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2. A summary of approximately 50 words must accompany the paper, and a set of conclusions must end it.

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4. The author's full name, Society membership grade, and footnote reference stating present employment must appear on the first page of the paper. (Authors need not be Society members).

5. Mathematics are recomposed from the copy that is submitted. Because of this, it is necessary that letters be drawn carefully, and that special symbols be properly identified. The letter symbols used should be defined where they first appear, in the illustrations or in the text, and arranged alphabetically in an Appendix.

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9. Technical papers must be written in the third person.

10. A list of key words and an informative abstract should be provided for information retrieval purposes. (Information on preparation available on request.).

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Proceedings of the American Society of Civil Engineers

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(over)

Note.—Part 2 of this Journal is the 1965-33 Newsletter of the Soil Mechanics Division.

The three preceding issues of this Journal are dated January, 1965, March, 1965, and May, 1965.

DISCUSSION

INFORMATION RETRIEVAL

The key words, abstract, and reference "cards" for each article in this Journal represent part of the ASCE participation in the EJC information retrieval plan. The retrieval data are placed herein so that each can be cut out, placed on a 3 x 5 card and given an accession number for the user's file. The accession number is then entered on key word cards so that the user can subsequently match key words to choose the articles he wishes. Details of this program were given in an August 1962 article in CIVIL ENGINEERING, reprints of which are available on request to ASCE headquarters.

KEY WORDS: construction; consultants; earthwork; foundations; professional practice; risk; safety; soil mechanics

ABSTRACT: The meaning of the term "calculated risk" is first explored and the terms "unknown risk" and "human risk" are introduced. Several case histories are then reviewed for the purpose of demonstrating the importance of risks in earthwork and foundation engineering. The final section deals with the question of how to cope with risks, with emphasis on the use and abuse of Boards of Consultants for projects involving great hazards to life and property.

REFERENCE: Casagrande, Arthur, "Role of the 'Calculated Risk' in Earthwork and Foundation Engineering," Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 91, No. SM4, Proc. Paper 4390, July, 1965, pp. 1-40.

KEY WORDS: clay (material); permeability; pore pressure; seepage; shear strength; soil compaction; soil mechanics; soil structure; testing; thixotropy

ABSTRACT: The effects of molding water content, density, degree of saturation, method of compaction, and thixotropic hardening on the permeability of compacted silty clay have been determined. The formation of a dispersed structure in samples compacted wet of optimum may result in a coefficient of permeability two or three orders of magnitude less than for the same soil compacted dry of optimum. The actual decrease in permeability wet of optimum appears to correlate well with the degree of shear strain applied to the soil during compaction. In line with this, it was found that for samples compacted wet of optimum kneading compaction gave significantly lower values of permeability than did static compaction. Thixotropic hardening was accompanied by an increase in permeability, a result compatible with the concept that thixotropic hardening involves a change to a more flocculent structure. As much as a five-fold increase in permeability may accompany an increase in saturation from the as-compacted state to the fully saturated condition. Because of the great variability in permeability with compaction conditions, selection of an appropriate value for use in problems involving seepage or pore pressure dissipation will be difficult.

REFERENCE: Mitchell, James K., Hooper, Don R., and Campenella, Richard G., "Permeability of Compacted Clay," Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 91, No. SM4, Proc. Paper 4392, July, 1965, pp. 41-65.

KEY WORDS: borings; density; evaluation; penetration; samplers; soil mechanics; soils (types); testing

ABSTRACT: In foundation test borings, the recovery of soil samples with a 2-in. OD split sample spoon driven with a 140-lb weight falling 30 in. has been in use for more than 30 yr. Recording the number of blows required to drive the spoon 12 in. has been called the "Standard Penetration Test." It provides an approximation of soil densities in situ. It adds little to the cost of boring operations but adds considerably to the evaluation of the results when it is properly performed and it's are limitations recognized. This paper traces the history of the test, the modifications that have been introduced, the variables inherent in the test, and sets forth the factors that affect the results. Applications in granular and cohesive soils are examined.

REFERENCE: Fletcher, Gordon F. A., "Standard Penetration Test: Its Uses and Abuses," Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 91, No. SM4, Proc. Paper 4395, July, 1965, pp. 67-75.

APPENDIX.—NOTATION

The following symbols have been adopted for use in this paper:

a = distance from the outside edge of the column footing to the column center, in feet;
b = distance from the outside edge of the wall footing to the column center, in feet;
c = width of wall footing, in feet;
K = pressure of the wall footing on the column footing, in kips per square foot;
K' = safe net upward soil pressure due to the loads on the column, wall, and wall footing, in kips per square foot;
L = dimension of the column footing parallel to the wall, in feet;
S = dimension of the column footing perpendicular to the wall, in feet; and
W = column load, in kips.

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STABILITY OF SLOPES IN ANISTROPIC SOILS

By Kwan Yee Lo,[1] A. M. ASCE

INTRODUCTION

Stability of slopes has been the subject of numerous publications in the literature of soil mechanics. In most of these papers, the soil is usually treated as a homogeneous isotropic material with constant strength throughout the slope under consideration, or in the layers into which the slope is divided arbitrarily. Even in the simple case of "$\phi = 0$" analysis, it is only recently that a rigorous solution for the case of strength increasing linearly with depth has been obtained.[2,3]

In nature, most soils are probably somewhat anisotropic because of the mode of deposition, the stress metamorphosis after deposition, or both. Consequently, the shear strength of some soils will vary with the direction of the slip plane with respect to physical vertical, for instance. It is important, therefore, to examine the effect of anisotropy on the conventional factor of safety in the design of earth slopes and cuts.

A few attempts have been made to study the anisotropy of undrained strength of soils.[4,5,6] Casagrande and Carrillo[4] suggested an expression for the

Note.—Discussion open until December 1, 1965. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Soil Mechanics and Foundations Division, Proceedings of the American Society of Civil Engineers, Vol. 91, No. SM4, July, 1965.

1 Supervising Foundation Engr., Dept. of Highways, Ontario, Canada.

2 Gibson, R. E., and Morgenstern, N., "A Note on the Stability of Cuttings in Normally-Consolidated Clays," Geotechnique, Institution of Civ. Engrs., London, England, Vol. 12, No. 3, 1962, pp. 212-216.

3 Kenney, T. C., "Stability of Cuts in Soft Soils," Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 89, No. SM5, Proc. Paper 3647, 1963.

4 Casagrande, A., and Carrillo, N., "Shear Failure of Anisotropic Soils, Journal of the Boston Society of Civil Engineers, Contribution to Soil Mechanics 1941-1953, 1944.

5 Hansen, J. Brinch, and Gibsen, R. E., "Undrained Shear Strength of Anisotropically Consolidated Clays," Geotechnique, Institution of Civ. Engrs., London, England, Vol. 1, 1948, pp. 189-204.

6 Jakobson, B., "Influence of Sample Type and Testing Method on Shear Strength of Clay Samples," Proceedings of the Royal Swedish Geotechnical Institute, Stockholm, No. 8, 1954.

strength in any direction in terms of the strengths in the principal directions. Hansen and Gibson[5] and Jakobson[6] considered the anisotropy of the initial state of stresses but assumed the strength (in terms of effective stresses) and deformation properties of the soil to be isotropic. Based on similar and additional assumptions, Schmertmann[7] compared the undrained shear strength determined from a field vane test and the in-situ strength existing in a slope. However, the inherent anisotropic property of the soil has not been considered. Recently, Ladd[8] furnished some data on the effect of rotation of principal stresses on the undrained strength of a clay.

Only the undrained case corresponding to the conventional $\phi = 0$ analysis will be considered herein. A general method of analysis that will take into account any random variation of strength of soil is described. Where the strengths in different directions can be defined by an expression involving the angle of deviation from the vertical and the "principal strengths," explicit solutions have been obtained for cases in which (1) the vertical principal

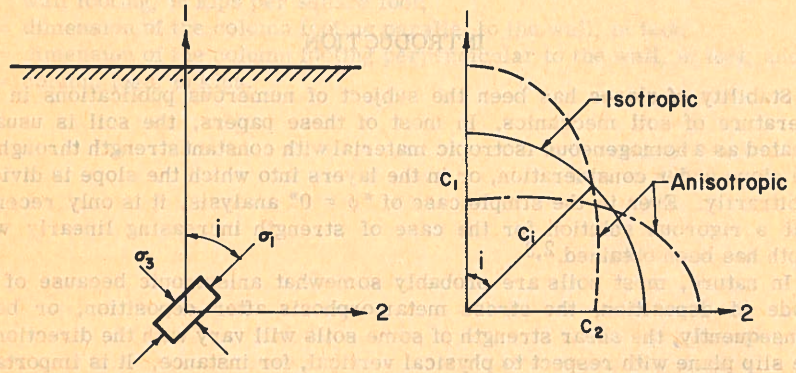

FIG. 1.—DEFINITION OF STRENGTH VARIATION WITH DIRECTION

strength, C_1, is constant, and (2) the vertical principal strength C_1, increases linearly with depth.

It is shown that the solutions obtained by Taylor[9] and Gibson and Morgenstern[2] for isotropic soils represent special forms of the solutions of foregoing cases, respectively. The stability numbers of these two cases have been calculated by an electronic computer for a range of soil parameters and slope angles. The practical significance of the results is considered briefly.

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7 Schmertmann, J. H., discussion of "Stability of Cuts in Soft Clays," by T. C. Kenney, Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 90, No. SM4, Proc. Paper 1964, pp. 183-189.

8 Ladd, C. C., correspondence in Geotechnique, Institution of Civ. Engrs., London, England, Vol. 14, 1964, pp. 353-358.

9 Taylor, D. W., "Stability of Earth Slopes," Journal of the Boston Society of Civil Engineers Contributions to Soil Mechanics 1925-1940, 1937.

Notation.—Letter symbols adopted for use in this paper are defined where they first appear and listed alphabetically in the Appendix.

ANISOTROPY OF CLAY

Definitions.—The term anisotropy is used exclusively herein to describe the variation of undrained shear strength with direction; the directional variation of other soil properties, such as compressibility or permeability, is not considered.

The definitions of shear strengths with direction are shown in Fig. 1. The physical vertical and horizontal directions (these usually coincide with lines perpendicular and parallel to the bedding planes of a soil deposit) are the principal directions. If a sample is tested with the direction of the major principal stress coinciding with the principal directions, the strengths thus determined are termed "principal strengths" and are designated as C_1 and C_2, respectively. When the major principal stress makes an angle, i, with the vertical, the strength determined will be denoted by C_i.

For isotropic material, $C_1 = C_i = C_2$; and the curve traced by C_i in a vertical plane is a circle. For anisotropic material, the locus of C_i can assume any form other than a circle. The ratio of the principal strengths C_2/C_1 will be termed the degree of anistropy and may be less or greater than one. For convenience, the former case will be referred to as, "M-anisotropy," and the latter case "C-anisotropy."

Origin and Types of Anisotropy.—The anisotropy of clays is intimately connected with their structure, which depends on the environmental conditions during which the soil is deposited as well as the stress changes subsequent to deposition. An excellent review of the concept of structure has been given by Rosenquist,[10] who demonstrated that clays laid down in salt water acquire an open card house structure with the particles randomly oriented. In a fresh water deposit, the structure is somewhat dispersed and a certain degree of parallelism is achieved between the clay particles. It is conceivable, therefore, that in the former case the clay is more or less isotropic in a macroscopic scale, while in the latter case, the clay will possess some inherent anisotropy. However, it is known that consolidation under deviatoric pressures tend to aline the clay particles. Under heavy overburden pressure, therefore, a clay that is initially isotropic may become anisotropic. The following examples illustrate three possible phenomena that may be commonly encountered.

1. Isotropy.—Jakobson[11] reported results of 34 unconfined compression tests performed on samples taken at right angles, diagonally, and parallel to a stratum of a post-glacial clay at a depth 10 ft from a site 75 miles northwest of Stockholm. The clay has an average liquid limit of 85%, plastic limit of 34%, and moisture content of 83%. It was found that the shear strengths in each type of test lie within the limit of the mean error of the other and are practically identical in all three directions as shown in Table 1. The clay is therefore isotropic.

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10 Rosenquist, I. Th., "Physico-Chemical Properties of Soils: Soil-Water Systems," Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 85, No. SM2, Proc. Paper 2000, 1959.

11 Jakobson, B., "Isotropy of Clays," Proceedings, European Conf. on Stability of Earth Slopes, Stockholm, Sweden, Vol. 1, 1955.

2. C-Anisotropy $C_2 > C_1$.—A large number of unconfined compression tests, 170 in all, were performed on block samples of London clay taken at depths varying from 55 ft to 160 ft from eleven sites in London.[12]

It was found that the horizontal strengths, C_2, are higher than the vertical strengths, C_1, at each site, except one site at which the clay is highly fissured. The ratio of C_2/C_1 was established to be 1.3 ± 0.1. Some tests were also performed with the major principal stress inclined to the vertical, but the angle of inclination was not clearly defined. Nevertheless, the anisotropic nature of the clay is well established. The higher strength exhibited in the horizontal samples may be related to the fact that London clay is heavily overconsolidated, and the horizontal stresses in the ground are considerably higher than the vertical stresses.[13]

It is important to note that the angle, f, between the failure plane and the plane normal to the direction of the major principal stress varies in its mean value between a relatively narrow limit of 50° to 60° for all tests at all sites, with an average of 56°. In fact, with few exceptions, the limit of variation is much smaller if results from each site are considered separately.

3. M-Anisotropy $C_1 > C_2$.—A comprehensive set of data was obtained in connection with the soils investigation for the construction of a tunnel in

TABLE 1.—RESULTS OF UNCONFINED COMPRESSION TESTS[11]

i	No. of tests	C_i, in kilograms per square centimeter
0	12	0.224 ± 0.033
45	12	0.254 ± 0.034
90	10	0.236 ± 0.025

Welland, Ontario. A shaft 5 ft in diameter was excavated 77 ft into the ground, and block samples were obtained at 4-ft intervals. More than 500 unconfined compression tests were performed on these samples as well as tube samples taken from the shaft. The complete results of the field and laboratory tests will be reported elsewhere.

For the purpose of investigating the strength variation with direction, tests were performed with the major principal stress inclined to the physical vertical at 0°, 15°, 30°, 45°, 60°, 75°, and 90°. End restraints were minimized by lubricating the platens with silicon grease. A typical set of results for samples D and H taken at depths of 42 ft and 60 ft, respectively, is shown in Fig. 2. The decrease in strength with the angle of rotation of the major principal stress is evident. The ratio of the undrained shear strengths C_2/C_1

12 Ward, W. H., Samuels, S. G., and Butler, M. E., "Further Studies of the Properties of London Clay," Geotechnique, Institution of Civ. Engrs., London, England, Vol. 9, No. 2, 1959, pp. 33-58.

13 Skempton, A. W., "Horizontal Stresses in an Overconsolidated Eocene Clay," Proceedings, 5th Internatl. Conf. on Soil Mechanics and Foundation Engrg., Paris, France, Vol. 1, 1961.

vary from 0.80 to 0.64 for all the block samples tested. The clay deposit is lightly overconsolidated with an overconsolidation ratio of 2.

Angle f, between the failure plane and the plane normal to the direction of the major principal stress, is plotted against the angle of ratation of the major principal stress in Fig. 2. It is evident that angle f is independent of angle i,

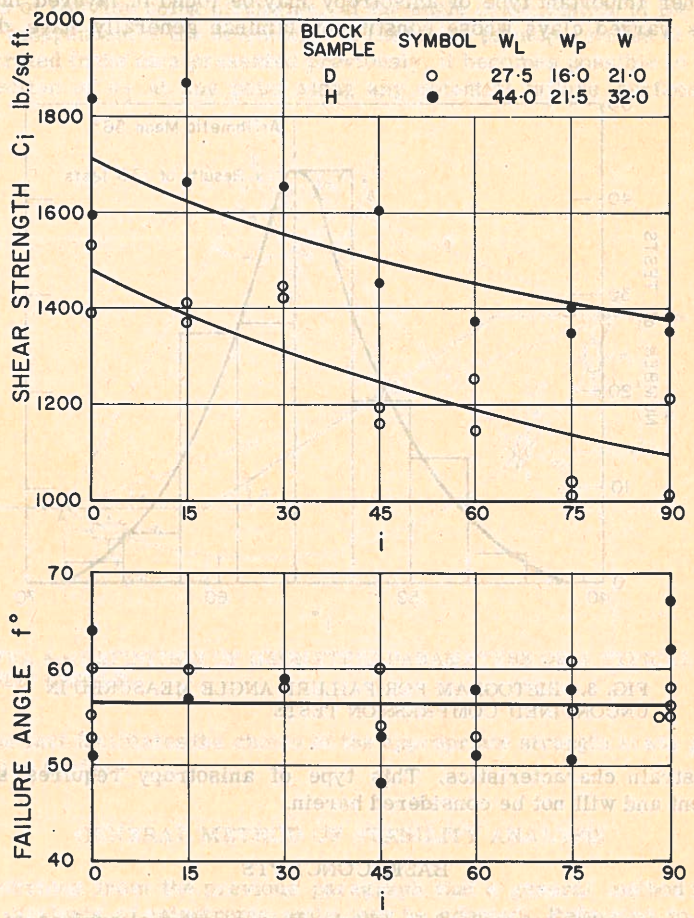

FIG. 2.—UNDRAINED SHEAR STRENGTH AND FAILURE ANGLE VS ANGLE OF ROTATION OF APPLIED MAJOR PRINCIPAL STRESS

as in the case of London clay. A statistical study of all the test results in which i is varied is shown in the form of a histogram in Fig. 3. The arithmetic mean of angle f is 56°.

These examples serve to illustrate some of the different types of anisotropy that may be encountered. Each type of anisotropy is closely connected, both with the environment of deposition of the clay and the stress changes to which it is subjected subsequent to deposition. The mode of variation of strength in example 3 would occur quite often. The methods of stability analysis examined in the following sections will deal mainly with this particular type of anisotropy, but the basic concepts and approach should be applicable to any phenomena of strength variations with direction.

Another important type of anisotropy may be found in layered materials such as varved clays whose constituent laminae generally have different

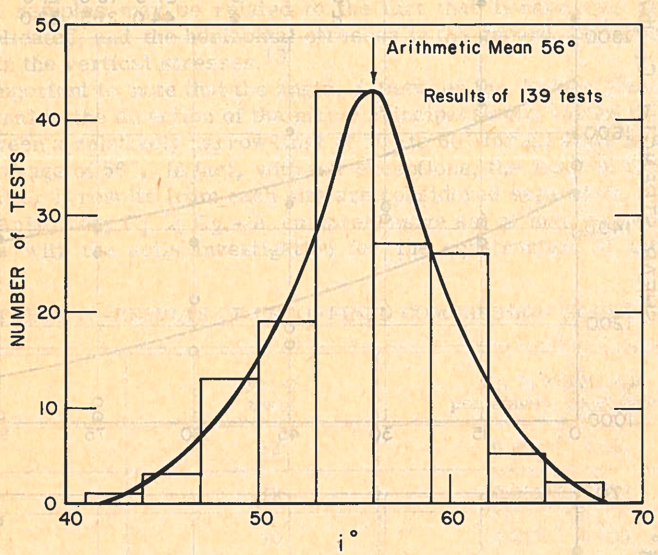

FIG. 3.—HISTOGRAM FOR FAILURE ANGLE MEASURED IN UNCONFINED COMPRESSION TESTS

stress-strain characteristics. This type of anisotropy requires separate treatment and will not be considered herein.

BASIC CONCEPTS

Consider a slope of angle β in which AB is a potential surface of rupture (Fig. 4). If failure of the slope is incipient, then every element of soil along AB must be in a state of limiting equilibrium. It will be assumed that AB is an arc of a circle. This assumption is justified in the undrained case, because both static and kinematic conditions are satisfied along the arc. Even when the particular nature of the soil is considered,[14] the absence of dilatancy dur-

14 Rowe, P. W., "The Stress-Dilatancy Relation for Static Equilibrium of an Assembly of Particles in Contact," Proceedings, Royal Soc. of London, Series A, Vol. 269, 1962.

ing shear in the undrained case makes a circular surface theoretically possible.

In the conventional methods of stability analysis, the effect of the rotation of principal stresses along the failure arc is not considered. However, near the top of the slope, the direction of the major principal stress, σ_1, is nearly vertical, while near the toe, σ_1 acts in an almost horizontal direction—the direction of rotation being clockwise. If the angle of inclination of the failure plane is independent of the orientation of the major principal stress at failure, as illustrated in the data presented previously, it becomes possible to specify the direction of σ_1 at any point along any potential failure surface. This

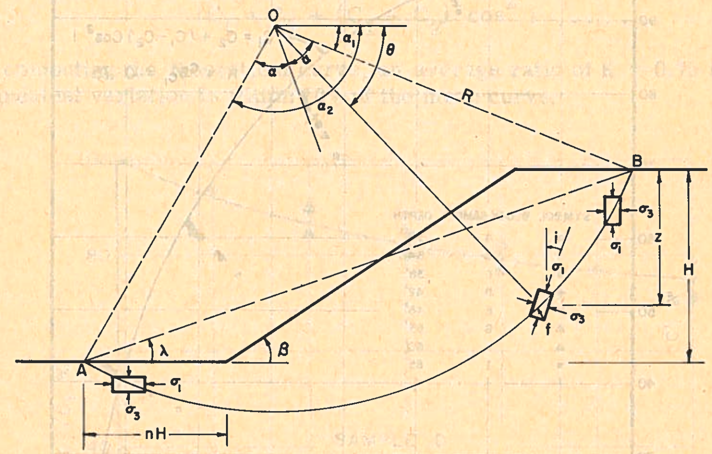

FIG. 4.—DEFINITION OF GEOMETRIC PARAMETERS OF A TYPICAL SLIP CIRCLE

important fact facilitates the choice of the appropriate strength at any point of the rupture surface.

GENERAL METHOD OF STABILITY ANALYSIS

It is obvious from the previous paragraph that a general method for the stability analysis of anisotropic soils may be adopted. Referring to Fig. 4 for the definition of geometric variables, the angle of inclination, i, of the major principal stress to the vertical is given by

$$i = f + \theta - \frac{\pi}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

For random variation of the undrained shear strength, C_i, both with respect to depth and orientation, the usual method of slices may be used. The appropriate shear strength applicable to each slice is simply chosen by averaging i along the failure arc in that slice, and C_i is then assumed to correspond to the average value of i from experimental data. The procedure then follows the usual method, and the analysis may be worked out by hand or by electronic computer.

RELATIONSHIP BETWEEN C_i AND i

While it is necessary to obtain a general, though approximate, method for stability analysis to account for the variability of natural soils, it is also both

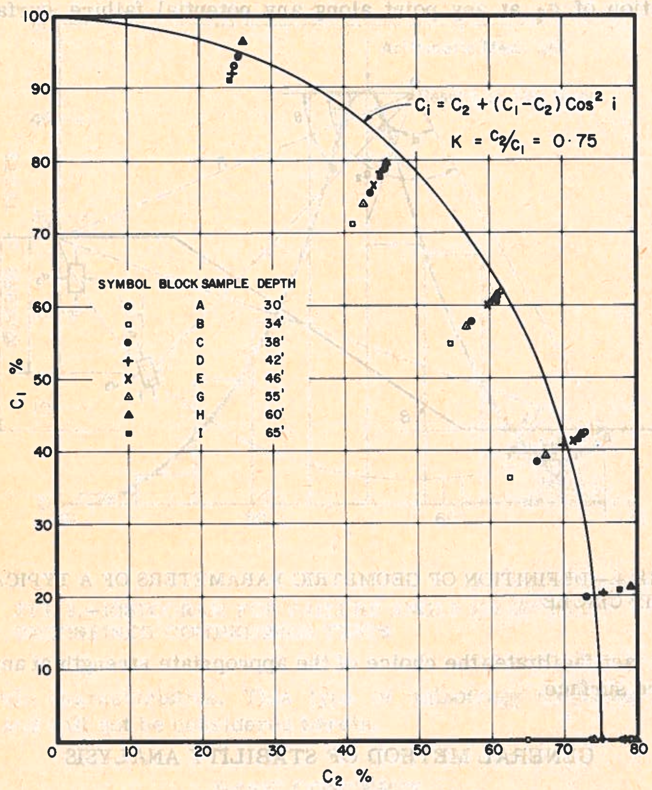

FIG. 5.—POLAR DIAGRAM OF EXPERIMENTAL STRENGTH VARIATION WITH ORIENTATION OF APPLIED MAJOR PRINCIPAL STRESS

interesting and important to arrive at rigorous solutions in order to study the consequences of the introduction of additional variables to the problem. It should, of course, be realized that the rigorous solutions obtained only apply when the basic idealized assumptions are satisfied, at least within reasonable limits.

To obtain a rigorous, solution, it is necessary to establish a mathematical relationship between the directional strength, C_i, and the angle of deviation, i, of the major principal stress at failure to the vertical. In order to examine whether such a relationship exists, the results of unconfined compression tests on Welland clay are plotted in a polar diagram (Fig. 5), in which the length of the radius is proportional to the shear strength measured at an angle, i.

For convenience of presentation, the percentage ratios of C_i/C_1, instead of the absolute values of shear strength are plotted. Each point represents the average of at least two tests. It is seen that the results of the eight block samples at different depths lie close to a theoretical curve represented by

$$C_i = C_2 + (C_1 - C_2) \cos^2 i \quad \text{.............} (2)$$

In computing the theoretical curve, an average ratio of K = 0.75 is taken. The greatest variation is within 10% of the mean curve.

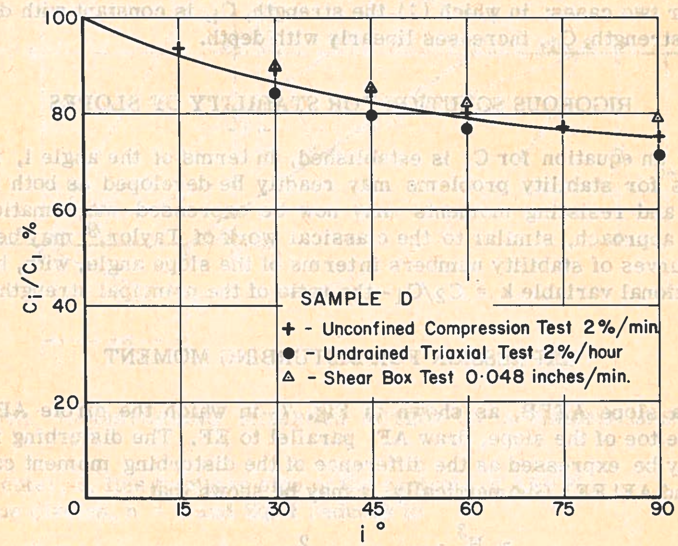

FIG. 6.—COMPARISON OF DIRECTIONAL STRENGTH VARIATIONS

In order to assure that the relationship between the strength C_1 and the angle i is not significantly influenced by the type of tests or rate of strain, shear box tests and unconsolidated, undrained triaxial tests were performed. The shear box used was of the conventional Bishop type, and i is taken as the angle of rotation of the bedding planes from the vertical so that the results can be compared with those from triaxial compression tests. The time to failure in the shear box tests was generally less than 3 min, while the triaxial tests were carried out at a strain rate of 2% per hr—i.e., one-sixtieth the strain rate of the unconfined compression tests. The results of these tests are plotted in Fig. 6. It is evident that Eq. 2 is not significantly affected by the type of test or rate of strain.

Eq. 2 was proposed by Casagrande and Carrillo[4] on intuitive rather than theoretical or experimental grounds. However, in addition to the experimental evidence cited previously, it is interesting to note that the effective stress in the ground, σ'_{it}, at any time, t, after the soil was deposited is given by an expression of identical form

$$\sigma'_{it} = \sigma'_{3t} + (\sigma'_{it} - \sigma'_{3t}) \cos^2 i \qquad (3)$$

Because, at a given time, a soil would have a given physical-chemical composition, the structure of the clay would be dependent on the effective stress history. Eq. 2 may therefore have some physical significance which at the present (1965) is not fully explored. The equation, of course, may not be valid for all soils, but it is conceivable that it may be applied to some soils commonly encountered. Therefore, Eq. 2 will be used to obtain rigorous solution for two cases; in which (1) the strength, C_i, is constant with depth and (2) the strength, C_1, increases linearly with depth.

RIGOROUS SOLUTION FOR STABILITY OF SLOPES

After an equation for C_i is established, in terms of the angle i, rigorous solutions for stability problems may readily be developed as both the disturbing and resisting moments may now be expressed mathematically. A general approach, similar to the classical work of Taylor,[9] may be used to obtain curves of stability numbers in terms of the slope angle, with, however, the additional variable $k = C_2/C_1$—the ratio of the principal strengths.

EXPRESSION FOR DISTURBING MOMENT

For a slope AEFB, as shown in Fig. 7, in which the circle AB passes below the toe of the slope, draw AF' parallel to EF. The disturbing moment, W d, may be expressed as the difference of the disturbing moment caused by AF'B and AF'FE. Geometrically, it may be shown that

$$W\,d = \frac{\gamma H^3}{12}\,[(1 - 2\cot^2\beta + 3\cot\lambda\cot\beta + 3\cot\alpha\cot\lambda - 3\cot\alpha\cot\beta) - 6n\,(n + \cot\beta - \cot\lambda + \cot\alpha)] \qquad (4)$$

writing

$$Y = 1 - 2\cot^2\beta + 3\cot\lambda\cot\beta + 3\cot\alpha\cot\lambda - 3\cot\alpha\cot\beta \qquad (5)$$

and

$$Z = 6\,n\,(n + \cot\beta - \cot\lambda + \cot\alpha) \qquad (6)$$

Then

$$W\,d = \frac{\gamma H^3}{12}\,(Y - Z) \qquad (7)$$

Eq. 7 is derived purely from geometric considerations and no assumption has been made. In deriving the expression for the disturbing moment, Taylor[9] imposed the restriction that the center of the critical circle be on the vertical passing through the midpoint of the slope. This is equivalent to requiring that

$$n = \frac{1}{2}\,(\cot\lambda - \cot\alpha - \cot\beta) \qquad (8)$$

This condition is relaxed in Eq. 4, although as will be shown subsequently, both methods lead to identical results for k = 1. Eq. 4 is valid whether or

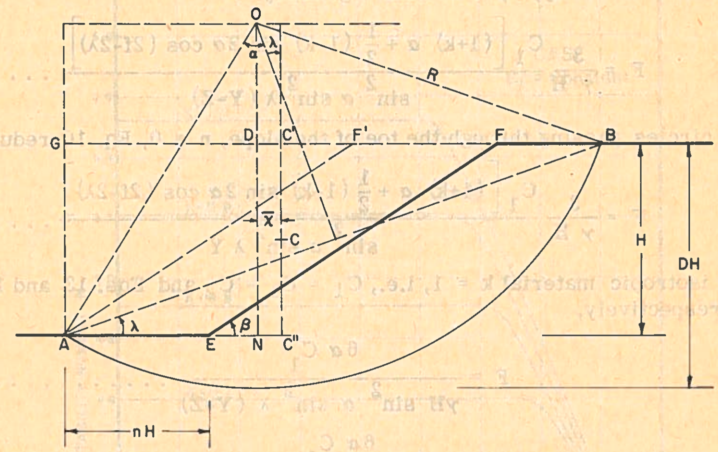

FIG. 7.—DISTURBING MOMENTS ACTING ON A SLIDING MASS

not the center, 0, lies to either side of the centroid of AEFF'.

For toe circles, n = 0 and Eq. 4 reduces to

$$W\,d = \frac{\gamma H^3}{12}\,(1 - 2\cot^2\beta + 3\cot\lambda\cot\beta + 3\cot\alpha\cot\lambda - 3\cot\alpha\cot\beta) \qquad (9)$$

Case 1.—The principal strengths C_1 and C_2 are constant with depth.

The simplest case to be dealt with is that the principal strengths C_1 and C_2 are constant with depth, and therefore the ratio of C_2 to C_1 is a constant. From Fig. 4, the resisting moment, M_R, is given by

$$M_R = R \int_{\alpha_1}^{\alpha_2} C(\theta, Z)\, R\, d\theta \qquad (10)$$

From Eqs. 1 and 2,

$$M_R = R^2 \int_{\alpha_1}^{\alpha_2} [C_2 + (C_1 - C_2) \cos^2 (\alpha + \theta)] \, d\theta \;\ldots\ldots (11)$$

in which a = f - π/2.

Eq. 11 may readily be integrated, and the explicit expression for M_R is

$$M_R = R^2 \left[(1 + k) \, C_1 \, \alpha + \frac{1}{2} (1 - k) \, C_1 \sin 2\alpha \cos (2f - 2\lambda) \right] \;\ldots\ldots\ldots (12)$$

The factor of safety, F, is therefore given by

$$F = \frac{3}{\gamma H} \; \frac{C_1 \left[(1+k) \; \alpha + \frac{1}{2} \; (1-k) \; \sin 2\alpha \cos (2f-2\lambda) \right]}{\sin^2 \alpha \sin^2 \lambda \, (Y-Z)} \;\ldots\ldots (13)$$

For circles passing through the toe of the slope, n = 0, Eq. 10 reduces to

$$F = \frac{3}{\gamma H} \; \frac{C_1 \left[(1+k) \; \alpha + \frac{1}{2} (1-k) \; \sin 2\alpha \cos (2f-2\lambda) \right]}{\sin^2 \alpha \sin^2 \lambda \; Y} \;\ldots\ldots (14)$$

For isotropic material k = 1, i.e., $C_1 = C_i = C_2$ and Eqs. 13 and 14 become, respectively,

$$F = \frac{6\alpha \, C_1}{\gamma H \sin^2 \alpha \sin^2 \lambda \, (Y-Z)} \;\ldots\ldots\ldots\ldots (15)$$

and

$$F = \frac{6\alpha \, C_1}{\gamma H \sin^2 \alpha \sin^2 \lambda \; Y} \;\ldots\ldots\ldots\ldots\ldots (16)$$

Eqs. 15 and 16 are identical with Taylor's solutions using the friction circle method for the case ϕ = 0 (Eqs. 20 and 18 in Taylor's paper[9]). The solution, Eq. 13, may be conveniently written as

$$F = \frac{C_1}{\gamma H} \, N \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

in which N is termed the stability number

$$N = \frac{3 \left[(1+k) \; \alpha + \frac{1}{2} (1-k) \; \sin 2\alpha \cos (2f-2\lambda) \right]}{\sin^2 \alpha \sin^2 \lambda \, (Y-Z)} \;\ldots\ldots (18)$$

It is obvious that the minimum factor of safety is obtained by minimizing the stability number, N, with respect to α and λ, so that

$$\left. \begin{array}{l} \dfrac{\delta N}{\delta a} = 0 \\[2ex] \dfrac{\delta N}{\delta \lambda} = 0 \end{array} \right\} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

The foregoing operations may be carried out by a computer, and N minimum is solely a function of k, f, and β, so that

$$N = g \; (k, f, \beta) \;\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

Given values of k and f, N is then a function of the slope angle, β, alone.

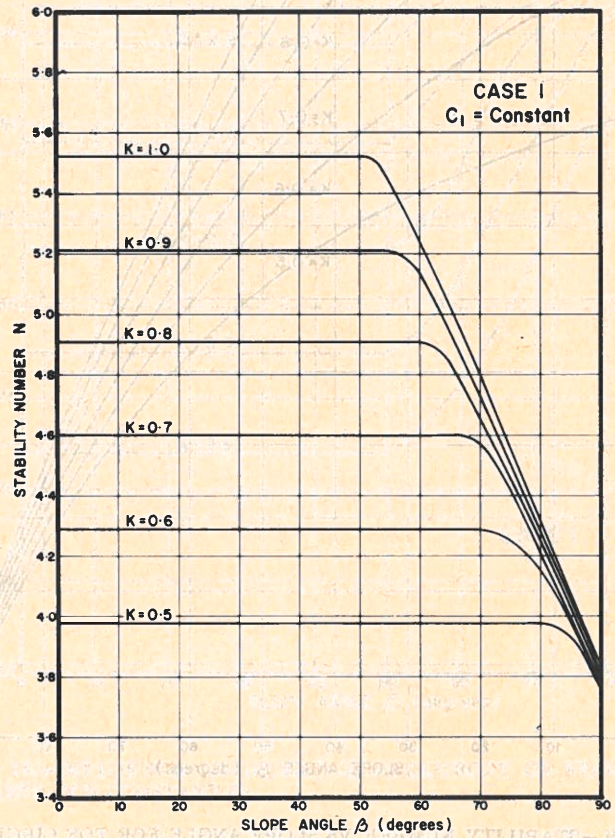

FIG. 8.—STABILITY NUMBER VS SLOPE ANGLE FOR CONSTANT C_1

Computations have been carried out for f = 55° and different values of k from 0.5 to 1.

The variation of the stability number, N, with slope angle for values of k ranging from 0.5 to 1 is shown in Fig. 8. As a direct consequence of the assumption that the shear strength is constant with depth, two distinct cases emerge. For values of β greater than a certain value, β_c, depending on k, the critical circles pass through the toe of the slope (n = 0). For $\beta < \beta_c >$

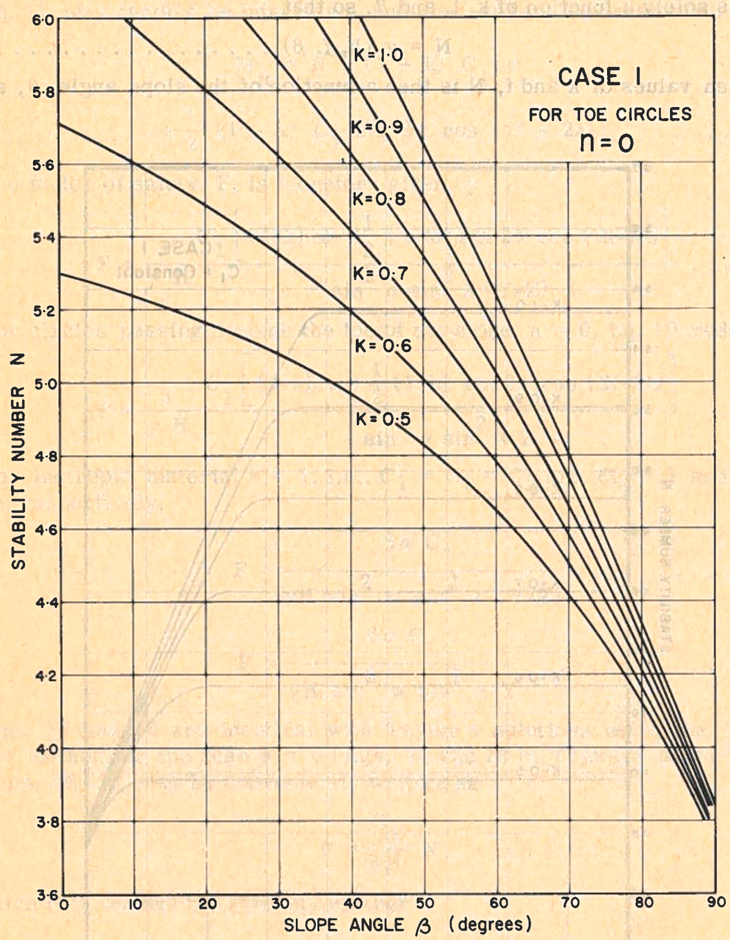

FIG. 9.—STABILITY NUMBER VS SLOPE ANGLE FOR TOE CIRCLES

the critical circles pass below the toe at infinite depth (λ = 0, n = ∞, D = ∞) and N is independent of β. In the latter case, the existence of a stronger stratum or limiting conditions, at or beyond the toe, that exercise restrictions to the slip circles have to be considered. The stability numbers for toe circles only are plotted in Fig. 9.

In order to examine the effect of anisotropy on the stability number, the ratio of N_k for a certain degree of anisotropy k and $N_k = 1$ for isotropic soils, ($N_k/N_k = 1$) is plotted aganist β in Fig. 10. It will be seen that the difference is significant for moderate degree of anisotropy, especially for $\beta < 50°$. For steep slopes, the ratio approaches unity. Further details on the implications on current methods of design of slopes will be examined subsequently.

Case 2.—The principal strengths, C_1 and C_2, increase linearly with depth simultaneously. In normally-consolidated clay deposits, the undrained shear

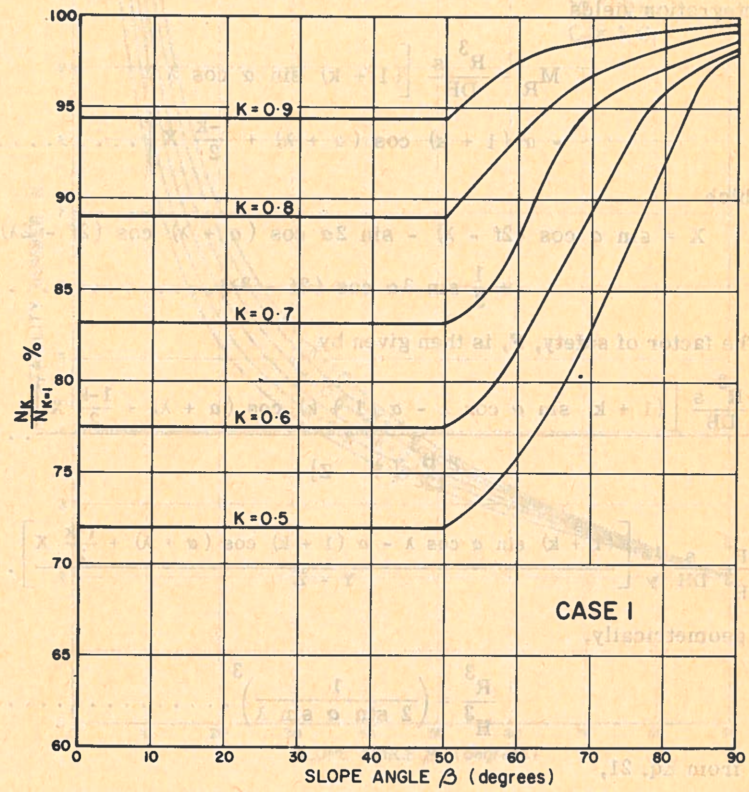

FIG. 10.—INFLUENCE OF DEGREE OF ANISOTROPY ON STABILITY NUMBER FOR CONSTANT C_1

strength increases linearly with depth, except in the zone of desiccation. In this case, it is assumed that both principal strengths, C_1 and C_2, increase in the same manner so that

$$C_1 = \frac{s\ z}{D\ H} \qquad (21a)$$

and $$C_2 = k\,C_1 \quad \text{(21b)}$$

in which s is the rate of increase of strength with depth and D denotes the depth factor.

The resisting moment is then given by

$$M_R = R^2 \int_{\alpha_1}^{\alpha_2} \left[\frac{k\,s\,z}{DH} + (1-k)\,\frac{sz}{DH}\cos^2(\alpha+\theta) \right] d\theta \quad \text{(22)}$$

Integration yields

$$M_R = \frac{R^3\,s}{DH}\Big[(1+k)\sin\alpha\cos\lambda - \alpha\,(1+k)\cos(\alpha+\lambda) + \frac{1-k}{2}\,X\Big] \quad \text{(23)}$$

in which

$$X = \sin\alpha\cos(2f-\lambda) - \sin 2\alpha\cos(\alpha+\lambda)\cos(2f-2\lambda) + \frac{1}{3}\sin 3\alpha\cos(2f-3\lambda) \quad \text{(24)}$$

The factor of safety, F, is then given by

$$F = \frac{\dfrac{R^3\,s}{DH}\left[(1+k)\sin\alpha\cos\lambda - \alpha\,(1+k)\cos(\alpha+\lambda) + \dfrac{1-k}{2}\,X\right]}{\dfrac{\gamma H^3}{12}\,[Y-Z]} \quad \text{(25)}$$

or

$$F = \frac{R^3}{H^3}\,\frac{s}{DH}\,\frac{12}{\gamma}\left[\frac{(1+k)\sin\alpha\cos\lambda - \alpha\,(1+k)\cos(\alpha+\lambda) + \dfrac{1-k}{2}\,X}{Y-Z}\right] \quad \text{(26)}$$

but, geometrically,

$$\frac{R^3}{H^3} = \left(\frac{1}{2\sin\alpha\sin\lambda}\right)^3 \quad \text{(27)}$$

and, from Eq. 21,

$$\frac{s}{D\,H} = \frac{C_1}{z} \quad \text{(28)}$$

Therefore,

$$F = \frac{3\,C_1}{\gamma z}\,\frac{\left[\dfrac{1+k}{2}\cot\lambda + \dfrac{\alpha\,(1+k)}{2}(1-\cot\alpha\cot\lambda) + \dfrac{1-k}{4\sin\alpha\sin\lambda}\,X\right]}{\sin^2\alpha\,\sin^2\lambda\,(Y-Z)} \quad \text{(29)}$$

For k = 1,

$$F = \frac{3\,C_1}{\gamma\,z}\,\frac{[\cot\lambda + \alpha\,(1-\cot\alpha\cot\lambda)]}{\sin^2\alpha\,\sin^2\lambda\,(Y-Z)} \quad \text{(30)}$$

which is the solution obtained by Gibson and Morgenstern[2] for isotropic soils.

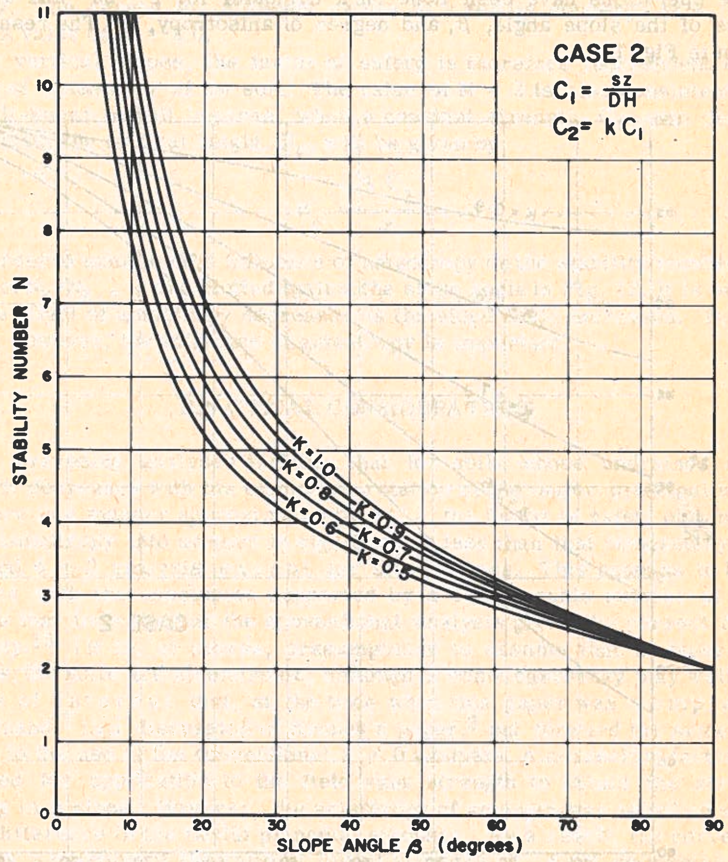

FIG. 11.—STABILITY NUMBER VS SLOPE ANGLE FOR C_1 INCREASING LINEARLY WITH DEPTH

The expression for the factor of safety can be written in the form

$$F = \frac{C_1}{\gamma z}\,N \quad \text{(31)}$$

in which the stability number, N, is given by

$$N = \frac{3\left[\frac{1+k}{2}\cot\lambda + \frac{\alpha\,(1+k)}{2}(1 - \cot\lambda\cot\alpha) + \frac{1-k}{4\sin\alpha\sin\lambda}\,X\right]}{\sin^2\alpha\,\sin^2\lambda\,[Y - Z]} \quad \ldots (32)$$

The stability number, N, is therefore a function of k, f, and β. For given values of k, f, and β, the minimum value of N may be computed as in Case 1. These operations have been done on a computer for f = 55° and different values of the slope angle, β, and degree of anisotropy, k. The results are shown in Fig. 11.

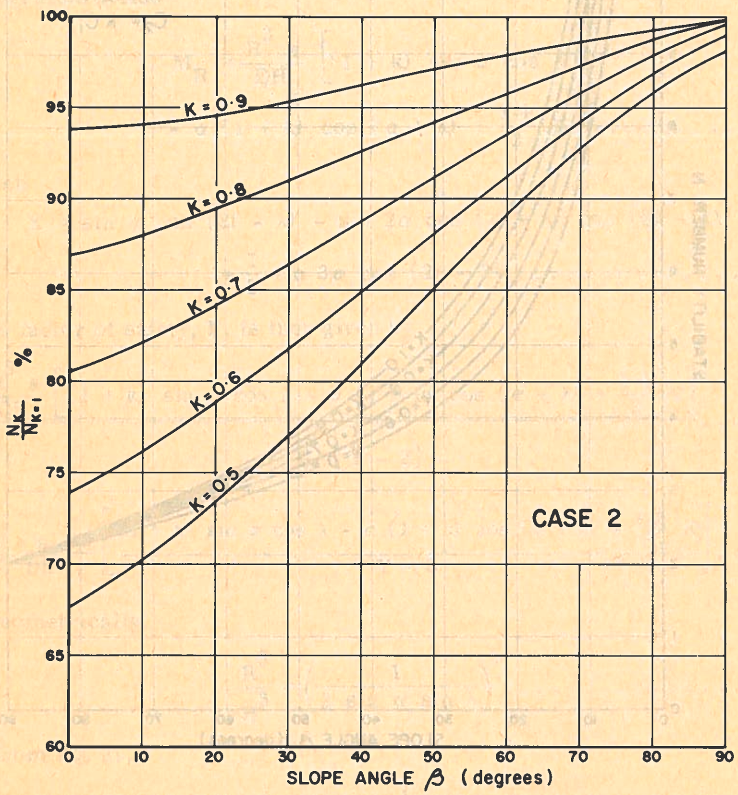

FIG. 12.—INFLUENCE OF DEGREE OF ANISOTROPY ON STABILITY NUMBER FOR C_1 INCREASING LINEARLY WITH DEPTH

The results of numerical analyses show that the minimum value of N, for given values of k and β, is given by the case n = 0. The slip surfaces are therefore an infinite number of circles passing through or above the toe of the slope, defined by a set of values of λ and α. Because the term C_1/γ_z is a constant for normally-consolidated clays, the factor of safety is therefore

independent of the height of the slope, H—a conclusion arrived at previously for isotropic soils.[2]

In Fig. 11, the stability numbers, N, are plotted against the slope angle, β, for different values of k. As β approaches zero, the stability numbers, N, and therefore the factors of safety, approach infinity; N decreases rapidly as β increases, but the rate of reduction decreases as the slope angle becomes steeper. The curves for different values of k finally converge at β = 90°, giving a value of N = 2.0.

For vertical slopes, the factor of safety is therefore independent of the degree of anisotropy of the soil. The value of N = 2 is also consistent with the well-known result because, when a constant strength, C_1, with depth is substituted, the critical height, H_c, will be given by

$$H_c = \frac{4\,C_1}{\gamma} \quad \ldots\ldots\ldots\ldots\ldots (33)$$

In order to examine the influence of anisotropy on the stability number, the ratios of $N_k/N_{k=1}$ are plotted against the slope angle in Fig. 12. It is evident that the effect of anisotropy decreases as the slope angle increases. For flat slopes, however, the influence of anisotropy is important.

PRACTICAL CONSIDERATIONS

The preceding analyses indicate that for soils whose undrained shear strength decreases with the clockwise rotation of the major principal stress at failure in a manner approximated by Eq. 2, the factor of safety obtained by taking anisotropy into account is significantly less than that obtained by conventional ϕ = 0 analysis except for steep slopes. This appears to be at variance with the conclusion supported by a considerable number of case records that indicate that the conventional analysis gives the correct factor of safety.[15] (It is, of course, presumptuous to assume that in these case records, the soils are all isotropic, although in some cases they may well be.)

It is of interest that, at the time when this paper was completed, Schertmann,[7] in a discussion of Kenney's paper,[3] put forward the same contention to the use of the conventional ϕ = 0 analysis. A correction factor was proposed for application to the field vane strength to obtain the strength existing in a slope. However, the anisotropy of strength was ascribed solely to the difference in the initial principal stresses. As a result, the correction factor is a function of the ratio of initial stresses, K. As K decreases from unity, the degree of anisotropy and the correction factor increases, the correction on the factor of safety (for isotropic soils, is therefore qualitatively in the same direction as in the present investigation.

To resolve the anomaly between the results of the present study and the apparent success of the conventional total stress analysis for slopes, careful consideration must be given to the shear strength determined and used in the analysis. An examination of the analyses of slope failures published in the literature shows that the undrained shear strengths used in the calculation of factors of safety in most, if not all, case records are invariably determined

15 Bishop, A. W., and Bjerrum, L., "The Relevance of the Triaxial Test to the Solution of Stability Problems," ASCE Research Conf. on Shear Strength of Cohesive Soils, Boulder, Colo., 1960.

from samples obtained from borehole or from in-situ vane tests. The strengths thus determined can in fact, be considerably less than the actual strength of the soil—the extent of the discrepancy being dependent on the type of soil.

The results of measurements of undrained shear strengths at Welland from (1) block samples, C_{1B}; (2) in-situ vane tests, C_V, and (3) borehole samples taken by 3-in. diameter piston sampler, C_{IS}, are shown in Table 2. It is evident that the strengths measured decrease from (1) to (3). The average ratio of C_V/C_{IB} is 0.80, while that of C_{IS}/C_{IB} is 0.71.

Results of shear strength determinations from conventional 2-in., thin wall Shelby tubes are less extensive, but the ratio of strengths is on the order of 0.5 to 0.6. A similar result has been reported for the discrepancy between block and tube samples for London clay.[12] These results illustrate that the strengths determined by borehole samples as well as in-situ vane tests are too low. The discrepancy between strengths determined from block and borehole samples may be attributed to disturbance caused by sampling. The reason for the anomaly between results of block samples and vane tests is obvious when the anisotropic nature of the soil is considered. In the vane test, the major principal stress at failure is horizontal for the cylindrical failure zone adjacent to the vane blades. At the ends, the stress conditions are more complex and obscure. However, for standard dimensions of the vane, it may be easily shown that the vane measures the strength of the soil along the cylindrical surface closely. The vane strength should, therefore, correspond approximately to the strength C_2, from the block samples. This hypothesis is supported by a comparison of C_V and C_{2B} in Table 2. It is clear, therefore, that both vane tests and unconfined compression tests on borehole samples do not necessarily give the right undrained shear strength and agreement between these two tests depends not only on sampling disturbance but also on the anisotropic nature of the clay. However, in the case of soils possessing M-anisotropy, the errors introduced by using shear strengths from unconfined

TABLE 2.—COMPARISON OF UNDRAINED SHEAR STRENGTHS

Sample	Depth, in feet	C_{1B}, in pounds per square foot	C_V, in pounds per square foot	C_{IS}, in pounds per square foot	$\frac{C_V}{C_{1B}}$	$\frac{C_{1S}}{C_{1B}}$	C_{2B}, in pounds per square foot	k, in feet
A	30-31	2150	-	-	-	-	1700	0.79
B	34-35	1400	1250	-	0.893	-	900	0.64
C	38-39	1600	1250	1050	0.781	0.656	1180	0.74
D	42-43	1460	1050	1300	0.719	0.890	1100	0.75
E	46-47	1500	1200	1150	0.800	0.766	1180	0.79
G	55-56	1650	1200	850	0.728	0.515	1200	0.73
H	60-61	1700	1300	1200	0.765	0.706	1350	0.80
I	65-66	1400	1300	1000	0.929	0.714	1260	0.78

compression tests on borehole samples or vane tests, and assuming the soil being isotropic, are compensating.

CONCLUSIONS

1. Depending on the environmental conditions of deposition and subsequent stress changes during geological history, soils may be isotropic or anisotropic.

2. In macroscopically homogeneous soils, the shear strength may vary with the direction of the applied major principal stress. A review of some reliable data obtained from block samples indicates that there are at least two types of anisotropy. The vertical principal strength may be greater or less than the horizontal principal strength.

3. An extensive study on one lightly overconsolidated glacial clay shows that the angle between the failure plane and the plane normal to the direction of the applied major principal stress is independent of the direction of the latter. While this conclusion may not necessarily be valid for all clays, it is supported by the reported comprehensive data on London clay.

4. Based on the foregoing empirical fact, a general method of stability analysis for anisotropic soils has been developed for the undrained case.

5. In the special case when the directional strength can be described by a mathematical expression, rigorous solutions have been obtained for the cases in which (a) The vertical strength is constant with depth, and (b) The vertical strength increases linearly with depth.

Stability numbers for the foregoing two cases have been computed by an electronic computer for a range of degree of anisotropy and slope angle. These numerical results show that for steep slopes, the effect of anisotropy is small. However, for flatter slopes, the influence of anisotropy on the stability conditions is significant.

ACKNOWLEDGMENTS

The numerical results of the stability numbers were obtained by the Electronic Computing Branch Dept. of Highways, Ontario, and the writer is indebted to A. E. Goodwin, Director of Electronic Computing Branch, K. Y. Shen and J. G. C. Ashford, for their cooperation.

The paper is published with the permission of H. W. Adcock, Assistant Deputy Minister (Engineering) of the Department of Highways, Ontario, Canada.

APPENDIX.—NOTATION

The following symbols have been adopted for use in this paper:

a = $f - \pi/2$;

C-Anisotropy = a type of anisotropy in which $C_2 > C_1$;

C_i = shear strength when the major principal stress at failure is inclined at angle i to the vertical;
C_1, C_2 = principal strengths in directions of principal stresses;
D = depth factor;
F = factor of safety;
f = angle between failure plane and the plane normal to the direction of the major principal stress;
H = height of slope;
i = angle of rotation of major principal stress from vertical, measured clockwise;
K = ratio of initial principal stresses;
k = degree of anisotropy = C_2/C_1;
M-Anisotropy = a type of anisotropy in which $C_1 > C_2$;
M_R = resisting moment;
N = stability number;
N_K = stability number for any value of k;
$N_{K=1}$ = stability number for k=1;
n = geometric parameter;
R = radius of slip circle;
s = rate of increase of strength with depth;
X = $\sin \alpha \cos (2f - \lambda) - \sin 2 \alpha \cos (\alpha + \lambda) \cos (2f - 2\lambda) + 1/3 \sin 3 \alpha \cos (2f - 3\lambda)$;
Y = $1 - 2 \cot^2 \beta + 3 \cot \lambda \cot \beta + 3 \cot \alpha \cot \lambda - 3 \cot \alpha \cot \beta$;
Z = $6 n (n + \cot \beta - \cot \lambda + \cot \alpha)$;
z = ordinate measured from top of slope;
α = geometric parameter = $\alpha_1 - \alpha_2/2$;
α_1, α_2 = geometric parameters;
β = angle of inclination of slope from horizontal;
β_c = β at a certain value;
θ = geometric parameter;
λ = geometric parameter; and
δ = stress.

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SHOCK WAVES IN GRANULAR SOIL

By Robert D. Stoll,[1] M. ASCE, and Ibrahim A. Ebeido,[2] A. M. ASCE

INTRODUCTION

Stress waves of small amplitude whose characteristics may be described to a good approximation using the linear theory of elasticity have been studied in detail for many years in considering earthquake disturbances. Although considerable information is currently (1965) available regarding seismic wave propagation in a variety of different types of soil, no general theory has been established to describe propagation of waves with larger stress amplitudes where the theory of elasticity is no longer applicable. This type of information is of great importance in considering close and intermediate distance phenomena associated with any large disturbance. It is this region between the low stress levels usually considered in seismic studies and the high stress levels where degradation and "locking" type of behavior may occur that is of principal interest herein.

Several major difficulties arise when studying wave propagation at stress amplitudes greater than those that permit the use of a linear elastic theory. First, there are no generally accepted constitutive equations that describe the behavior of a soil material under a prescribed history of stress and strain. Some progress has been made by investigators such as R. D. Mindlin,[3] H. Deresiewicz,[4] and their co-workers in describing the behavior of aggregations of like spheres using the linear theory of elasticity; however, most work has been concentrated on a phenomenological approach to description of the behavior of a statistically homogeneous granular mass with arbitrary gradation. This type of description is generally based on experimental investiga-

Note.—Discussion open until December 1, 1965. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Soil Mechanics and Foundations Division, Proceedings of the American Society of Civil Engineers, Vol. 91, No. SM4, July, 1965.
1 Assoc. Prof. of Civ. Engrg., Columbia Univ., New York, N. Y.
2 Lecturer, Dept. of Civ. Engrg., Alexandria Univ., Alexandria, U.A.R.
3 Duffy, J., and Mindlin, R. D., "Stress-Strain Relations and Vibrations of Granular Medium," Journal of Applied Mechanics, ASME, New York, N. Y., Vol. 24, 1957.
4 Deresiewicz, H., "Mechanics of Granular Matter," Advances in Applied Mechanics, Academic Press, New York, N. Y., 1958, Vol. IV, pp. 233-306.

factor expressing the change of angle on one side of one column only); and (2) the moment of exterior forces is connected to the radius of curvature of the deformation by a simple relationship.

The moment is linearly proportional to 1/R. Therefore it is to be expected that the relationships connecting the strains and crack-width in wall with the radius of curvature will be simpler that those connecting these values with the gradient.

The experimental results[4] confirmed this assumption. In masonry walls of different materials, the maximum crack-width variated linearly with 1/R. Moreover, a critical value was found to exist for R. When R decreased under 1,500 m (5,000 ft) in all types of masonry tested, the rate of crack-widening increased considerably. This radius could be considered as a "yield point" for the walls tested.

STABILITY OF SLOPES IN ANISOTROPIC SOILS[a]

Discussion by George G. Meyerhof

GEORGE G. MEYERHOF,[16] F. ASCE.—The author has shown that the estimated stability of slopes in clays under undrained conditions is considerably affected by anisotropy of the soil for the customary flat slopes. New stability numbers have therefore been derived for use in practice. It may be noted that these results can be approximated by using the average principal strength, $(C_1 + C_2)/2$, in the previous solutions for isotropic soils.

However, the greater the degree of anisotropy of the soil, the more the potential failure surface will depart from the assumed cylindrical form and become elongated in the direction of the smaller shear strength. Consequently, a solution of this problem would be expected to yield even smaller stability numbers for anisotropic soils than have been derived.

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[a] July, 1965, by Kwan Yee Lo (Proc. Paper 4405).

[16] Dean, Nova Scotia Tech. Coll., Halifa, Nova Scotia, Canada.

energy per blow would require that the two men raise a 350-lb weight—which they could not do.

6. Boston's original reason for determining and recording the number of blows was to better soil classification and perhaps aid in estimating the cost of hand excavating belled caissons. Only later were formulas mentioned by which it was alleged that footings could be proportioned as could the necessary embedment of piles, for support of a given load, by inserting the number of blows, noting the type of soil, and solving for the answer.

Throughout the body of the paper, Fletcher repeatedly notes the many serious pitfall involved in the use of the SPT. In his Conclusions, he states that "SPT has had 30 yr of generally successful application," and then follows with an Appendix citing thirteen reasons that can affect the accuracy of the reported number of blows.

There are eight other reasons far more detrimental to accuracy than those mentioned by Fletcher.

1. *Boring Contractors*.—This is business, the primary purpose of which is profit.

2. *Boring Foremen and Helpers*.—The human element in a rough and hidden operation.

3. *Boring Inspectors*.—This is the question of whether there should be one inspector per boring crew, one for a number of crews, or none at all. Inspectors quick enough to keep abreast of fickle men in many boring crews are rare indeed.

4. *Pumps of Various Types and Sizes, Independently Power-Driven or Directly Connected to a Power-Driven Boring Machine*.—The boring foreman who, of his own volition, regulates the volume of circulating liquid to suit the type of soil being loosened and removed from the borehole, is rare indeed. The writer has yet to encounter one such foreman except where a hand powered pump was used. What would be the reaction of those fully acquainted with the principles of proper boring procedure as presently understood on learning from the superintendent of borings for a large concern that he is using single stage 4-in. centrifugal pumps, on an active operation, when working through 2-1/2 in. casing, because he cannot make the estimated footage per day with smaller pumps?

5. *Power Driven Equipment for Making Borehole*.—The success of the manufacturer depends on greater footage of hole per day than can be produced by other machines.

6. *Specifications*.—Some specifications are so contrived that the author thereof could not comply. Specifications will not improve the reliability or accuracy of the reported number of blows.

7. *Soil Handling*.—The mishandling of soils during construction and the brutal driving of piles can and do void reasonably accurate predictions.

8. *Engineering Eccentricities*.—Pet requirements of some engineers confuse the most cooperative of boring crews to the extent of producing wide variations in the reported number of blows.

Considering the thirteen reasons cited by the author and the additional eight presented herein that can affect the number of blows, how can the engineer possibly determine, from the information submitted to him, the accuracy of the reported results?

The writer believes it wrong to infer or imply that the so-called SPT or any similar data are or can be made reliable and consistent. With no better supporting evidence of reliability, then as now, he believes it a mistake to use the term "Standard." His experience proves these data should be used with great restraint.

STABILITY OF SLOPES IN ANISOTROPIC SOILS[a]

Discussions by Peter Lumb and John H. Schmertmann

PETER LUMB,[17] M. ASCE.—In this paper, the basis of the analysis is the relationship between the directional strength and the angle of inclination, i, to the vertical, as given by Eq. 2. Although the experimental results of Fig. 5 agree with the predicted values, there is no theoretical justification for the assumed relationship, as is mentioned by the author.

A rational interpretation of anisotropic strength can, however, be made by using a modification of the maximum shear strain energy failure criterion, which has been described by Hill.[18] This failure criterion also agrees with the experimental values but leads to a completely different conclusion as to the stability of anisotropic soils than that reached by the author.

For brevity, Casagrande and Carrillo's Eq. 2 will be called Criterion A and the modified maximum shear strain energy criterion will be called Criterion B.

For purely cohesive anisotropic materials, Criterion B assumes that failure occurs by yielding and that the yield criterion is a quadratic function of the stress components σ_x, σ_y, σ_z, τ_{xy}, τ_{yz}, and τ_{zx}, in which the x, y, and z axes are considered as the principal axes of anisotropy. In the present case, it will also be assumed that the anisotropy is symmetrical about the vertical z axis.

Following Hill, the failure criterion for the cohesive anisotropic soil can be written as

$$\left(\sigma_y - \sigma_z\right)^2 + \left(\sigma_z - \sigma_x\right)^2 + \left(2\,n^2 - 1\right)\left(\sigma_x - \sigma_y\right)^2 + 2\,m^2\,\tau_{yz}^2 + 2\,m^2\,\tau_{zx}^2 + 2\left(4\,n^2 - 1\right)\tau_{xy}^2 = 2\,p_1^2 \qquad (34)$$

[a] July, 1965, by Kwan Yee Lo (Proc. Paper 4405).

[17] Senior Lecturer in Civ. Engrg., Univ. of Hong Kong, Hong Kong.

[18] Hill, R., "The Mathematical Theory of Plasticity," Clarendon Press, Oxford, England, 1950, Chapter XII.

in which p_1 = compression strength in vertical direction; p_2 = compression strength in horizontal direction; s_1 = shear strength in horizontal plane; s_2 = shear strength in vertical plane; and

$$p_1 = n \cdot p_2 = m \cdot s_2 = \left(4\,n^2 - 1\right)^{\frac{1}{2}} \cdot s_1 \quad \ldots\ldots\ldots (35)$$

The strength is thus characterized by the three independent parameters p_1, n, and m. For complete isotropy, n = 1 and m = $\sqrt{3}$.

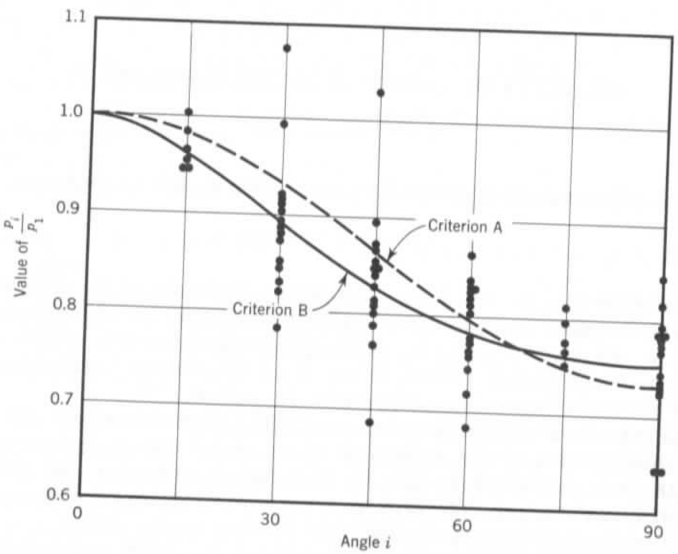

FIG. 13

The triaxial or unconfined compression strength, p_i, of a cylindrical sample cut with its axis inclined at an angle i to the vertical can easily be shown to be given by

$$p_i = p_1 \left[\cos^2 i\left(\cos^2 i - \sin^2 i\right) + n^2 \sin^4 i + m^2 \sin^2 i \cos^2 i\right]^{-\frac{1}{2}} \quad \ldots (36)$$

According to Criterion A, the corresponding equation is

$$p_i = p_1 \left[\cos^2 i + \frac{1}{n} \sin^2 i\right] \ldots\ldots\ldots\ldots\ldots (37)$$

Fig. 13 shows the experimental results of Fig. 5 together with additional results furnished by Lo. From these results, maximum likelyhood estimates of the parameters and their standard deviations have been calculated as:

$$n = 1.367 \pm 0.045 \quad \text{for Criterion A}$$

$$\left.\begin{aligned} n &= 1.327 \pm 0.0185 \\ m &= 2.022 \pm 0.042 \end{aligned}\right\} \text{for Criterion B}$$

There is no statistically significant difference between Criterion A or B, because of the large scatter of the experimental results, although the mean values for a particular angle i agree rather more closely with Criterion B than with Criterion A. It is of interest to note that for this soil the results are consistent with the particular values

$$p_2 = \frac{3}{4} \cdot p_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots (38a)$$

$$s_2 = \frac{1}{2} \cdot p_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots (38b)$$

The Vane Test.—On the basis of Criterion B, it is easy to show that the ratio of the shear strength as measured by a vertical vane test to the shear strength as measured by an unconfined compression test on a vertical sample is given by

$$\frac{Cv}{C_1} = \frac{\left(s_1 + s_2 \cdot \frac{D}{3\,H}\right)}{s_2\left(1 + \frac{D}{3\,H}\right)} = \left[\frac{m}{\left(4\,n^2 - 1\right)^{\frac{1}{2}}} + \frac{D}{3\,H}\right]\left(1 + \frac{D}{3\,H}\right)^{-1} \ldots\ldots (39)$$

in which D = the diameter of the vane and H = the length.

Taking n = 4/3 and m = 2, the ratio Cv/C_1 is calculated as

D/H	0	2/3	1	3/2
C_v/C_1	0.809	0.844	0.857	0.873

The experimental values of Table 2 give Cv/C_1 = 0.802 ± 0.080, and consequently, it is evident that there is no statistically significant difference between the actual ratio and that predicted by Criterion B.

Plane Strain.—The actual results agree with either Criterion A or B but the implications of the two criteria are completely different when the case of plane strain is considered.

Assuming the soil to be incompressible, Criterion B for plane strain reduces to

$$\frac{2\,n^2 + 1}{4\,n^2}\left(\sigma_z - \sigma_x\right)^2 + m^2\,\tau_{xz}^2 = p_1^2 \quad \ldots\ldots\ldots (40)$$

since

$$\sigma_y = \frac{\sigma_z + \left(2\,n^2 - 1\right)\sigma_x}{2\,n^2} \quad \ldots\ldots\ldots\ldots (41a)$$

and

$$\tau_{xy} = \tau_{zy} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (41b)$$

The plane strain compression strength p_i' for a sample cut at an angle i to the vertical is given by

$$p_i' = p_1 \cdot \frac{2}{m}\left[\frac{2n^2+1}{m^2 n^2}\cos^2 2i + \sin^2 2i\right]^{-\frac{1}{2}} \quad \ldots\ldots (42)$$

and this can be re-written as

$$\frac{p_i'}{p_1} = \frac{2}{m}\left\{\frac{1 - X}{1 - X \sin^2 2i}\right\}^{\frac{1}{2}} \quad \ldots\ldots (43)$$

in which X, a new measure of the anisotropy, is defined by

$$X = 1 - \frac{m^2 n^2}{2n^2 + 1} \quad \ldots\ldots (44)$$

It is evident that the average plane strain strength is larger or smaller than the vertical triaxial or unconfined compression strength when X is negative or positive, respectively. In the present case, X is negative so the plane strain

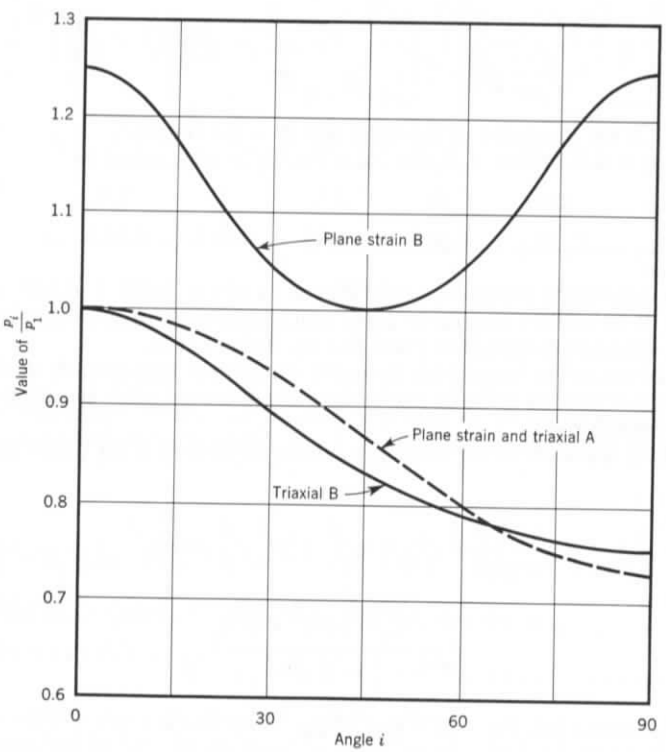

FIG. 14

strength is greater than, or at least equal to, the unconfined compression strength. This is illustrated in Fig. 14 for n = 4/3 and m = 2. Criterion A, of course, implies that the plane strain strength is identical with the triaxial or unconfined strength.

A suitable test for determining which failure criterion best characterizes the strength of an anisotropic soil would be a comparison of plane strain strength, p_2', and unconfined strength, p_2, for samples cut horizontally. The ratio of p_2'/p_2 is predicted as: $p_2'/p_2 = 1$ for Criterion A; $p_2'/p_2 = 2n^2(2n^2 + 1)^{-1/2} = 2.22$ for n = 4/3 for Criterion B.

The difference between the two predictions is sufficiently great for a clear decision to be obtained.

Application to Stability Problems.—Criterion B agrees with the experimental results and has the advantage of a rational basis. If it is granted that this failure criterion could be applied to anisotropic soils, then the implications with regard to plane strain cannot be ignored.

For the present case, the plane strain compression strength is greater than or at least equal to the vertical triaxial or unconfined compression strength, and the plane strain shear strength is identical with the triaxial shear strength. This leads to the rather paradoxical conclusion that the stability of a slope in such an anisotropic soil must be greater than or at least equal to the stability of a similar slope in an isotropic soil for the same value of the vertical compression strength. The fact that the horizontal compression strength is less than the vertical is irrelevant, for the case of plane strain.

Confirmation of this conclusion is given by an exact solution for the bearing capacity of a long strip foundation (Hill[18]), corresponding roughly to the stability of a slope of zero angle of inclination. The ratio between the anisotropic bearing capacity, Q', and the isotropic bearing capacity, Q, for equal strengths p_1 can be written as

$$\frac{Q'}{Q} = \frac{(1 - X)^{\frac{1}{2}} + E(X)}{1 + \frac{\pi}{2}} \quad \ldots\ldots (45)$$

in which X = the previously defined measure of anisotropy and E(X) = the complete elliptic integral of the second kind, with modulus X. Taking n = 4/3 and m = 2, Eq. 45 yields the ratio Q'/Q as 1.17; that is, there is a predicted increase in bearing capacity resulting from anisotropy of 17%.

JOHN H. SCHMERTMANN,[19] M. ASCE.—Lo's paper is a significant attempt to carry forward slope design capability to include the effect of undrained strength anisotropy. He presents graphs (Figs. 8 through 12) of potentially immediate practical usefulness in the solution of such a problem. The writer will discuss herein three points that should be considered when evaluating the significance of these graphs.

1. The author merely assumes inherent anisotropy;

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[19] Prof. of Civ. Engrg., Univ. of Florida, Gainesville, Fla.

2. carrying the author's recommendations through when using in situ vane test strengths does not result in a reasonable compensation of errors; a large correction factor is still needed;

3. the possible significance of shear stress direction on a given failure plane.

Inherent Anisotropy versus Effective Stress.—Although Lo presents excellent data further proving the reality of the anisotropic nature of undrained shear strength, he then assumes that this anisotropy reflects "an inherent anisotropic property of the soil." Because he makes a special point of distinguishing this from anisotropy in effective stresses, the writer assumes that Lo means the soil would exhibit anisotropic strength characteristics other than those directly attributable to anisotropy of effective stress at failure. This might well be true. However, this is a key point in the paper and it is not proven by any of the author's tests. These were all undrained without pore pressure measurements. For the sake of discussion, it could be argued that strength in terms of effective stress is isotropic but that the apparent anisotropy of undrained strength is caused by different pore pressures at failure depending on failure plane direction. As the angle i increases, so does the total rotation of principle planes during the strength test. Pore pressures in normally consolidated clays are likely to increase as this rotation increases, effective stresses will be reduced, and, therefore, the undrained strength will be reduced.

A paper by Aas[20] also presents data that further demonstrates the anisotropy of undrained strength. His investigation was done in normally consolidated clays by using vanes with different height to diameter ratios. As noted by Aas, the strength anisotropy he reports appeared to be directly related to the in situ principle effective stress ratio, K_o. As detailed by the writer in a previous discussion,[7] it is not necessary to resort to assuming anisotropic behavior in terms of effective stress to explain these test results. Perhaps it is also not necessary in an explanation of the author's tests.

The author is well aware of the possible special importance of effective stress to the anisotropy of undrained strength. The discussion centered around Eq. 3, which is the same as the writer's[7] Eq. 31, conjectures on this possibility. It would be most valuable if interested investigators could obtain anisotropic strength data in terms of effective stress so that the relative importance of considerations other than effective stress can be evaluated.

There is an unfortunate confusion of symbols in the paper that will enter any consideration of the above discussion. In Fig. 8 through 12, the symbol K is lettered in upper case. According to the listed Notations this means "ratio of initial principle stresses." According to the sense of the paper, these should all be lower case k, with meaning "degree of anisotropy = C_2/C_1." However, if prior effective stresses do control anisotropic undrained strength, then the now erroneous K would be a correct alternative symbol to use in these figures (for normally consolidated clays.)

Correction When Using Vane Test Strengths is Still Large.—At the end of his paper, the author makes an important point regarding use of in situ vane

20 Aas, G., "A Study of the Effect of Vane Shape and Rate of Strain on the Measured Values of In-Situ Strength of Clays," Proceedings, 6th Internatl. Conf. on Soil Mechanics and Foundation Engrg., Montreal, Canada, 1965, Vol. I, p. 143.

test strengths. The presently (1965) conventional method of applying vane test strengths is to use them for the strength he denotes as C_1 and also to neglect strength anisotropy by using the stability numbers for k=1. He notes these are both errors but are compensating. Lest a reader be misled, it should be noted that, although compensating, they far from cancel, as detailed below.

The following applies to a Case-2 strength distribution with depth, with the soil exhibiting M-anisotropy. The correct factor of safety for the slope, using C_1 and the proper N_k, is herein denoted F_c and according to Eq. 31 can be expressed as

$$F_c = \frac{C_1}{\gamma Z} N_k \quad \dots\dots\dots\dots (46)$$

Following the discussion in the preceding paragraph, and denoting the vane strength by C_v, the conventional factor of safety obtained when using the vane strengths, herein denoted F_v, is

$$F_v = \frac{C_v}{\gamma Z} N_{k=1} \quad \dots\dots\dots\dots (47)$$

The ratio F_c/F_v then expresses the net correction factor, according to Lo's analysis, to be applied to a conventional slope analysis based on vane strengths.

As noted by both Lo and the writer,[7] the field vane measures approximately the in situ undrained strength denoted by Lo as C_2. For the purposes of this discussion, the writer suggests that the vane strength be expressed as

$$C_v = C_2 + \frac{C_1 - C_2}{X} \quad \dots\dots\dots\dots (48)$$

in which X = a factor depending on the height to diameter ratio of the vane. The value X=9 is suggested as reasonable for the standard vane in which h/d = 2. Combining Eqs. 46, 47, and 48 then gives the correction factor

$$\frac{F_c}{F_v} = \frac{\left(\frac{N_k}{N_{k=1}}\right)}{\left(k + \frac{1-k}{9}\right)} \quad \dots\dots\dots\dots (49)$$

The writer has substituted into Eq. 49 using Fig. 12 and the k values of 0.50 and 0.75; the results are presented graphically by the solid lines in Fig. 15.

Fig. 15 demonstrates that the net correction factor, based on Lo's analysis, is still significant. The need for such a factor is not evident from practical experience. This does not mean that the theory behind the factor is necessarily incorrect, because other errors in determining the proper strength value to use in the analysis may satisfactorily compensate. Some likely possible errors are neglecting residual shear stresses in the soil mass because of anisotropic consolidation, rate-of-strain effects, and progressive failure. The writer used this same argument when he suggested[7] similar correction factors. These factors were based solely on anisotropy in effective stress and are also shown in Fig. 15 with light, dashed lines. The two sets of factors, Lo's and the writer's, are similar in magnitude but different in direction with increasing slope angle. A possible significance of this difference is presented below.

Possibility of Undrained Strength Anisotropy with Shear Stress Direction.— The most important practical assumption in the $\phi = 0$ (or total stress) method of stability analysis as now (1965) practiced is that the strength measured in the undrained test used is the strength at failure in the field. The need for any correction factors, such as Lo's $(N_k/N_{k=1})$ or the (F_c/F_v) in Fig. 15, follows from the fact that present methods of testing do not fulfill this assumption; the greater the success in measuring field strength the smaller any correction need be.

It could be argued that, when considering only M-anisotropy and vane tests, any correction factor should decrease with increasing slope angle. The stand-

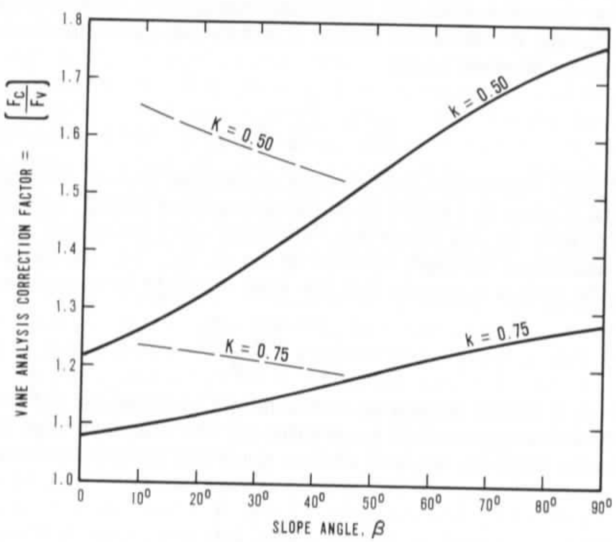

FIG. 15.—VANE ANALYSIS CORRECTION FACTOR STILL REMAINING AFTER APPLICATION OF LO'S RESULTS

ard vane with h/d=2 essentially measures the plane strain, undrained strength on a near-vertical plane. The more closely the failure plane in the field approaches the vertical, the more correctly would the vane measure field strength. Failure surfaces in the field more closely approach the vertical as the slope angle increases. Therefore, any correction factor should decrease with increasing β. This is the result previously obtained by the writer,[7] which is also partly replotted in Fig. 15.

However, in this previous work, the writer missed one aspect of the problem that has been brought to light as a result of Lo's paper. In-situ vane tests produce shearing stresses that have a 90° directional difference when compared to shear along a slope failure surface. Consider a set of x-y-z coordinates with x horizontal and perpendicular to the slope, y horizontal and parallel to the slope, and z vertical in the plane of the slope section. Shear along the

vane perimeter acts tangentially in the x-y plane, but shear along the failure surface acts in the x-z plane. It can now be argued (below) that correction factors could increase with β, as suggested in Fig. 15.

There is a given soil structure on a given plane. Inherent directional anisotropy on a given plane is probably unimportant. Any undrained strength differences with differences in shear direction will depend primarily on differences in effective stress at failure and this means pore pressure differences. Although evidence on this point is needed, to reach a conclusion conforming to Lo's predictions it will be assumed that greater principle plane rotations during shear result in greater pore pressures at undrained failure and consequently lesser undrained strength. (It seems likely that the validity of this assumption will depend on whether the slope results from a cut or a fill and also on the over-consolidation ratio of the clay.) In a vane test, this rotation is 90° (horizontal to vertical). Along a failure surface in the field, the average rotation is less than 90° and decreases as the slope angle increases. Differences in pore pressures between vane and failure surfaces, hence effective stress, hence undrained strength, and hence correction factors, would then increase with slope angle and agree with Lo in Fig. 15.

The possibility that undrained strength is not only anisotropic with failure plane inclination, but also with shear stress direction, is, at present, only a theoretical one. However, as explained by Lo and briefly re-examined above, it seems an important and likely possibility. Every effort should be made to check this point experimentally. If this proves to be true, then it will greatly affect our design and interpretation of shear tests and the profession will be indebted to the author for this contribution.

within the elastic range when the applied lateral load is small. When the proportional limit is exceeded the lateral deflections increase rapidly. The lateral deflections can be calculated when the applied load approaches the failure load by assuming that the load-deflection relationship is a parabola (shown in Fig. 11). The initial slope of this relationship can be calculated by using the coefficient of subgrade reaction mentioned above.

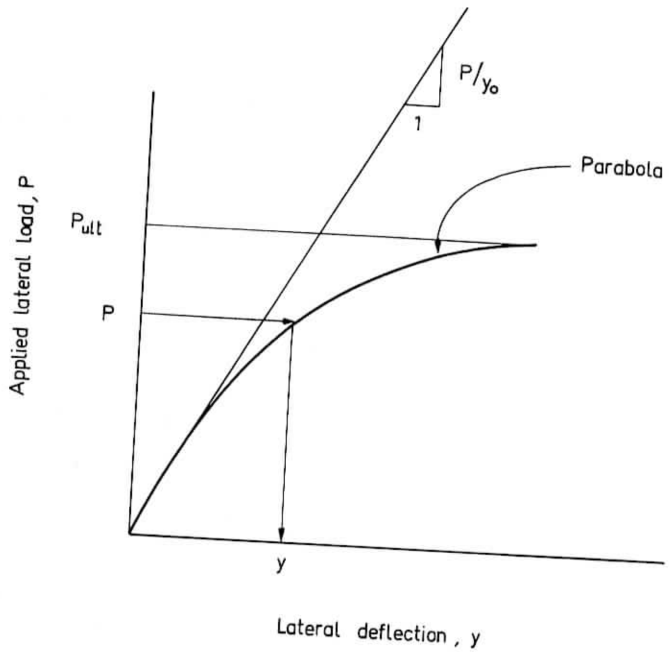

FIG. 11.—LOAD-DEFLECTION RELATIONSHIP FOR A LATERALLY-LOADED PILE

The coefficient of horizontal subgrade reaction has been calculated by assuming that this coefficient is the same as the coefficient of vertical subgrade reaction for cohesive soils. The accuracy of this assumption is not known. Available test data indicate, however, that this is a safe assumption. It must be emphasized that the number of test data available is limited and that additional data are needed.

STABILITY OF SLOPES IN ANISOTROPIC SOILS[a]

Closure

KWAN YEE LO,[21] A. M. ASCE.—The discussers consider three main points: (1) The choice of failure criterion for anisotropic soils; (2) the distinction between inherent or intrinsic anisotropy and anisotropy caused by a difference in initial effective stresses; and (3) the interpretation of in situ vane test in anisotropic soils in relation to the importance of the direction of shear stresses.

Lumb made an interesting application of the theory of perfectly plastic materials to the analysis of the test data, using Hill's yield criterion for cohesive anisotropic soils. For isotropic soils, Eq. 34 reduces to

$$J_2 = (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + (\sigma_x - \sigma_y)^2 + 6\,\tau_{xz}^2 + 6\,\tau_{zx}^2 + 6\,\tau_{xy}^2 = 2\,p_1^2 \quad ..(50)$$

which is the von Mises criterion of failure. However, none of the failure criteria commonly used in the theory of plasticity (e.g., von Mises or Tresca) has been established on a sufficiently firm basis to be generally valid for soils. Even for isotropic soils some of the basic postulates of plastic theory are not satisfied and some of its limitations have already been examined.[22] The validity of plasticity theory as applied to soils is, therefore, doubtful and Hill's criterion cannot be supposed to have a more rational basis than the simpler criterion used by the writer. A consequence of the basic assumptions made in the plastic theory may well be the prediction from Hill's theory that the stability of a slope must be greater than or equal to the same slope in isotropic soil having the same value of C_1.

It is gratifying to note from Lumb's analysis that there is no statistically significant difference between the two criteria in the cases studied. At the same time it must be reemphasized that the data on the Well and clay presented in the paper represents only one type of anisotropy. As more information becomes available different types of anisotropy will be discovered and defined. It is unlikely that either of the criteria discussed will be able to describe all the different types of anisotropy in nature. It appears necessary, therefore, to retain a simple approach to account for different conditions, and the basic concept expounded in the paper is more important than any rigorous analysis of any one type of anisotropy.

Intrinsic, Induced, and Stress Anisotropy.—At the beginning of the paper, the writer distinguished between two types of anisotropy which caused the di-

[a] July, 1965, by Kwan Yee Lo (Proc. Paper 4405).

[21] Professeur Auxiliaire, Départment de Génie Civil, Université Laval, Québec, Canada; formerly, Superv. Foundation Engr., Dept. of Highways, Ontario, Canada.

[22] Bjerrum, L., and Lo, K. Y., discussion of "Mechanics of Triaxial Test for Soils," by R. M. Haythornthwaite, Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 87, No. SM2, Proc. Paper 2804, April, 1961, pp. 173-176.

rectional dependence of strength: the inherent anisotropy a soil possesses as an intrinsic part of its structure, and the anisotropy of the stress system act-

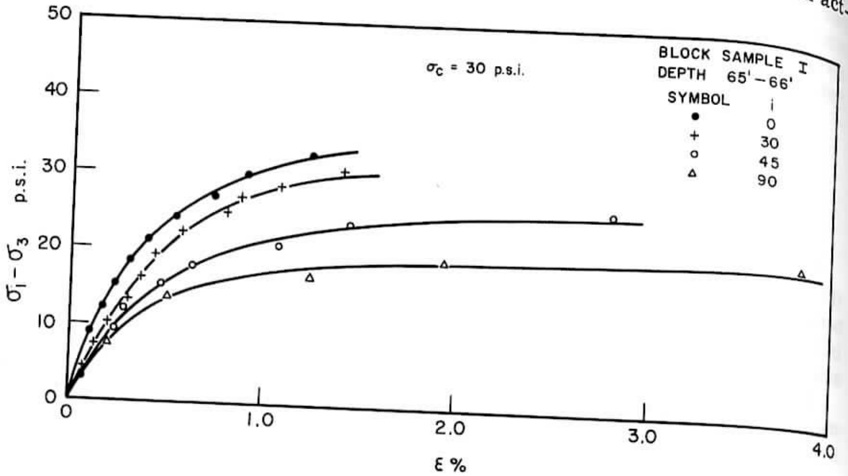

FIG. 16.—STRESS-STRAIN RELATIONSHIP IN CIU TESTS FOR DIFFERENT VALUES OF i

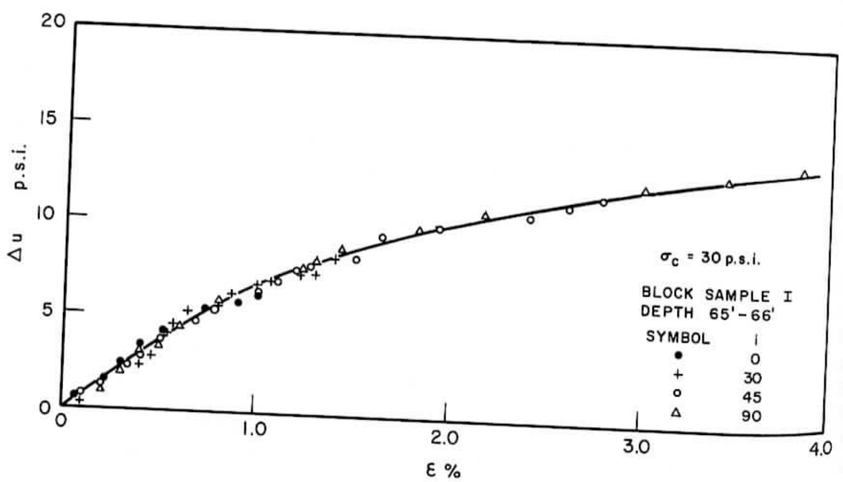

FIG. 17.—PORE PRESSURE-STRAIN RELATIONSHIP IN CIU TESTS FOR DIFFERENT VALUES OF i

ing on the soil in the ground (stress anisotropy). This distinction is necessary because the two types of anisotropy not only arise from separate physical bases, but also different types of conventional shear tests measure the effects

of either one, or both, types of anisotropy. In practice either effect, or both, may have to be considered, depending on the nature of the analysis used and the method of measurement of shear strength.

The directional variation of undrained strength caused by anisotropy of the initial stress system, or stress anisotropy, has been examined by Hansen and Gibson.[5] Although their theory is confined by the limitation of the λ theory, it may be used to assess the significance of the effect of this type of anisotropy.

Each time a sample is extracted from the ground, the sample is subjected to a negative pore pressure. Therefore, the initial effective stress in the sample is isotropic and the stress anisotropy is no longer present. Unconfined compression tests and triaxial tests on samples consolidated isotropically in the laboratory, therefore, must measure inherent anisotropy.

It is now clear that intrinsic anisotropy is a physical fact rather than an assumption; it has been supported by the test data of the unconfined compression tests. In terms of effective stresses either, or more likely, both, the cohesion and friction term may be expected to be anisotropic; there appears to be no reason why the pore pressure should be dependent on the direction of stresses at a given strain. The following data support this view.

TABLE 3.—RESULTS OF C.I.U. TESTS

i, in degrees	W_i, in %	W_f, in %	$(\sigma_1 - \sigma_3)_f$, in pounds per square inch	ϵ_f, in %	Δ_u, in pounds per square inch
0	32.6	31.0	36.0	1.3	7.8
30	28.6	28.6	30.8	1.4	8.4
45	30.3	29.6	26.0	2.6	11.5
90	31.8	31.0	19.9	4.0	13.5

A series of isotropically consolidated, undrained tests with pore pressure measurements were performed with i = 0, 30, 45, and 90, respectively, on a block sample I of the clay from the same site as the clay used in the paper. The stress-strain curves and induced pore pressures Δu for specimens consolidated at an ambient pressure of 30 psi are plotted in Figs. 16 and 17, respectively. Further details on the test results are contained in Table 3.

The anisotropic nature of the clay is evident in Fig. 16 in which the maximum stress difference and modulus of deformation decrease with increase in angle i may be seen. However, the pore pressures plot uniquely on a single curve in Fig. 17. Therefore, the difference in pore pressure at failure is caused by the difference of strain at failure which increases from 1.3% to 4.0% with i (shown in Table 3).

Unfortunately, the data is not sufficiently complete to determine the values of the effective cohesion, c', and the effective angle of shearing resistance, φ', at different values of i; but an estimate of ϕ_i' can be made. From tests on vertical samples, it is known that c' is nearly zero and φ' = 27°. Assuming c' = 0, the Mohr-Coulomb criterion of failure may be written in terms of principal stresses

$$(\sigma_1 - \sigma_3)_f = \frac{2\,(\sigma_3 - u)\sin\phi'}{1 - \sin\phi'} \qquad (51)$$

Assuming ϕ' is isotropic and equal to 27°, the values of $(\sigma_1 - \sigma_3)_f$ are calculated using Eq. 51 and shown in the third column of Table 4.

It is apparent from Table 4 that the difference in pore pressure caused by difference in failure strain accounts for only part of the strength decrease with an increase in i. The required ϕ_i' is shown in the last column of Table 4. For this particular sample, ϕ_i' is reduced by 5° when i changes from 0° to 90°. The effective angle of shearing angle must, therefore, be anisotropic. It is also apparent that the dependence of pore pressure on the strain at failure is also an intrinsic behavior of anisotropic soils and is not a result of the initial stress system.

Further data on the anisotropy of ϕ' of remolded clay has been presented by Broms and Casbarian[23] and has been mentioned elsewhere.[24]

In addition to the two basic types of anisotropy discussed above, anisotropy may arise from another source. During deformation, the shear strains tend to aline the particles. This type of anisotropy may be termed "induced anisotropy," which can only be defined at a particular stage of a test but cannot be measured independently because the over-all effect is reflected in a shear test. However, because this source of anisotropy is inherent in soil during deformation it may be considered as part of the intrinsic anisotropy and need not be studied separately at the present stage.

TABLE 4.—ANISOTROPY OF EFFECTIVE ANGLE OF SHEARING RESISTANCE

i	$(\sigma_1 - \sigma_3)_f$ observed	$(\sigma_1 - \sigma_3)_f$ calculated $\phi' = 27°$	ϕ_i' required
0	36.0	36.8	27
30	30.8	35.8	24.5
45	26.0	30.7	24
90	19.9	27.4	22

Vane Test.—Near the end of the paper, the writer stated that: ". . . for standard dimensions of the vane, it may be easily shown that the vane measures the strength of the soil along the cylindrical surface closely. The vane strength should, therefore, correspond approximately to the strength C_2 . . ." Because the discussers showed considerable interest (with good justification) in the interpretation of the vane test in anisotropic soils, a more detailed analysis of the vane test appears necessary.

The following common assumptions regarding the vane test will be made:

1. The test is performed at a depth sufficiently below the bottom of the borehole or housing (for push, in the Swedish or Norwegian vane) so that the initial state of stresses in the ground at the point where the test is performed is not altered.
2. The soil fails on a vertical cylindrical surface and on horizontal end surfaces.
3. The distribution of shear stresses is uniform at the ends.

It follows from the first assumption that the vane test, in contrast to the unconfined compression test, measures the over-all effects of intrinsic and stress anisotropy. It will be shown subsequently that, in this particular case, the stress anisotropy is not important. With these assumptions, it may easily be shown that the correct expression for the vane test is

$$\frac{C_v}{C_1} = \frac{\dfrac{C_2}{C_1} + \dfrac{D}{3\,H}}{1 + \dfrac{D}{3\,H}} \qquad (52)$$

Eq. 52 is identical with the expression derived by Hansen and Gibson.[5] The essential difference lies in the interpretation of C_1 and C_2. From Eq. 52, it may also be seen that deviations from assumption 3 is relatively unimportant, for $H \geqq 2D$.

For $k = C_2/C_1 = 0.75$, the ratio C_v/C_1 is calculated for different vane dimensions.

$\frac{D}{H}$	0	$\frac{1}{2}$	1	2	4	8
$\frac{C_v}{C_1}$	0.750	0.785	0.813	0.850	0.895	0.933

For the standard case of H = 2D, $C_v/C_1 = 0.785$ which compares closely with $C_2/C_1 = 0.75$ and field average of 0.80 (Table 2). Although the slightly higher value of C_v/C_1 of the field average theoretically can be attributed to stress anisotropy with full justification, such agreement between theory and practice is simply fortuitous. It may, therefore, be concluded that intrinsic anisotropy is the dominating factor for this clay.

Schmertmann suggested that the vane strength be expressed as in Eq. 48. Combining Eqs. 48 and 52 yields

$$X = 1 + \frac{3\,H}{D} \qquad (53)$$

For H = 2D Schmertmann suggested X = 9, without stating the underlying reasons. But a proper value should be 7, as given by Eq. 53. However, the change in the value of X decreases the factor F_c/F_v (shown by the solid lines in Fig. 15) only by a few per cent.

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[23] Broms, B. B., and Casbarian, A. O., "Effect of Rotation of the Principal Stress Axes and of the Intermediate Principal Stress on the Shear Strength," Proceedings, 6th Internatl. Conf. on Soil Mechanics and Foundations, Montreal, Que., Canada, Vol. 1, 1965.

[24] Lo, K. Y., discussion of "Effect of the Rotation of the Principal Stress Axes and of the Intermediate Principal Stress on the Shear Strength," by B. B. Broms and A. O. Casbarian, Proceedings, 6th Internatl. Conf. on Soil Mechanics and Foundations, Montreal, Que., Canada, Vol. 3, 1965.

The increase in the ratio of F_c/F_v with slope angle may be explained in the following manner. As the slope angle β increases, the average rotation of principal stresses decreases. In the limit, the slip circle approaches a plane, so that stability is controlled by C_1 as given by Eq. 33. The factor F_c/F_v therefore increases.

The results of vane tests reported by Aas[20] may be interpreted consistently by using Eq. 52. The importance of stress anisotropy may be assessed by the expressions given by Hansen and Gibson.[5] Substituting some reasonable values of K, ϕ, and the ratio of compressibility, λ, in the Hansen and Gibson expressions, both the inherent and stress anisotropy were found to be of equal importance, depending on the values of the parameters assumed. The vane tests therefore measure both types of anisotropy, but the variation of strength between i = 0° to 90° cannot be determined from vane test results alone. The degrees of anisotropy for the four clays tested by Aas are interpreted to be approximately 1, 0.65, 0.5, and 0.65 for Aserum, Drammen, Lierstranda, and Manglerud, respectively.

The writer agrees with Meyerhof that as the degree of anisotropy increases, the potential failure surface will depart from the cylindrical form. Any form of assumed slip surface may be analyzed by the approach used in the paper.

Errata.—The following corrections should be made in the original paper:

Pages 97 through 102, Figs. 8 through 12: K should read k
Page 90, Fig. 3: The abscissa should read f, instead of i

CONSOLIDATION OF NORMALLY CONSOLIDATED CLAY[a]

Discussions by Gerald P. Raymond, P. N. Sundaram, and Gerald A. Leonards and A. G. Altschaeffl

GERALD P. RAYMOND,[22] A. M. ASCE.—Barden and Berry have obtained some excellent test results in support of their theory and the theory presented by Davis and Raymond.[23] However, there are three aspects of the paper which need amplification.

Unfortunately, the authors do not indicate how the value of b and n can be obtained and, as a result, laboratory interpretation and field predictions still appear to require Terzaghi's theory or, for large load increments and thin clay layers, the modification presented by Davis and Raymond.[23]

[a] September, 1965, by Laing Barden and Peter L. Berry (Proc. Paper 4481).
[22] Asst. Prof. of Civ. Engrg., Queen's Univ., Kingston, Ont., Canada.
[23] Davis, E. H., and Raymond, G. P., "A Non-Linear Theory of Consolidation," Geotechnique, London, England, Vol. 15, 1965, pp. 161-173.

The writer has been working on an extension to the solution developed by Davis and Raymond.[24,25] The initial assumptions are similar to those made by the authors. The fundamental equation was expressed as

$$e = I \log_{10}\left(\frac{k}{k_n}\right) = -C \log_{10}\left(\frac{\sigma}{\sigma_n}\right) \quad (27)$$

in which e = the void ratio, I = a constant, k = the coefficient of permeability, k_n = the value of k when e = 0, C = a constant, σ = the effective stress, σ_n = the value of σ when e = 0.

It has been shown[24,25] how C and I may be obtained from standard laboratory tests (instantaneous loading and a constant increment ratio). Having obtained C and I the third required parameter is

$$k \cdot \sigma^a = k_n \cdot \sigma_n{}^a \quad (28)$$

in which

$$a = \frac{C}{I} \quad (29)$$

is obtained from the time factor and the time to reach 50% consolidation. When C = I the solutions become identical to the Davis and Raymond solutions.

For deep deposits of soil the effect of depth should not be ignored. Solutions may be obtained to the equations previously presented[23] which account for the depth effect. Except for the case of C = I the Terzaghi coefficient of consolidation varies with the load. Because of the weight of the soil itself this results in a variation with depth. Because the case of a constant coefficient of permeability is equivalent to the case of I = ∞ in Eq. 1, it is doubtful whether the authors' Conclusions 5 (last sentence) and 6 are valid, especially for deep beds of clay. No doubt the authors were only considering the consolidation of thin beds of clay, as their equations are based on the assumption of no variation of total load with depth. At present the writer is obtaining a large number of solutions accounting for the depth effect and hopefully will publish them in a form suitable for use in practical problems.

In examining the derivation of the authors' solutions it should be noted that the compression index is obtained from a plot of void ratio versus the natural logarithm of effective pressure. It would appear that the compression index used on Page 33 was obtained from logarithms to the base of ten. If the result shown in Eq. 26 is multiplied by 0.434 it would agree with the predictions of Leonards and Altschaeffl[26] and the writer[24,25].

P. N. SUNDARAM.[27]—The paper presented by Barden and Berry is one more link in the chain of theories on consolidation. The authors confine their

[24] Raymond, G. P., "Rate of Settlement and Dissipation of Pore-Water Pressure During Consolidation of Clays Subjected to Simple Loading and One-Dimensional Drainage," thesis presented to the University of London, at London, England, in 1965, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.
[25] Raymond, G. P., "The Consolidation of Some Normally Consolidated Fine Grained Soils Subjected to Large Load Ratios and One-Dimensional Drainage," Report No. 103, Dept. of Highways, Ontario, Toronto, Ont., Canada, 1966.
[26] Leonards, G. A., and Altschaeffl, A. G., "Compressibility of Clay," Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 90, No. SM5, Proc. Paper 4049, September, 1964, pp. 133-155.
[27] Indian Inst. of Tech., Bombay, India.