0999
12	3.08	3.08	3.90	0.444	0.2166	0.1771	0.1377	0.0983
13	3.93	3.93	4.66	0.342	0.1720	0.1301	0.0882	0.0462
15	3.51	3.51	4.18	0.426	0.1972	0.1553	0.1135	0.0717
16	3.69	3.69	4.00	0.465	0.2040	0.1580	0.1120	0.0660
19	3.10	3.10	4.05	0.423	0.2104	0.1718	0.1335	0.0946
(e) Pore Pressure By Bishop's Method; $\gamma_{sat} = 1.8^a$								
4	3.18	2.73	5.61	0.306	0.1486	0.1189	0.0893	0.0597
7	3.20	2.35	5.022	0.375	0.1590	0.1209	0.0828	0.0447
8	3.10	2.28	4.22	0.465	0.1940	0.1520	0.1070	0.0636
12	3.08	2.44	4.33	0.444	0.1911	0.1510	0.1115	0.0716
13	3.93	3.07	5.178	0.342	0.1501	0.1072	0.0642	0.0212
15	3.51	2.71	4.444	0.426	0.1722	0.1290	0.0858	0.0426
16	3.69	2.73	4.444	0.465	0.1761	0.1272	0.0783	0.0294
19	3.10	2.40	4.50	0.423	0.1852	0.1481	0.1110	0.0739
(f) Pore Pressure By Bishop's Method; $\gamma_{sat} = 2.2^a$								
4	3.18	2.73	4.59	0.306	0.1757	0.1335	0.0913	0.0491
7	3.20	2.35	4.11	0.375	0.1918	0.1398	0.0978	0.0358
8	3.10	2.28	3.45	0.465	0.2300	0.1705	0.1105	0.0506
12	3.08	2.44	3.55	0.444	0.2264	0.1704	0.1152	0.0596
13	3.93	3.07	4.24	0.342	0.1762	0.1164	0.0566	-0.0032
15	3.51	2.71	3.80	0.426	0.2032	0.1432	0.0832	0.0232
16	3.69	2.73	3.64	0.465	0.2076	0.1403	0.0730	0.0057
19	3.10	2.40	3.68	0.423	0.2174	0.1627	0.1081	0.0535

<sup>a</sup>  $\frac{U}{s} = 1$  on 2.5,  $\frac{F}{s}$  is vertical at 0.25 H.

<sup>b</sup>  $\frac{U}{s} = 1$  on 2.5,  $\frac{F}{s}$  is vertical at 2.50 H.

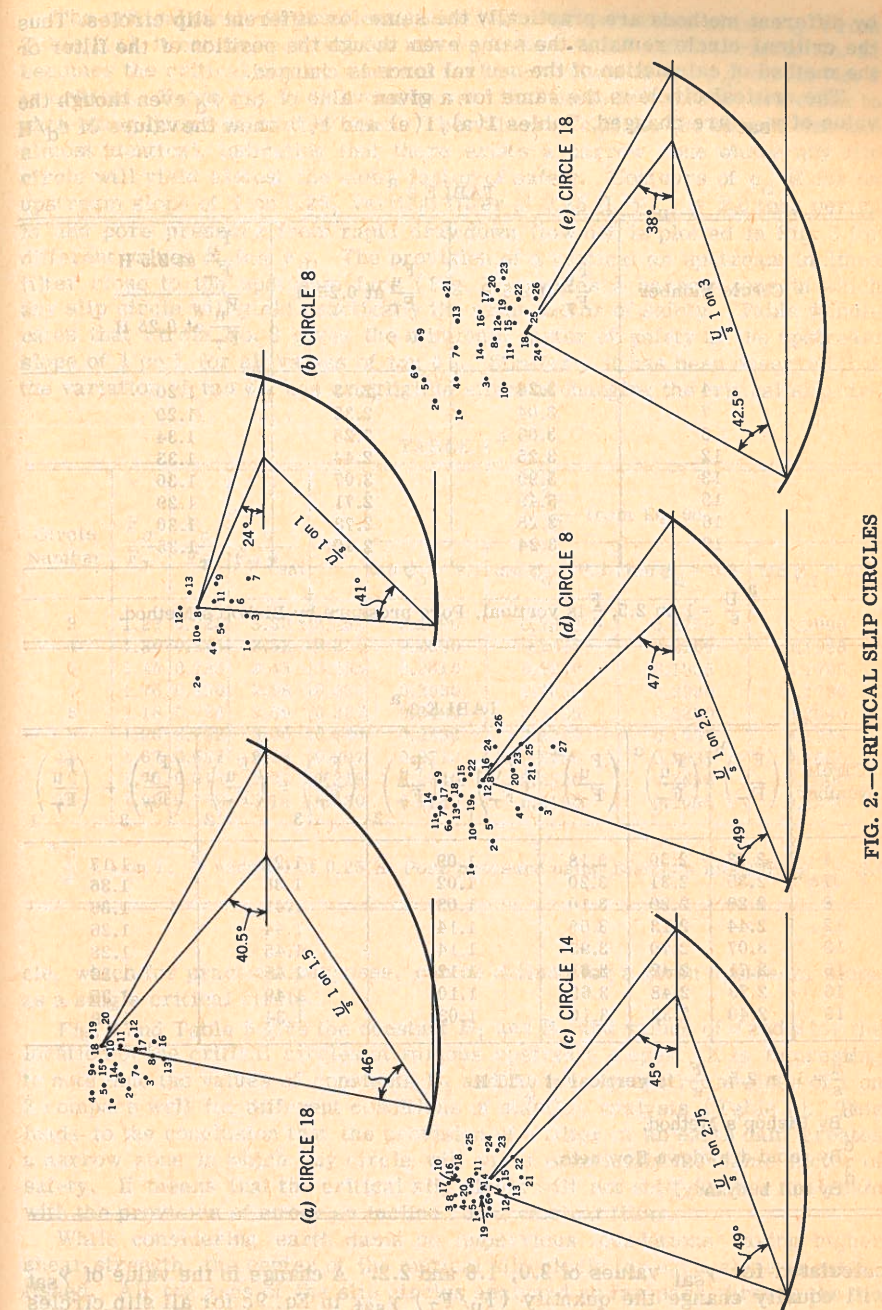


FIG. 2.—CRITICAL SLIP CIRCLES



by different methods are practically the same for different slip circles. Thus the critical circle remains the same even though the position of the filter or the method of calculation of the neutral force is changed.

The critical circle is the same for a given value of  $\tan \phi_d$  even though the value of  $\gamma_{sat}$  are changed. Tables 1(a), 1(e) and 1(f) show the values of  $c_d/H$

TABLE 2<sup>a</sup>

Circle Number	$\frac{F_u}{F_T}$ at 2.5 H	$\frac{F_u}{F_T}$ at 0.25 H	$\frac{F_u}{F_T}$ at 2.5 H
			$\frac{F_u}{F_T}$ at 0.25 H
4	3.28	2.73	1.20
7	3.04	2.35	1.29
8	3.06	2.28	1.34
12	3.25	2.44	1.33
13	3.99	3.07	1.30
15	3.49	2.71	1.29
16	3.28	2.73	1.20
19	3.24	2.40	1.35

<sup>a</sup>  $\frac{U}{S} = 1$  on 2.5,  $\frac{F}{S}$  is vertical, Pore pressure by Bishop's Method.

TABLE 3<sup>a</sup>

Circle Number	$\left(\frac{F_u}{F_T}\right)^b$	$\left(\frac{F_u}{F_T}\right)^c$	$\left(\frac{F_u}{F_T}\right)^d$	$\left(\frac{F_u}{F_T}\right)_1 \div \left(\frac{F_u}{F_T}\right)_2$	$\left(\frac{F_u}{F_T}\right)_3 \div \left(\frac{F_u}{F_T}\right)_2$	$\left(\frac{F_u}{F_T}\right)_3 \div \left(\frac{F_u}{F_T}\right)_1$
4	2.73	2.50	3.18	1.09	1.27	1.17
7	2.35	2.31	3.20	1.02	1.38	1.36
8	2.28	2.20	3.10	1.03	1.41	1.36
12	2.44	2.13	3.08	1.14	1.44	1.26
13	3.07	2.70	3.93	1.14	1.45	1.28
15	2.71	2.42	3.51	1.12	1.45	1.30
16	2.73	2.48	3.69	1.10	1.49	1.35
19	2.40	2.32	3.10	1.03	1.34	1.29

<sup>a</sup>  $\frac{U}{S} = 1$  on 2.5,  $\frac{F}{S}$  is vertical at 0.25 H

<sup>b</sup> By Bishop's Method.

<sup>c</sup> By rapid drawdown flow nets.

<sup>d</sup> By full buoyancy.

calculated for  $\gamma_{sat}$  values of 2.0, 1.8 and 2.2. A change in the value of  $\gamma_{sat}$  will equally change the quantity  $(F_u/F_T) \gamma_{sat}$  in Eq. 9c for all slip circles thus yielding the same critical circle.

The critical circle is practically identical, with variations of  $\tan \phi_d$ . From Table 1(a) it can be seen that for  $\tan \phi_d$  values of 0.3 and 0.4, circle No. 12 becomes the critical circle, whereas for lower values of  $\tan \phi_d$ , circle No. 8 is critical. From Fig. 2(d) it can be seen that these two circles are close to each other lying in a narrow zone. The values of  $c_d/H$  for the two circles are almost identical, indicating that there exists a narrow zone where any slip circle will yield almost the same factor of safety. Contours of  $c_d/H$  for an upstream slope of 1 on 1.25, vertical filter at 0.25 H,  $\gamma_{sat} = 2.0$  tons per cu m and pore pressure from rapid drawdown flow net is plotted in Fig. 3 for different values of  $\tan \phi_d$ . The provision of a vertical or upstream inclined filter close to the upstream face (Fig. 3) creates a narrow zone in which any slip circle will yield practically the same factor of safety. Table 4 indicates that circle No. 8 gives the minimum factor of safety at the upstream slope of 1 on 1 for all values of  $\tan \phi_d$ . Similarly it has been observed that the variation of  $\tan \phi_d$  has a negligible effect on changing the critical slip cir-

TABLE 4<sup>a</sup>

Circle Number	$\frac{F_u}{F_T}$	$\frac{F_u}{F_T}$	$\frac{b}{F_T \gamma_{sat}}$	$F_T$	$\frac{c_d}{H}$ from Eq. 9c			
					$\tan \phi_d = 0.1$	$\tan \phi_d = 0.2$	$\tan \phi_d = 0.3$	$\tan \phi_d = 0.4$
3	1.22	0.725	3.30	0.252	0.2790	0.2505	0.2250	0.2090
4	1.30	0.875	3.52	0.216	0.2600	0.2340	0.2100	0.1925
5	1.48	0.785	3.45	0.255	0.2610	0.2270	0.1955	0.1795
7	1.76	0.890	2.98	0.330	0.2960	0.2550	0.2090	0.1780
8	1.18	0.756	2.69	0.357	0.3460	0.3125	0.2820	0.2640
9	1.96	0.752	3.00	0.336	0.2830	0.2280	0.1750	0.1355
11	1.67	0.638	3.25	0.297	0.2710	0.2250	0.1835	0.1665
12	1.75	0.614	3.07	0.318	0.2840	0.2320	0.1850	0.1600
13	1.95	0.788	3.14	0.330	0.2740	0.2190	0.1695	0.1370

<sup>a</sup>  $\frac{U}{S} = 1$  on 1;  $\frac{F}{S}$  vertical of 0.25 H; Pore pressure using Bishop's method;  $\gamma_{sat} = 2.0$

cle, which for practical purposes, can be defined for a given upstream slope as a single critical circle.

Fig. 5 and Table 5 give the constant  $B_1$  and  $B_2$  and values of  $\alpha$  and  $\beta$  for the location of the critical circles at various upstream slopes. It is interesting to note that the values of constants  $B_1$  and  $B_2$  for the upstream slope of 1 on 2 compare well for different conditions of stability analysis (Table 6). This leads to the conclusion that the provision of a filter in an earth dam creates a narrow zone in which any circle will yield practically the same factor of safety. It means that the critical slip circle will not shift further upstream with the provision of either an inclined or a vertical filter.

While considering earth dams on impervious foundations having higher shear strength, the center of the critical slip circle for each slope is established. All the arcs of the slip circles analyzed so far pass through the toe of the dam. The validity of this assumption with special reference to earth



dams having filters close to the upstream slope should be established. In general, three cases can arise:

Case 1.—The foundation material is weaker (has a low shear strength) than the material comprising the impervious zone of the earth dam. In this case,

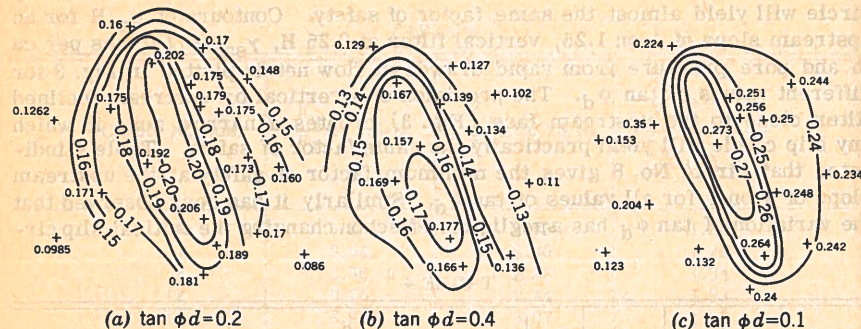


FIG. 3.—CONTOUR LINES OF  $c_d/H$  FOR SLOPE AND FILTER SHOWN IN FIG. 4

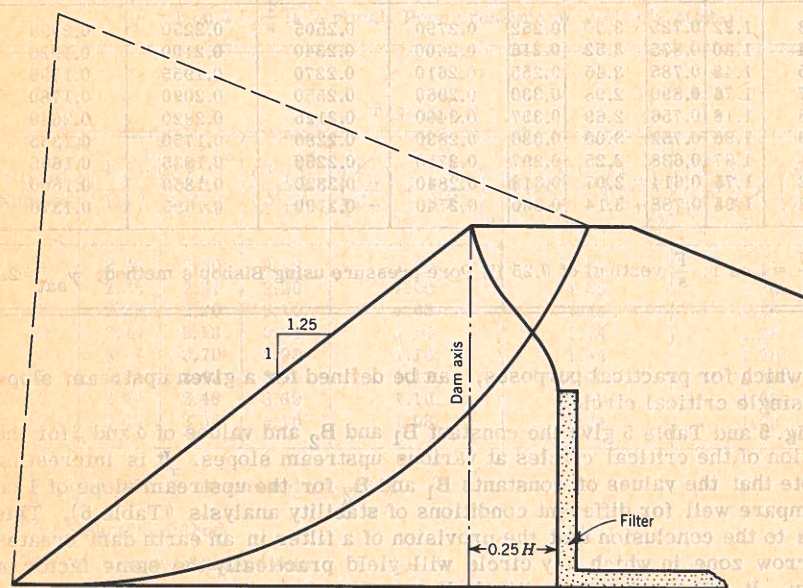


FIG. 4

it is more likely that the failure surface will be "deep" i.e. passing through the foundation material and not through the upstream toe.

Case 2.—The foundation materials and material comprising the dam are the same. In this case, for slope less than  $53^\circ$ , the critical circle will most probably pass through the toe of the upstream face.

Case 3.—The foundation material is stronger (has a higher shear strength) than the material in the impervious zone. The foundation material may be pervious or impervious. It is already established that the pore pressure decreases considerably and hence the factor of safety increases as the foundation material becomes more pervious, with higher shear strength, 4,7,11,12

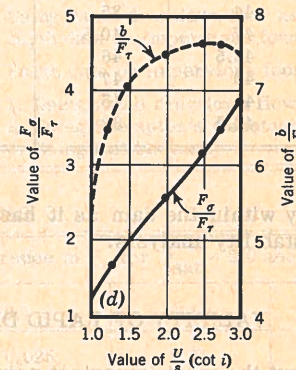
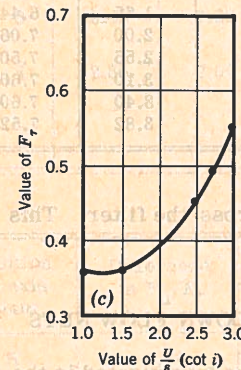
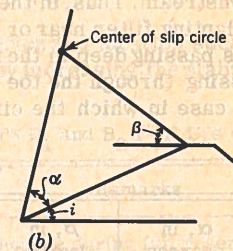
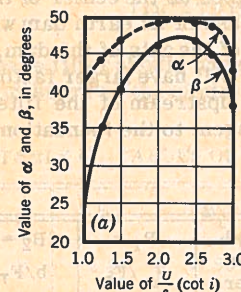


FIG. 5

and hence the impervious material in the foundation, having a higher shear strength represents the worst condition. This case will be considered herein.

The material used in the dam will generally be available at the site, thus the material in the dam, is usually the same as in the foundation (Case 2). Thus Case 1 hardly occurs and does not represent a general condition in earth

11 Patel, V. J., "Designing of Earth Dams with Upstream Slanting Filter," *Proceedings, First Asian Conf. on Soil Mechanics and Foundation Engrg.*, New Delhi, India, February, 1960.

12 Reinus, E., "Stability of Upstream Slope of Earth Dams," *Swedish State Committee for Building Research, Bulletin 12*, Stockholm, 1949.



dams. The critical slip circle may be tangent to the foundation surface if the foundation material has the higher shear strength. Thus Case 1 and Case 3 necessitate taking a few more circles, some tangent to the base of the dam or lying entirely within the dam in the case of the foundation material being stronger, and some passing deep in the foundation and cutting the ground upstream of the upstream toe of the dam. Any slip circle, passing deep in the foundation will cut the filter and hence the total percentage of dry material in the sliding mass will increase, thus reducing the pore pressure. Also the center of these circles will be downstream of that for circles passing through the upstream toe. But the shearing stress decreases as the center of the slip circle moves downstream. Thus, in the special case of an earth dam with vertical or upstream slanting filter near or upstream of the axis of the dam, it is likely that the circles passing deep in the foundation will have larger factors of safety than those passing through the toe and lying upstream of the filter. This is similar to the case in which the circle is tangent to the foundation surface or

TABLE 5

$\frac{U}{s}$	$\alpha$ , in degrees	$\beta$ , in degrees	Critical Circle Number	$B_1 = F_\sigma / F_\tau$	$B_2 = b / F_\tau$	$F_\tau$
1 on 1	41	24	8	1.18	5.38	0.357
1 on 1.25	44	35	4	1.65	6.44	0.350
1 on 1.5	47	40.5	18	2.00	7.06	0.357
1 on 2	48.5	46	14	2.55	7.50	0.390
1 on 2.5	49	47	8	3.10	7.60	0.465
1 on 2.75	49	45	14	3.40	7.60	0.510
1 on 3	42.5	38	18	3.82	7.52	0.552

lies, entirely within the dam as it has to cross the filter. This fact is also verified by stability analysis.

#### VALIDITY OF RAPID DRAWDOWN FLOW NETS

The last but the most important point to be investigated is the effect of the nonvalidity of the rapid drawdown flow nets, on the factor of safety using materials that are not freely draining. Referring to Eq. 4b it is clear that the difference in the factor of safety of the earth dam for two types of soils, namely, freely draining and one that is not freely draining can be given as

$$\text{Difference in } F_s = \frac{(\text{difference in } F_u / F_\tau) \tan \phi}{\gamma_{\text{sat}}} \dots (11)$$

The change in the value of  $F_u / F_\tau$  and the factor of safety for the upstream slope of 1 on 3 for freely draining and non-freely draining material is given in Table 7.

The change in the factor of safety when the filter is placed at 0.25 H only affects the third significant figure and hence is quite negligible as the factor

of safety is normally specified to two significant figures only. Thus the controversy of the validity (or non validity) of the rapid drawdown flow net hardly affects the factor of safety, leading to the conclusion that the rapid drawdown flow net for freely draining material can also be used for materials that are not freely draining when the filter is placed within 0.25 H of the axis because the curves of  $F_u / F_\tau$  for the freely and non freely draining conditions show a tendency to meet as the filter is moved upstream (Fig. 6).

#### LOCATION OF INCLINED FILTER

A drainage filter should have an inclination and location such that it reduces the pore pressure to the minimum, thereby increasing the stability of the dam.

TABLE 6.—VALUES OF CONSTANTS  $B_1$  and  $B_2$  FOR U/s OF 1 ON 2

	$B_1$	$B_2$	Remarks
1	2.14	7.50	Earthquake during full reservoir <sup>13,14</sup>
2	2.77	7.29	Using rapid drawdown flow net for earth dam on previous foundation <sup>7</sup>
3	2.49	7.50	Using rapid drawdown flow net for earth dam on impervious foundation <sup>4</sup>
4	2.55	7.50	Earth dam on impervious foundation 1. Using rapid drawdown flow net 2. Using Skempton's Constants 3. Using full buoyancy

TABLE 7

Filter position from the axis of the dam	Difference in $F_u / F_\tau$	Difference in $F_s$ for $\gamma_{\text{sat}} = 2.0$ and values of $\tan \phi$		
		0.1	0.2	0.3
0.25 H	0.50	0.025	0.05	0.075
1.00 H	0.70	0.035	0.07	0.105
2.50 H	1.50	0.75	0.15	0.225

This can be achieved by providing an inclined filter in the vicinity of the critical slip circle. The arcs of the critical slip circles located for different upstream slopes pass through a narrow region (Fig. 7), suggesting that the location of the filter for the maximum pore pressure reduction is approximately the same for all upstream slopes. Table 5 lists the values of  $\alpha$  and  $\beta$

<sup>13</sup> Patel, V. J., and Bokil, S. D., "Effect of Earthquake on Pore Pressure in Earth Dams," *Proceedings*, 2nd Symposium on Earthquake Engrg., University of Roorkee, Roorkee, India, November, 1962.

<sup>14</sup> Patel, V. J., and Bokil, S. D., "Earthquake Resisting Designs of Earth Dams," *Proceedings*, 2nd Asian Conf. on Soil Mechanics and Foundation Engrg., Tokyo, Japan, May, 1963, pp. 301-305.



for different slope value used in Fig. 10. The centers of the critical circles for various upstream slopes lie almost in a straight line meeting the axis at a distance  $0.5 H$  below the base of the dam. An inclined filter (slope 1 on 2) located at  $-0.6 H$  effectively reduces the pore pressure on the different slip circles for the various upstream slopes. The criterion for the location of the filter is that it be parallel and close to that part of the arc of the critical slip circle lying between the base and one-half the height of the dam.

The equipotential lines are traced by electrical analogy on "Teledeltos" paper for full reservoir and rapid drawdown conditions for upstream slopes of 1 on 1.25, 1 on 1.5, 1 on 2, 1 on 2.5, 1 on 2.75, and the upstream inclined filter (slope 1 on 2) placed  $-0.6 H$  from the axis of the dam. The neutral forces on the critical slip circle have been computed for different upstream slopes for both the freely draining material and the non freely draining ma-

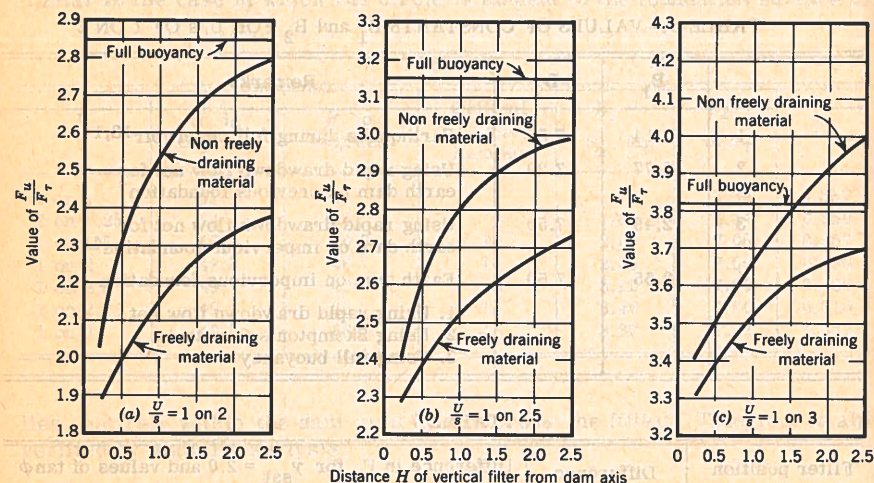


FIG. 6

terial [Fig. 8(a)]. Bishop's method yields smaller neutral forces than the rapid drawdown flow nets. This is due to the narrow zone of impervious material in which the values of  $h_c$  will be less than  $h'$  (Eq. 6) for most of the critical circle thereby resulting in a negative pore pressure. Hence to be on the safe side, the neutral forces obtained from the rapid drawdown flow nets have been used in the calculations [Fig. 8(b)].

## STABILITY CHART

As the values of constants  $B_1$ ,  $B_2$  and  $B$  for the various upstream slopes and for the inclined slope (1 on 2) and position (-0.6 H) of the filter are evaluated, it is possible to calculate the value of  $c_d H$  of Eq. 9c for various upstream slopes and a filter slope of 1 on 2 at -0.6 H from the axis of the dam,

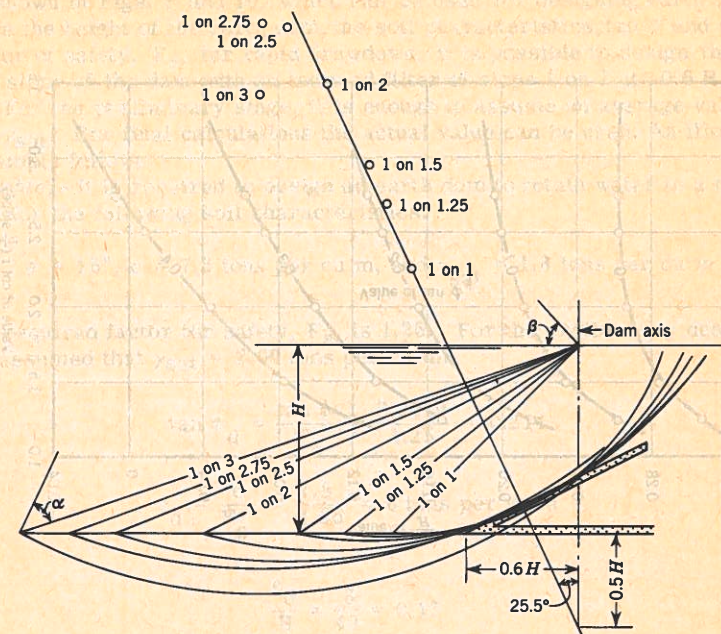


FIG. 7

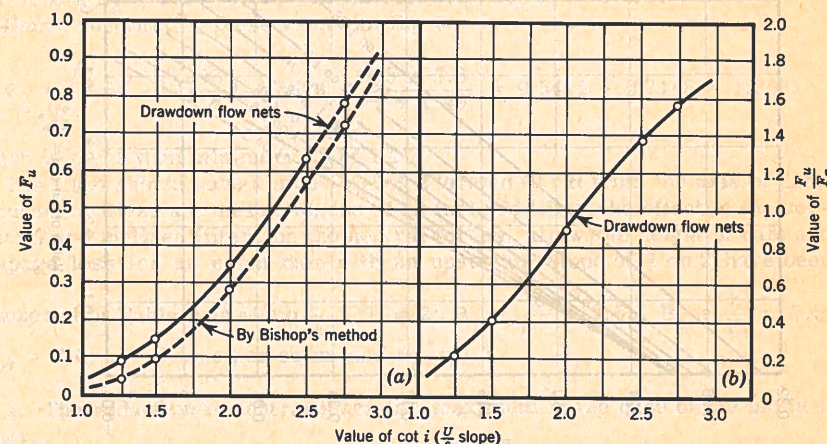


FIG. 8



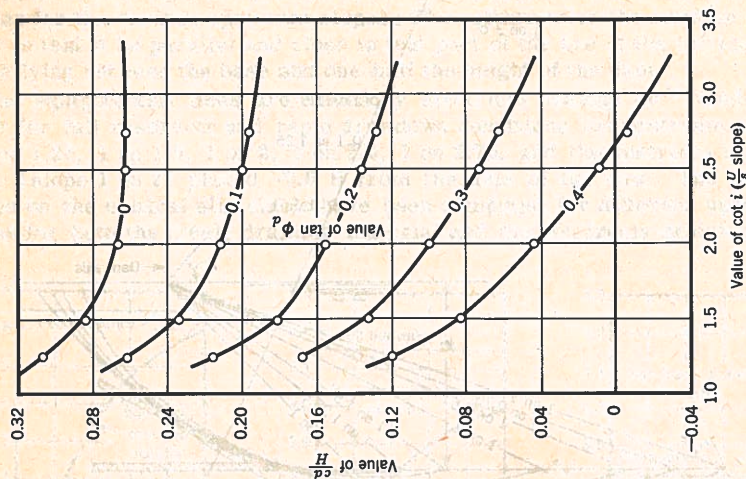


FIG. 10

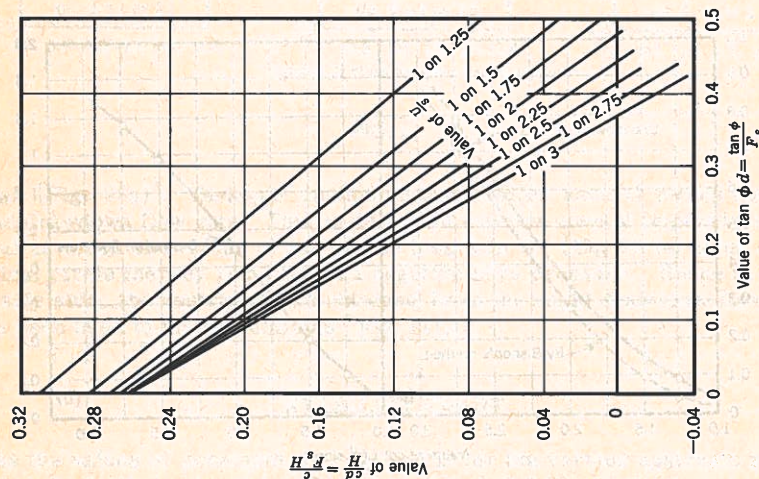


FIG. 9

and for an average  $\gamma_{\text{sat}}$  value of 2.0 and  $\tan \phi_d$  values of 0.0, 0.1, 0.2, 0.3 and 0.4 as shown in Figs. 9 and 10, which can be used for designing earth dams.

Given the height of the earth dam, the soil characteristics,  $\tan \phi$  and  $c$ , and the factor of safety,  $F_s$ , for rapid drawdown, it is possible to design the upstream slope of the dam with an inclined filter of slope 1 on 2 at  $-0.6 H$  from Fig. 8. For the preliminary stage, it is enough to assume an average value of 2.0 for  $\gamma_{\text{sat}}$ . For final calculations the actual value can be used. An illustrative example follows.

*Example.*—It is required to design an earth dam to retain water to a height of 40 m for the following soil characteristics:

$$\phi = 15^\circ, c = 7.5 \text{ tons per cu m, and } \gamma_{\text{sat}} = 1.8 \text{ tons per cu m}$$

The required factor for safety,  $F_s$ , is 1.25. For the preliminary design it can be assumed that  $\gamma_{\text{sat}} = 2.00$  tons per cu m.

$$\tan \phi_d = \frac{\tan \phi}{F_s} = \frac{0.2679}{1.25} = 0.214$$

$$c_d = \frac{c}{F_s} = \frac{7.5}{1.25} = 6 \text{ tons per sq m}$$

$$\frac{c_d}{H} = \frac{6}{40} = 0.15$$

For  $c_d/H = 0.15$  and  $\tan \phi_d = 0.214$ , the upstream slope of the earth dam from Fig. 8 is 1 on 2. Now the factor of safety can be calculated with  $\gamma_{\text{sat}} = 1.8$  from Eq. 9c. For an upstream slope of 1 on 2, from Table 5  $B_1 = F_\sigma/F_7 = 2.55$  and  $B_2 = b'/F_7 = 7.50$ . From Fig. 7(b)  $F_u/F_7 = 0.898$ .

Hence substituting these values in Eq. 9c

$$F_s = \left( 2.55 - \frac{0.898}{1.8} \right) 0.2679 + \frac{7.5 \times 7.5}{1.8 \times 40} = 0.5495 + 0.7188 = 1.2683$$

which is more than minimum  $F_s$  of 1.25.

The filter should have a slope of 1 on 2 located  $-0.6 H$  from the axis of dam. In order to illustrate the usefulness of an inclined filter, the effect of the horizontal and inclined filters on the neutral forces, allowable height of dam and seepage loss for an earth dam with an upstream slope of 1 on 2 have been

compared in Table 8, in which  $\frac{U}{s} = 1$  on 2;  $B_1 = \frac{F_\sigma}{F_7} = 2.55$ ;  $B_2 = \frac{b'}{F_7} = 7.5$ ;  $\gamma_{\text{sat}} = 2.0$ ;  $c = 7$  tons per sq m, and  $F_s = 1.3$ .

1. The reduction in neutral force is a maximum in the case of the inclined filter.

2. The allowable height of the dam is appreciably increased by the inclined filter for values of  $\tan \phi$  equal to or more than 0.4. Hence for higher values of  $\tan \phi$  it will be advantageous to use an inclined filter.



3. The quantity of seepage loss is not great in the case of an inclined filter when compared to a horizontal filter. It has also been proved<sup>15</sup> that the increase in quantity of seepage by provision of a vertical or upstream inclined filter forms the negligible part of the total water loss.

4. The provision of an upstream inclined filter makes it possible to design upstream and downstream slopes much steeper than in conventional design with a central impervious core. This will result in saving of earth material making the design more economical.<sup>15,16</sup>

### DESIGN OF THE FILTER

The filter should be thick enough to discharge the seepage through the dam. The seepage will be a maximum under full reservoir conditions.

TABLE 8<sup>a</sup>

Comparison of various effects	Horizontal filter at - 0.5 H	Inclined filter (1 on 2) at - 0.6 H
Neutral forces, ( $F_u/F_T$ )	1.30	0.898
Allowable height of dam, in meters, for		
$\tan \phi = 0.20$	28.2	30.1
$\tan \phi = 0.30$	36.3	39.5
$\tan \phi = 0.40$	45.6	57.6
Seepage quantity $q/kH$	1.23	1.38

$$^a \frac{U}{s} 1:2; B_1 = F_\sigma / F_T = 2.55, B_2 = b/F_T = 7.5, \gamma_{sat} = 2.0, c = 7 \text{ t/m}^2, F_s = 1.3$$

If  $q$  is the quantity of seepage through the dam per unit length, the thickness required for the inclined filter (Fig. 11) is

$$t_1 = \frac{q}{k_f} \sin \delta \dots \dots \dots (12)$$

in which  $k_f$  is the coefficient of permeability of the filter material and  $\delta$  is the inclination of the filter with the base of the dam measured counter clock-wise from downstream.

The thickness of the horizontal filter,  $t_2$  is calculated by treating the flow through the section ABCD of the horizontal filter as the flow through a dam

<sup>15</sup> Patel, V. J., and Krishnayya, A. V. G., "Quantity of Seepage in Earth Dams with Upstream Slanting Filter," Vishwakarma, Calcutta-12, India, May, 1962, pp. 55-62.

<sup>16</sup> Patel, V. J., "Economy Resulting from the New Technic in Earth Dam Design," Vishwakarma, Calcutta-12, February, 1962, pp. 21-27.

with upstream and downstream vertical faces separated by a distance,  $L$ , and with tail water depth,  $t_3$ . The quantity of seepage is accurately given by

$$q = \frac{k_f}{2L} (t_2^2 - t_3^2) \dots \dots \dots (13)$$

With a factor of safety of 2 and negligible tail water depth

$$q = \frac{k_f}{4L} t_2^2 \dots \dots \dots (14)$$

the thickness,  $t_2$ , required for the horizontal filter can be calculated from Eq. 14 after computing the quantity of seepage,  $q$ , from the flow nets.

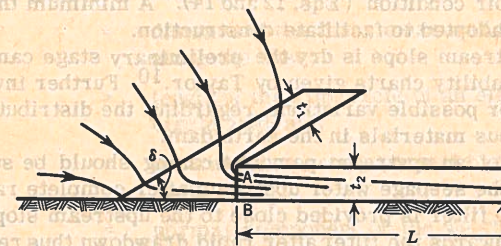


FIG. 11.—FILTER CROSS SECTION

A minimum thickness of 2 m for the filter is recommended to facilitate construction. A transition zone should be provided around the filter to avoid migration of fine particles.

### CONCLUSIONS AND RECOMMENDATIONS

1. There exists only one critical slip circle during rapid drawdown, full reservoir or earthquake during full reservoir condition for a given upstream slope irrespective of the height of the earth dam, soil characteristics and methods of calculating the pore pressures.
2. The provision of a vertical or inclined filter creates a narrow zone in which the factor of safety for different slip circles scarcely varies.
3. The critical slip circle will not move upstream with the provision of a filter. Thus the critical slip circle is a function of the upstream slope only.
4. The validity (or non validity) of rapid drawdown flow nets for materials that are not freely draining does not change the factor of safety appreciably when the vertical filter is placed at a distance of 0.25 H or less from the axis of the dam. Therefore, the rapid drawdown flow net can also be used for materials that are not freely draining.
5. The difference in pore pressures and hence the factors of safety calculated using full buoyancy and Skempton's constants as suggested by Bishop is negligible when the filter is placed at or beyond 2.5 H.



6. The critical slip circles for different upstream slopes located from the values of  $\alpha$  and  $\beta$  (Table 3) pass through a narrow region indicating the same slope and position of the drainage filter for all upstream slopes. An upstream inclined filter (slope 1 on 2) located  $-0.6 H$  from the axis of the earth dam effectively reduces the pore pressure on different critical slip circles for the various slopes.

7. The stability charts and nomographs could be constructed for calculating the factor of safety easily for earth dams with a vertical or inclined filter as the constants  $B_1$ ,  $B$ , and  $B_2$  in Eq. 9a are determined. The preliminary results thus obtained by using Fig. 9 can be further investigated for stability with possible variations such as a pervious casing and saturated density of the soil. While investigating further, the values of  $\alpha$  and  $\beta$  given in Table 3 can be used for the location of the preliminary critical circle. This will enable the designer to locate the actual critical slip circle in a few trials.

8. The thickness of the filter should be designed to drain the seepage safely during the full reservoir condition (Eqs. 12 and 14). A minimum thickness of two meters should be adopted to facilitate construction.

9. When the downstream slope is dry the preliminary stage can be determined by the use of stability charts given by Taylor.<sup>10</sup> Further investigation would be necessary for possible variations regarding the distribution of the pervious and impervious materials in the earth dam.

10. The thickness of an upstream pervious casing should be such that it will be able to drain the seepage water upstream after complete rapid drawdown. Since a drainage filter is provided close to the upstream slope, a number of flow lines run towards the filter after rapid drawdown thus reducing the quantity of seepage towards the upstream slope. Moreover when the drainability of the impervious clay core is poor a small quantity of seepage can be expected upstream. It has been found that a nominal thickness of 3 m will be sufficient for the pervious casing to drain the seepage as well as to protect the clay core from the effect of drying and wetting.<sup>17</sup>

11. A slope protection consisting of riprap should be provided over the upstream pervious casing to protect the slope from wave action.

12. This information can be used in designing earth dams with inclined zones and rock-fill dams if the upstream inclined filter is extended to the top of the dam and material downstream of the filter is the same pervious material as that of the filter.

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<sup>17</sup> Patel, V. J., and Krishnayya, A. V. G., "Rational Design of the Upstream Pervious Casing in Earth Dams," 2nd Asian Conf. on Soil Mechanics and Foundation, Engrg., Tokyo, Japan, May, 1963, pp. 316-320.

#### APPENDIX.—NOTATION

The following symbols have been adopted for use in this paper.

$B, B_1$ and $B_2$	=	constants;
$b'$	=	length of the arc of the slip circle in the impervious zone per unit height of the dam;
$c$	=	cohesive strength of the soil;
$c_d$	=	cohesive strength divided by the factor of safety with respect to the strength of soil;
$d$	=	distance of the filter measured from the axis of the dam along the base;
$dl$	=	length of the slip circle of an elemental slice in the slip circle analysis;
$du$	=	neutral force acting normal to the slip surface over an area, $dl$ ;
$d\sigma$	=	normal component of the weight of the slice acting on the area, $dl$ ;
$d\tau$	=	tangential component of the weight of the slice acting on the area, $dl$ ;
$F_s$	=	factor of safety with respect to the strength of the soil;
$F_u$	=	coefficient of neutral force or pore pressure;
$F_\sigma$	=	coefficient of normal force;
$F_\tau$	=	coefficient of tangential force;
$H$	=	depth of water retained on the upstream face, which will be referred to as the height of dam in this paper;
$h_c$	=	the vertical intercept between the upstream face of the impervious zone and the point at which the pore pressure is required;
$h_r$	=	the depth of the pervious material;
$h'$	=	difference between the full depth of water, $H$ , and head corresponding to the potential under full reservoir condition of the point where the pore pressure is required;
$k$	=	coefficient of permeability;
$k_f$	=	coefficient of permeability of the filter material;



L	=	length of the horizontal filter;
l	=	length of slip circle in the impervious zone;
m	=	meter length = 100 cm;
n	=	porosity;
q	=	quantity of seepage per unit length of earth dam;
t	=	metric ton = 1000 kg;
$t_1, t_2$	=	thickness of the drainage filter;
u	=	pore pressure;
$\gamma_w$	=	unit weight of water;
$\gamma_{sat}$	=	unit saturated weight of soil;
$\delta$	=	angle of inclination of the filter with the base of dam measured counter clockwise from downstream side;
$\sigma$	=	normal force;
$\tau$	=	tangential force;
$\phi$	=	angle of internal friction;
Axis of dam	=	the vertical line passing through the point of intersection of the upstream slope and the full reservoir water level;
d/s	=	downstream side;
F/P	=	filter position;
F/s	=	filter slope; and
U/s	=	upstream slope.

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## UNWATERING AKOSOMBO COFFERDAMS

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### SYNOPSIS

The Akosombo Dam is of the rockfill type, with a central vertical core of impervious material. Unwatering of the foundation area required cofferdams both upstream and downstream. Though the cofferdams were high, particularly the downstream one, they had to be built practically entirely under water. It was necessary to allow one season's flood to pass over the uncompleted cofferdams, and they could be sealed only after the flood had passed. The upstream cofferdam was then sealed in the conventional manner, by dumping sealing material on its upstream face from its crest. Because of the characteristics of material available, sealing the downstream cofferdam in similar fashion would have been expensive and time-consuming, and the time available was short. A method of sealing was therefore devised and used that took advantage of the special circumstances existing and required no equipment that was not already available. Artificial silting was induced by dredging clay dumped into the river, and discharging it near and along the downstream face. This method was entirely successful, and resulted in saving the contractor both time and money.

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