VOL.97 NO.SM1. JAN. 1971

PROCEEDINGS OF THE AMERICAN SOCIETY OF CIVIL ENGINEERS



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Station, Vicksburg, Miss. 1966.

The performance of the loundation

where the boyles are. The decleation of the ring learn was calculated assuming different values for the subgress Olzu LOKOO. These remute are shown in The following symbols are used in this paper:

= width of foundation:

= dilational wave velocity;

= shear wave velocity:

= Young's modulus of soil;

= Young's modulus of beam;

= shear modulus:

= moment of inertia for beam:

K = coefficient;

= subgrade modulus;

subgrade modulus for foundation 1 ft wide:

vertical spring constant:

rotational spring constant:

length of straight beam or of segment of beam;

x =distance along beam from concentrated load;

deflection of beam;

yo = deflection of beam beneath concentrated load;

concentrated load acting on beam; activities and about any pure

λ = parameter in theory for beam on elastic foundation; and elastic and elastic foundation;

neer Waterways Experiment Station. Appreciation also to due to the Mainh M.

Parsons Company for making available their design calculations. The pre-

liminary studies described herein were carried out through T. W. Lambe &

Using the Latinity measured are sup vertical deflections, at a rel connection and midway triwern ribs, the proper value for the materade mindrales may be obtained by intermediation and a server of pages 7 and 2 were

which was not taken into account in the analysis.) The results are given in

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## SOIL MECHANICS AND FOUNDATIONS DIVISION

previous solution (1), and function for resulting from region AC Proceedings of the American Society of Civil Engineers

#### LIMIT ANALYSIS OF STABILITY OF SLOPES

By Wilfred F. Chen, 1 A. M. ASCE and M. W. Giger<sup>2</sup>

#### INTRODUCTION

The upper bound theorem of the generalized theory of perfect plasticity has been previously applied to obtain the critical height of an embankment (1). A rotational failure mechanism (logarithmic spiral) passing through the toe was assumed in the analysis (1). These limit analysis results were found to be in good agreement with existing limit equilibrium solutions. The case where the failure plane may pass below the toe, as for small values of friction angle  $\phi$  and embankment slope angle  $\beta$  (Fig. 1), was not considered. This will be described herein. The soil is assumed, as in Ref. 1, to be a perfectly plastic material which obeys the Coulomb yield criterion and its associated flow rule (2).

#### CRITICAL HEIGHT OF EMBANKMENT

The upper bound theorem of limit analysis states that the embankment shown in Fig. 1 will collapse under its own weight if, for any assumed failure mechanism, the rate of external work done by the soil exceeds the rate of internal energy dissipation. Equating external and internal energies for any such mechanism thus gives an upper bound on the critical height.

The rate of external work done by region ABC'CA can easily be obtained by first finding rates of work  $\dot{W}_1$ ,  $\dot{W}_2$ ,  $\dot{W}_3$ , and  $\dot{W}_4$  due to the soil weight in regions OBC'O, OABO, OAC'O, and ACC'A, respectively. The rate of external work for region ABC'CA is then obtained by the simple algebraic summation,  $W_1 - W_2 - W_3 - W_4$ . It is found, after performing some algebraic manipulations, that the total rate of external work due to the weight of the soil in re-

Note.-Discussion open until June 1, 1971. To extend the closing date one month, a written request must be filed with the Executive Director, ASCE. This paper is part of the copyrighted Journal of the Soil Mechanics and Foundations Division, Proceedings of the American Society of Civil Engineers, Vol. 97, No. SM1, January, 1971. Manuscript was submitted for review for possible publication on March 24, 1970.

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<sup>2</sup> Research Asst. of Geotechnical Div., Fritz Engrg. Lab., Lehigh Univ., Bethlehem, Pa.

gion ABC'CA is

in which  $\gamma$  = the unit weight of the soil and  $\Omega$  = the angular velocity of region ABC'CA. Functions  $f_1$ ,  $f_2$ , and  $f_3$  remain identical in their form, as in the previous solution (1), and function  $f_4$  resulting from region ACC' is expressed as

$$f_4 = \left(\frac{H}{r_o}\right)^2 \frac{\sin (\beta - \beta')}{2 \sin \beta \sin \beta'} \left[\cos \theta_o - \left(\frac{L}{r_o}\right) \cos \alpha - \frac{1}{3} \left(\frac{H}{r_o}\right) (\cot \beta' + \cot \beta)\right] \dots (2)$$

in which  $\theta_o$ ,  $\theta_h$ , and  $\beta'$  = angular variables, specifying the assumed mechanism completely; height = H; length of OB =  $r_o$ ; length of AB = L; and length of C'C = d (Fig. 1).

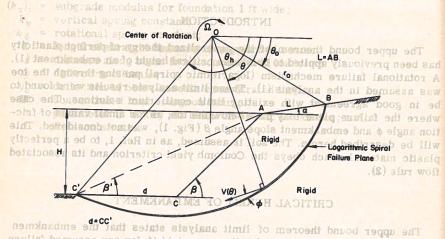


FIG. 1.—FAILURE MECHANISM FOR STABILITY OF SLOPES WITH FAILURE PLANE PASSING BELOW TOE

The internal dissipation of energy occurs along discontinuity surface BC'. The total internal dissipation of energy is identical to the expression given in Ref. 1. Indiew to a supply when a way we have to a supply the supply when a supply we have to a supply the supply when a supply we have to a supply the supply when a supply we have to a supply the supply when a supply we have to a supply the supply when a supply we have to a supply the supply when a supply we have to a supply the supply when a supply we have to a supply the supply when a supply when a supply we have the supply when a supply when a supply we have the supply when a supply when a supply we have the supply when a supply when a supply we have the supply when a supply when a supply we have the supply when a supply when a supply we have the supply when a supply when a supply we have the supply when a supply when a supply we have the supply when a supply when a supply we have the supply when a supply when a supply we have the supply when a supply when a supply we have the supply when a supply when a supply we have the supply when a supply we have the supply when a supply

Equating external rate of work, Eq. 1, with internal rate of energy dissipation yields

$$W_1 - W_2 - W_3 - W_4$$
. It is found, after performing some algebraic manipula (8) s. that, the total rate of external work the to the  $({}^{\alpha}\theta_{i}, {}^{\alpha}\theta_{i}, {}^{\alpha}\theta_{i}) + {}^{\alpha}\theta_{i} + {}^{\alpha}\theta_{i}) = (H_{i} + H_{i}) + H_{i} + H_{i$ 

in which  $f(\theta_h, \theta_o, \beta^i)$  is now defined as global like out to control configuration and

$$f(\theta_h, \theta_o, \beta') = \frac{\sin \beta' \left\{ \exp \left[ 2 \left( \theta_h - \theta_o \right) \tan \phi \right] - 1 \right\}}{2 \sin(\beta' - \alpha) \tan \phi \left( f_1 - f_2 - f_3 - f_4 \right)} \left\{ \sin \left( \theta_h + \alpha \right) \exp \left[ \left( \theta_h - \theta_o \right) \tan \phi \right] - \sin \left( \theta_o + \alpha \right) \right\} \dots \tag{4}$$

TABLE 1.—STABILITY FACTOR  $N_s = H_c$  ( $\gamma/c$ ) BY LIMIT ANALYSIS FOR SMALL VALUES OF  $\alpha$ ,  $\beta$ , and  $\phi$ 

Friction	Slope angle,	4	7	$\beta$ , in de	, in degrees				
angle, $\phi$ , in degrees (1)	α, in degrees (2)	50 (3)	45 (4)	40 (5)	35 (6)	30 (7)	25 (8)	20 (9)	15 (10)
0 5 5 5	0 0 0 12 5 0 0 0 0 0 0	5.52 6.92 <sup>a</sup> 6.76 <sup>a</sup>	5.53 7.35 7.18	5.53 7.84 <sup>a</sup> 7.64 <sup>a</sup>	5.53 8.41 <sup>a</sup> 8.19 <sup>a</sup>	5.53 9.13 8.83	5.53 10.02 9.65	5.53 11.46 10.99	5.53 14.38 13.71

a Failure through toe; all others failure below toe.

TABLE 2.—COMPARISON OF STABILITY FACTOR  $N_s=H_c$  ( $\gamma/c$ ) BY METHODS OF LIMIT EQUILIBRIUM AND LIMIT ANALYSES  $\alpha=0$ 

Slope angle, $\beta$ , in degrees		1:04:0	1610	CURVED FAILURE SURFACE						
		Friction angle, $\phi$ , in degrees		1988 1986Li 1988	Limit Analysis					
(1)	107.0	180 (2)	1520	Slices (3)	φ Circle (4)	Log-Spiral (5)	Log-Spiral			
90	1494	11.80	28463	3,83	3.83	3,83	3,83			
10.70	186.18	08.8 5	28885	4.19	4.19	4.19	4.19			
17.01	9:22	100.15	28538	5.02	5.02	7.402	5.02			
85.01	10018	25	201.5	6.06	6.06	2.08 6.06	6.06			
75	I Lake	10.00	37858	4.57	4.57	0 4.57	4.56			
38,81	115.49	98.9 5	53018	5.13	5.13	- 6	5.14			
13.24	19841	57.415	26726	6.49	6.52	10	6.57			
04,01	149,16	257 25	80486	8.48	8.54	15	8.58			
12,63	180.45	2. 1000	18888	61824	1600.在4	20				
60	10:30	24 A 4 0	34.118	5.24	5.24	35.24	5,25			
18.37	20,01	5	36605	6.06	6.18	08 6.18	6.16			
		15		8.33	8.63	8.63	8,63			
14.91	188 EL	25	CARGE	12.20	12.65	12.82	12.74			
69'91	77.EL	199.41	(0.00)		1.38	8				
45	18,68	15.4E 0	6時/度	5.88	5.88 <sup>a</sup>	5.88ª	5.53 <sup>a</sup>			
50.88	13,50	5	81738	7.09	7.36	GJ, 1	7.35			
26.25	12,42	15	ASSER	11.77	12.04	20	12,05			
16.07	96.61	25	明美地	20.83	22.73	25	22.90			
35,64	20,61	10.98	01.00 01.00		46.0	30 a 35				
30	20114	0	COVAL GRAD	6.41 <sup>a</sup>	6.41 <sup>a</sup>	6.41 <sup>a</sup>	5.53 <sup>a</sup>			
22,49	31,41	T8,645	ista di	8.77ª	9.09 <sup>a</sup>	0	9.13 <sup>a</sup>			
24.63	10.9	15	1 042 042	20.84	21.74	2	21.69			
68.49	16.91	25	1.664.101	83.34	111,1	125.0	119.93			
94.19	88.85	17.64	188.93	28.8	8.13	15				
15	87.0	ea.e. 0	1 96 90	6.90 <sup>a</sup>	6.90a	02 6.90a	5.53 <sup>a</sup>			
80.12	38.85	14.815	186.00	13.89 <sup>a</sup>	014.71	14.71 <sup>a</sup>	14.38 <sup>a</sup>			
87.08	48,84	88.810	1.66.00	8.25	43.62 <sup>a</sup>	80	45.49			

a Critical failure surface passes below toe.

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Friction angle,	Slope angle,	L Lower	marks to	Part of the	English of the	Erm vl	
φ, in degrees	α, in degrees	90	85	80	75	70	65
(1)05	(2) 38	(3)	(4)	(5)	(6)	(7)	(8)
POPI (A)	6 (2)	(0)02	(4)	(3)	(0)	30. (1) 63	(0)
0	0	3.83	4.08	4.33	4.56	4.80	5.03
eela 14efla Tid	01 8018 10.	# 1 See.	F 6 88 8	200.00	a . 0		.0
88.01 50.11 Pa	41" 5018 10.	4.19	4.50	4.82	5.14	5.47	5.81
17.EL   CB. GE. 6	e 2953 ei	4.14	4.44	4.74	5.05	5.37	5.69
10	0	4.58	4.97	5.37	5.80	6.25	6.73
	5	4.53	4.91	5.30	5.71	6.15	6.63
while en da	and 10 = ang	4.47	4.83	5.21	5.61	6.03	6.48
10 11 15	作品等的"社"。	5.02	5.50	6.01	6.57	7.18	7.85
TOU A A CHIE	5	4.97	5.44	5.94	6.49	7.08	7.75
	10	4.91	5.36	5.85	6.38	6.97	7.63
	15	4.83	5.27	5.74	6.26	6.82	7.46
FACE	The second second second second second	n Cirida 2-800-1			0.20	7.02	120
20	D PAILOURE SUI	5.50	6.10	6.75	7.48	8.30	9.25
Heald	5	5.46	6.04	6.68	7.40	8.21	9.16
	(210mmale	5.40	5.97	6.60	7.30	8.10	9.04
Auniysia	15	5.33	5.88	6.50	7.18	7.97	8.89
larina-pol	20	5.24	5.77	6.37	7.03	7.79	8.68
25	(d) 0	6.06	6,79	7.62	8,58	9.70	11.05
	5	6.01	6.73	7.56	8.50	9.61	10.96
28.8	38.4 10	5.95	6.67	7.48	8.41	9.51	10.84
4.19	15	5.89	6.58	7.38	8.30	9.38	10.70
20.2	20	5.80	6.48	7.26	8.16	9.22	10.51
80.8	80.8 25	5.70	6.35	7.10	7.97	9.00	10.26
3430	0-4.67	6.69	7.61	8.67	0 9.94	11.48	13.44
6.1.6	5	6.64	7.55	8.61	9.86	11.40	13.35
78.8 1.5.1	10	6.59	7.48	8.53	9.77	11.30	13.24
85.8	15	6.52	7.40	8.44	9.67	11.18	13.10
00.0	20	6.44	7.31	8.32	9.54	11.03	12.93
an a 10	25	6.35	7.19	8.18	9.37	10.83	12.70
6.25	30	6.22	7.04	7.99	9.14	10.56	12.37
FER.S FAIL	ALO SULLANIS	A SEEVER	1.038	no si	at we w	10.00	12.01
35		7.42	8.58	9.97	11.68	13.86	16.77
	5	7.38	8.52	9.90	11.60	13.77	16.68
Theodeternal	588 10	7.32	8.46	9.82	11.51	13.68	16.58
	The second second second second second	7.26	8.38	9.73	11.41	13.56	16.44
ne to intern	disa20 1101 d	7.19	8.29	9.63	11.29	13.42	16.29
22, 90 .19	25	7.10	8.18	9.49	11.13	13.23	16.07
Equating ext	30	6.99	8.04	9.33	10.93	12.99	15.78
atio sa bride	8 <sub>14.8</sub> 35 a	6.84	7.86	9.10	10.64	12.64	15.34
40	0	8.29	9.77	11.61	13.97	17.15	21,72
7 - F (A	A A .	8.24	9.71	11.54	13.89	17.07	21.63
20.811)	9.010	8.19	9.65	11.46	13.81	16.97	21.53
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william (0 4 ) a	20 20 20	8.06	9.49	11.27	13.59	16.72	21.25
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45,49	30	7.87	9.25	10.99	13.25	16.33	20.78
A Party of the Par	35	7.74	9.09	10.78	13.00	16.02	20.39
	40	7.56	8.86	10.50	12.64	15.55	19.77

tunction  $f(\theta_h, \theta_o, \theta')$  has a minimum and, thus, | SISYLANA TIMIL BY  $(\theta_h, \theta_o, \theta')$  and  $\theta'$  satisfy conditions

s	lope ang	le, β, in d	legrees		0 =	108 :0	6 186	;0 =-	100 a 0 0
60	55	50	45	40	35	30	25	20	15
(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
5.25	5.46	5.52	5.53	5.53	5.53	5.53	5.53	5.53	5.53
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6.16	6.53	6.92	7.35	7.84	8.41	9.13	10.02	11.46	14.38
6.03	6.38	6.76	7.18	7.64	8.19	8.83	9.65	10.99	13.71
- 00	7.84	8.51	9.31	10.30	11.61	13.50	16.64	23,14	45.49
7.26	7.72	8.38	9.16	10.13	11.42	13.28	16.37	22.79	44.95
7.14 6.99	7.54	8.18	8.93	9.87	11.11	12.89	15.84	21.96	42.90
6.33	1.01	0.10	0.00				2002	Alban B	mit a first
8.63	9.54	10.64	12.05	13.97	16.83	21.69	32.11	69.40	MAL SINGSAL
8.52	9,42	10.51	11.91	13.82	16.65	21.48	31.85	69.05	ESEMBLE FO
8.38	9.26	10.34	11.72	13.59	16.38	21.14	31.38	68.26	stopes o
8.19	9.04	10.09	11.42	13.23	15.92	20.49	30.25	65.17	le a #
	ruisted	- sull	s will	as needs		980.48	04.00	desigt	of such
10.39	11.80	13.63	16.16	19.99	26.66	41.22	94.63	98 .	
10.28	11.69	13.51	16.03	19.85	26.48	41.02	94.38	(J75):	Harrie II II The
10.16	11.54	13.35	15.85	19.64	26.23	40.69	93.78 92.90		
9.98	11.35	13.12	15.58	19.32	25.82	40.09			
9.74	11.07	12.79	15.17	18.77	25.01	38.64	88.63	PALLIN	FIG. 2
12.74	14.97	18.10	22,90	31.33	50.06	119,93		MILE	SLOPE A
12.64	14.86	17.98	22.77	31.19	49.89	119,70	all equ		
12.52	14.73	17.83	22,60	30.99	49.63	119.35	no and	seful.	It can be
12.36	14.55	17.62	22.35	30.69	49.23	118.79	limit	Inalys	
12.14	14.30	17.33	21.98	30.20	48.50	117.43			
11.84	13.92	16.85	21.35	29.24	46.76	112.07	oage.	age may	
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16.04	19.71	25.41	35.54	58.27	144.20 144.01	TOMAN		es of	
15.94	19.61	25.29	35.41	58.13 57.92	143.74	0=9			
15.82	19.48	25.15	35.25	57.63	143.74	1		N.C.	
15.67	19.31	24.96 24.68	35.01 34.67	57.16	142.54	and the		0 95	
15.47	19.08	24.27	34.11	56.30	140.54	1.4 B			
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20.94	27.45	39,11	65.52	166.38	THE PER	Faity.		1	
20.84	27.34	39.00	65.39	166.22	mer Di	ty Line	the mas	se fip	and Jan
20.73	27.22	38.85	65.22	166.00				18A	
20.58	27.05	38.66	64.70	165.72	48	1		ATS	
20.40	26.84	38.40	64.65	165.19	and the second			(0)	
20.14	26.53	38.02	64.12	164.30					
19.78	26.07	37.38	63.14	162.33					
19.21	25.27	36.15	60.80	154.98	-REPE	RENCES			
28.91	41.89	71.49	185.49	60	0.0	75	0	e	
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28.71	41.66	71,23	185.17	THE PROPERTY AND	COMMINS 2	DO LOS			
28.57	41.51	71.04	184.93						1 12 16 1
28.39	41.29	70.78	184.57	WAL M.	HOTESAN	ALTERIAL SECTION	F GRISTA SOI	the Share	The Paris of the
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27.82	40.58	69.81	183.01	and the same	10 /10	410.	er en =	W ## M	11842 0
27.32	39.88	68.73	180.81	Bul Mer	Bertin S. F	William I	ally six cel	1	
26.45	38.53	66.12	172.51	10 pp 1	7. 165 19	2		- DE	A SELECTION

function  $f(\theta_h, \theta_o, \beta')$  has a minimum and, thus, indicates a least upper bound when  $\theta_h$ ,  $\theta_o$ , and  $\beta'$  satisfy conditions

with  $\beta' \leq \beta$  (Fig. 1). The corresponding values for  $\theta_h$ ,  $\theta_o$ , and  $\beta'$  satisfying

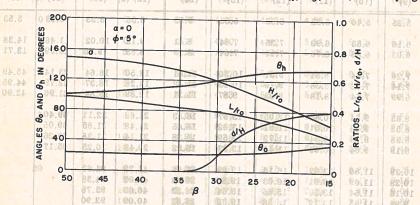


FIG. 2.—CRITICAL VALUES FOR  $\theta_h$ ,  $\theta_o$ ,  $L/r_o$ ,  $H/r_o$ , AND d/H AS FUNCTION OF SLOPE ANGLE β

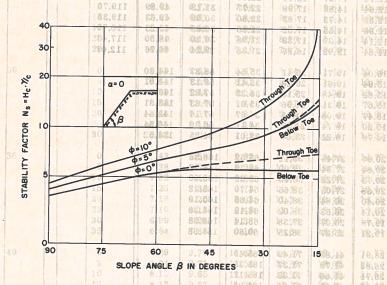


FIG. 3.—STABILITY FACTORS  $N_s$  AS FUNCTION OF  $\beta$ 

which is the ratio between distance d and critical height  $H_c$  (Fig. 1).

#### NUMERICAL RESULTS Proceedings of the Assessant Arbent Free folding the block of

A complete numerical solution to this problem has been obtained by numerical methods, the numerical work being performed on a CDC 6400 digital = function defined in Eq. 4, see Ref. 1: computer.

The results are tabulated numerically in Table 1 for small values of  $\alpha$ ,  $\beta$ , and  $\phi$ . For the case of  $\phi = 5^{\circ}$  and  $\alpha = 0^{\circ}$  the corresponding critical values of  $\theta_h$ ,  $\theta_o$ ,  $H/r_o$ ,  $L/r_o$ , and d/H are plotted in Fig. 2. Fig. 3 shows the transition zone where the most critical failure plane starts to pass below the toe, when  $\alpha = 0$ . Comparison of limit analysis results with already existing limit equilibrium solutions are given in Table 2. It may be seen that agreement is good. Table 3 tabulates the value of  $N_s$  for various combination of slopes  $\alpha$ and  $\beta$ . Because there are no existing solutions available for the case  $\alpha \neq 0$ , the tabulated results will be useful in the analysis and design of such problems. y = unit weight;

#### CONCLUSIONS be to signe noticitat = 0

θh, θo, β' = angular variables (Fig. 1);

The agreement between the limit analysis and limit equilibrium results relative to the stability of slopes proves both interesting and useful. It can be concluded, therefore, that the upper bound theory of limit analysis may be applied to predict the critical height of slopes. In many cases it may be much more convenient to use the upper bound method and it also places the matter of stability analysis on a much more logical ground.

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The research reported herein was supported by the National Science Foundation under Grant GK-14274 to Lehigh University.

The writers thank Mrs. Jane Lenner for typing the manuscript and John Gera for the figure drafting.

# APPENDIX I.-REFERENCES

A setumic wave travelling through a number of reclogic formations business

reaches an engineering wark would undergo a multitude or reflections, re-

wave effect outside the concerns and symbol might well be unticipated. Such an effect might also be particularly the entire at sites adjacent to a sloping rook

- 1, Chen, W. F., Giger, M. W., and Fang, H. Y., "On the Limit Analysis of Stability of Slopes," Soils and Foundations, The Japanese Society of Soil Mechanics and Foundation Engineering, Vol. IX, pp. 23-32, 1969.
- 2. Drucker, D. C. and Prager, W., "Soil Mechanics and Plastic Analysis or Limit Design," Quarterly of Applied Mathematics, Vol. 10, pp. 157-165, 1952.
- 3. Taylor, D. W., Fundamentals of Soil Mechanics, John Wiley and Sons, New York, 1948.

#### APPENDIX II.—NOTATION

The following symbols are used in this paper:

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marical methods, the numerical work being performed (noisedos (= 0) digital

f, = function defined in Eq. 4, see Ref. 1;

 $f_2$  = function defined in Eq. 5, see Ref. 1; because on eliment and

boulf = function defined in Eq. 6, see Ref. 1; d = \$10.000 and and a bour

notifie f = function defined in Eq. 2; letter gas Hybrids to the forth to a

nedw . fd = function defined in Eq. 4; alg eruffel is the most end end end

H, Hc = height and critical height of embankment, respectively;

at hL, d = lengths (Fig. 1); vam if . S sideT all newing our anotheries mutidil

saqNs = stability factor; are rot W to solve edt satalbear & sideT boog

 $r_0, r(\theta)$  = length variables of logarithmic spiral curve; and parabel a base

 $down V(\theta) = discontinuous velocity across failure plans (Fig. 1); because and$ 

 $\alpha$ ,  $\beta$  = slope angles:

 $\gamma$  = unit weight;

 $\theta_h$ ,  $\theta_o$ ,  $\beta'$  = angular variables (Fig. 1);

φ = friction angle of soil; and ΔΙΟΝΟΟ \* AND GAR AS FUNCTION OF

 $\Omega$  = angular velocity.

The agreement between the itanif analysis and limit equilibrium results relative to the stability of slopes proves both interesting and useful. It can be concluded, therefore, that the ruger bound theory of limit smalysis may be applied to predict the critical height of slapes. In usery cases, it may be much more convenient to use the upper bound method and it also places the matter of stability analysis on a much more logical ground.

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The research reported herein was supported by the National Solence Foundation under Grant GK-14274 to Lehigh University.

The writers though Mrs. Jane Lenser low typing the manuscript and John Gera for the figure drafting.

APPENDIX !.- REFERENCES

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1. Chen. W. F. Oiget. Mr. W. wild Farg. H. W. Ton'the Unite Analysis of Stability of Slapes."

Soils and Foundations. The Japanese Society of Soil Mechanics and Foundation Engineering.

Vol. His. Dp. 22 42 1959.

2. Drucket. D. C. and Frager. W. Soil Mechanics and Plastic Analysis of Lima Design. Cuor.

Letter of Applied Mathematics, Vol. 10, pp. 137-165, 1952
3 Taylor, D. W., Fundamentak of Soil Nathanitis, John Wiley and Sons, New York, 1948

<sup>2</sup> Prof. of Civ. Engrg., Univ. of Calif., Berkeley, Calif.

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## SOIL MECHANICS AND FOUNDATIONS DIVISION

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RESPONSE OF NONUNIFORM SOIL DEPOSITS TO TRAVELLING
SEISMIC WAVES

By Houshang Dezfulian, A. M. ASCE and H. Bolton Seed, M. ASCE

#### INTRODUCTION

Records of ground motions in recent earthquakes have provided convincing evidence of the important effects of local soil conditions on the amplitude and frequency characteristics of ground surface motions. Marked variations in surface motions may occur over relatively short distances due to variations either in depth or character of the underlying soil deposits. Such variations are likely to occur in areas adjacent to sloping rock surfaces such as that shown in Fig. 1, and methods of evaluating these variations by determining the response of the soil deposit to motions developed in the underlying rock have been presented. In these analyses, however, the rock motions have been treated as only a time-dependent phenomenon with the same motions being developed at all points in the rock at any given instant. Possible spacial variations in rock motions, due to wave propagation effects, have not been considered.

While the present state of knowledge with regard to spacial variations in seismic motions is quite deficient, the possibility of some type of travelling wave effect outside the epicentral region might well be anticipated. Such an effect might also be particularly important at sites adjacent to a sloping rock surface (as in Fig. 1) where the motions might propagate towards or away from the rock outcrop.

A seismic wave travelling through a number of geologic formations before it reaches an engineering work would undergo a multitude of reflections, re-

Note. - Discussion open until June 1, 1971. To extend the closing date one month, a

written request must be filed with the Executive Director, ASCE. This paper is part of the copyrighted Journal of the Soil Mechanics and Foundations Division, Proceedings of the American Society of Civil Engineers, Vol. 97, No. SM1, January, 1971. Manuscript was submitted for review for possible publication on January 7, 1970.

1 Asst. Prof. of Civ. Engrg., Univ. of Tehran, Tehran, Iran.