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DISCUSSION

Proc. Paper 7785

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[a] Discussion period closed for this paper. Any other discussion received during this discussion period will be published in subsequent Journals.

C = damping matrix;
d_s = distance travelled in direction of V_s;
d_x = distance travelled in direction of V_x;
K = stiffness matrix;
M = mass matrix;
$P_{eff}(t)$ = time-dependent vector of effective forces at nodal points;
R = influence coefficient matrix;
t = time;
u = vector of displacements of nodal points due to inertia forces;
u_c = vector of total displacements of base nodal points;
u_g = vector of prescribed displacements of base nodal points;
u_s = vector of static displacements of nodal points;
u_t = vector of total displacements of nodal points above base;
V_p = velocity of propagation of longitudinal waves;
V_s = velocity of propagation of shear waves;
V_x = velocity of propagation of wave front along x-axis; and
θ = angle between direction of wave propagation and x-axis.

Journal of the
SOIL MECHANICS AND FOUNDATIONS DIVISION
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PHREATIC SURFACE LOCATION AFTER DRAWDOWN

By Arthur H. Dvinoff,[1] A. M. ASCE and Milton E. Harr,[2] M. ASCE

INTRODUCTION

The stability of an earth embankment impounding a water reservoir is highly dependent upon the location of the phreatic surface within the embankment. Of particular interest is the stability during or immediately following a rapid drawdown of reservoir level. Because of the lack of a solution procedure, the assumption is often made that no drainage occurs within the embankment during drawdown. This contention may lead to an overly conservative slope design in some cases.

The equation governing two-dimensional flow in porous media has been found to be (3,4)

$$n\frac{\partial h}{\partial t} = k_x h \frac{\partial^2 h}{\partial x^2} + k_x\left(\frac{\partial h}{\partial x}\right)^2 + k_y h \frac{\partial^2 h}{\partial y^2} \quad \ldots\ldots\ldots\ldots (1)$$

in which n = effective porosity; h = total head at point x, y at time t; and k_x, k_y = coefficients of permeability in the x and y directions, respectively. Solution of Eq. 1 yields the phreatic surface at all points where total head equals elevation head. If the phreatic surface is relatively flat, it may be assumed that the gradients are small. Thus term $k_x(\partial h/\partial x)^2$ may be neglected. For an isotropic media, Eq. 1 then reduces to the linear form

$$\frac{\partial h}{\partial t} = \frac{k\bar{h}}{n}\left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2}\right) \quad \ldots\ldots\ldots\ldots (2)$$

in which $\bar{h}$ = the mean height of the phreatic surface and k = the isotropic permeability (2,3,10). Eq. 2 had been employed to locate the phreatic surface

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within the shell of a zoned earth embankment subjected to a rapid drawdown of reservoir level.

BOUNDARY CONDITIONS

Fig. 1 represents a zoned embankment with a vertical impervious core, resting on a horizontal impervious foundation. Initially, the reservoir level

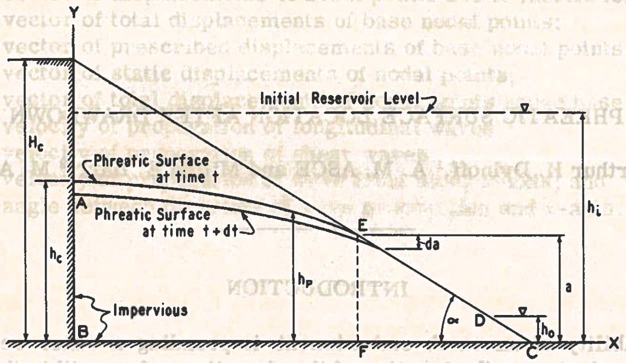

FIG. 1.—REGION OF FLOW

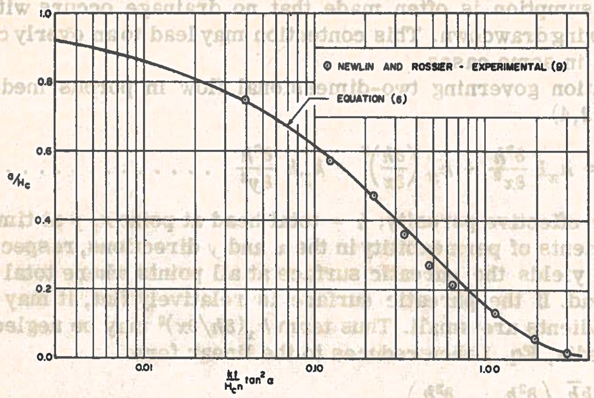

FIG. 2.—COMPARISON OF EQ. 6 WITH EXPERIMENTAL DATA OF NEWLIN AND ROSSIER (9)

was h_i and the embankment shell was fully saturated below that elevation. The reservoir level was rapidly lowered to h_o and the resulting phreatic surface is shown at some time t after onset of drawdown. The phreatic surface intersects the vertical core at level h_c above the foundation (Point A) and the embankment slope at height a (Point E). The intersection of the phreatic surface with the embankment slope will be referred to as the exit point.

The boundary conditions surrounding the region of flow are now completely

defined. Across the impervious boundaries there is no flow. Thus, along AB, $\partial h/\partial x = 0$ and, along BC, $\partial h/\partial y = 0$. Along the phreatic surface and the surface of seepage, ED, the total head is equal to the elevation head. Thus, along AE and ED, $h = y$. Finally, along the reservoir boundary, DC, total head is equal to the drawdown reservoir elevation, or $h = h_o = f(t)$, in which $f(t)$ may be any arbitrary function of time. For instantaneous drawdown, h_o is constant for $t > 0$.

With the boundary conditions known, it is possible to solve Eq. 2 and locate the phreatic surface at any time t. However, it is first necessary to locate the exit point at time t independently of Eq. 2.

LOCATION OF EXIT POINT

Assume that in the time interval dt the phreatic surface falls a distance da, such that the new position is parallel to the old position (Fig. 1). The volume of water that has been removed from a unit thickness of the section is

$$dV = n(H_c - a)\ \text{ctn}\ \alpha\ da + \frac{1}{2}\ n\ \text{ctn}\ \alpha\ (da)^2 \quad \ldots\ldots\ldots\ldots\ (3)$$

If the flow through section EFCE is essentially horizontal, from Darcy's Law, the discharge was found by Pavlovsky (5,7) to be

$$Q = -k(a - h_o)\left(1 + \ln\frac{a}{a - h_o}\right)\tan\alpha \quad \ldots\ldots\ldots\ldots\ (4)$$

Since the discharge during any time interval must equal the volume between the phreatic surface locations at the beginning and end of the interval, $Qdt = dV$, or, with second-order terms neglected

$$-\int_o^t \frac{k\tan^2\alpha}{n}\,dt = \int_{H_c}^{a} \frac{\dfrac{H_c - a}{a - h_o}}{1 + \ln\dfrac{a}{a - h_o}}\,da \quad \ldots\ldots\ldots\ldots\ (5)$$

If $h_o = 0$ for $t > 0$ (total instantaneous drawdown), Eq. 5 may be integrated directly with the result:

$$\frac{kt}{H_c n}\tan^2\alpha = \frac{a}{H_c} - 1 - \ln\frac{a}{H_c} \quad \ldots\ldots\ldots\ldots\ (6)$$

Eq. 6 is plotted in Fig. 2 along with data points from a model test conducted by Newlin and Rossier (9) for the case of total instantaneous drawdown from full pool ($h_i/H_c = 1.0$). Noted that the agreement is very close.

If h_o is not equal to zero, Eq. 5 is difficult to integrate directly. A numerical scheme, shown in Fig. 3, was employed instead with the second-order terms retained. Using this scheme, the exit point was located after instantaneous drawdown from full pool for various drawdown ratios (h_o/h_i). The results are shown in Fig. 4.

For rapid but not instantaneous drawdown, h_o is defined as a function of time. In particular, let $h_i = H_c$; $h_o = H_c - \sigma t$ when $t \le t_f$; and $h_o = H_c - \sigma t_f$ = constant when $t > t_f$; in which σ = constant rate of drawdown and t_f =

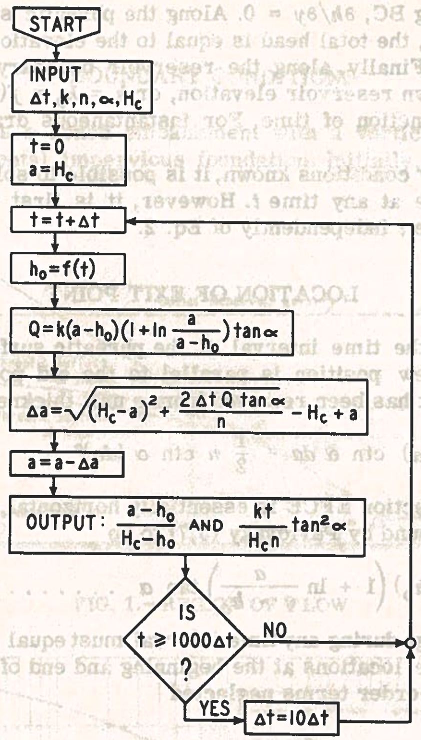

FIG. 3.—NUMERICAL PROCEDURE TO LOCATE EXIT POINT AFTER DRAWDOWN FROM FULL POOL

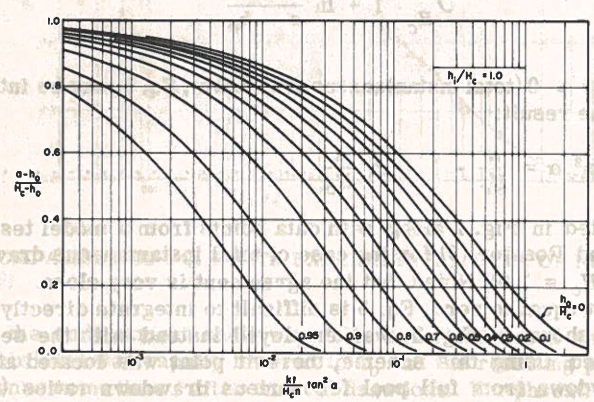

FIG. 4.—ELEVATION OF EXIT POINT AFTER INSTANTANEOUS DRAWDOWN FROM FULL POOL

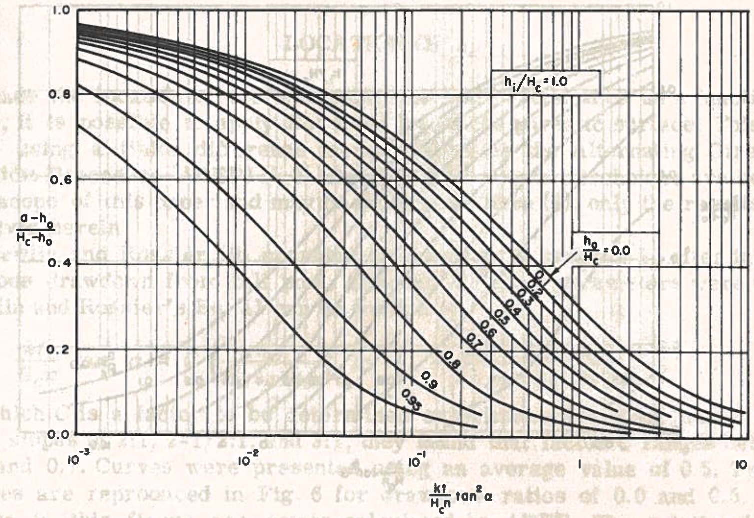

FIG. 5.—ELEVATION OF EXIT POINT DURING DRAWDOWN AT CONSTANT RATE FROM FULL POOL

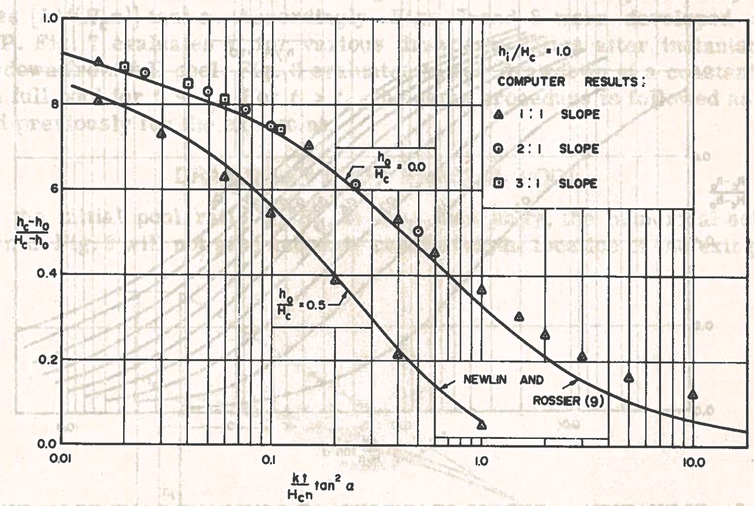

FIG. 6.—COMPARISON OF COMPUTER PREDICTION OF h_c AFTER INSTANTANEOUS DRAWDOWN WITH SOLUTION OF NEWLIN AND ROSSIER (9)

duration of drawdown. For this condition Fig. 5 locates the exit point during drawdown from full pool for any time $t \le t_f$. To locate the exit point for

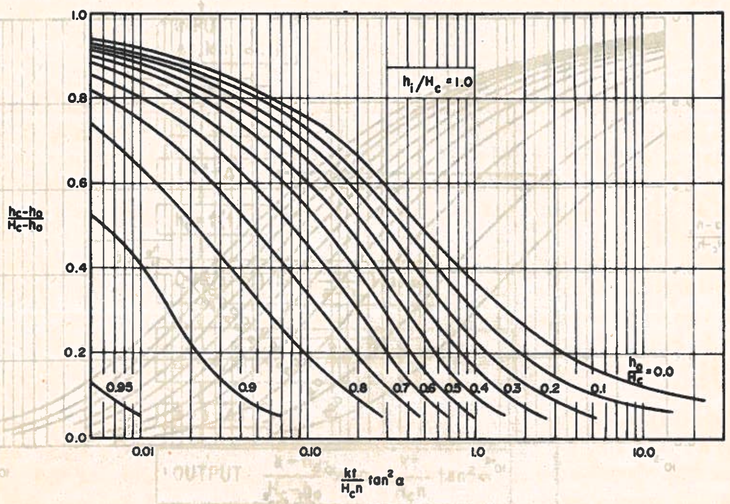

FIG. 7.—ELEVATION h_c AFTER INSTANTANEOUS DRAWDOWN FROM FULL POOL

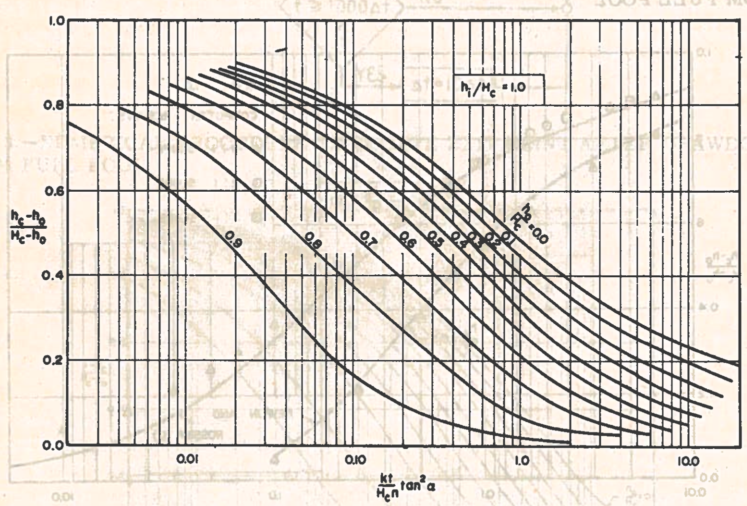

FIG. 8.—ELEVATION h_c DURING DRAWDOWN AT CONSTANT RATE FROM FULL POOL

$t > t_f$, it is necessary to return to Fig. 4. Fig. 4 is entered from the left with a value of $(a - h_o)/(H_c - h_o)$ computed using the elevation of exit point at t_f. A value of $[kt/(H_c n)] \tan^2 \alpha$ (known as T_f) is thus determined for a

given value of h_o/H_c. The value of $(a - h_o)/(H_c - h_o)$ for $t > t_f$ is now that ordinate corresponding to an abscissa of $T_f + [k(t - t_f)/(H_c n)] \tan^2 \alpha$ and the proper curve for h_o/H_c.

LOCATION OF h_c

Once the location of the exit point has been established as a function of time, it is possible to apply Eq. 2 and locate the phreatic surface. This was done using a finite difference method known as the Alternating Direction Explicit Procedure (ADEP) (1,8). Details of the solution technique are beyond the scope of this paper and may be found elsewhere (4); only the results will be given herein.

Newlin and Rossier (9) provided an equation to evaluate h_c after instantaneous drawdown from full pool. Although different parameters were used, Newlin and Rossier's Eq. 11 can be written as

$$\frac{kt}{H_c n} \tan^2 \alpha = C \left(\frac{h_o}{H_c - h_o}\right) \ln \left[\left(1 + \frac{h_c - h_o}{2H_c - h_o}\right) \frac{h_o}{h_c}\right] \qquad (7)$$

in which C is a factor to be determined experimentally. Using model tests with slopes of 2:1, 2-1/2:1, and 3:1, they found that factor C ranges between 0.3 and 0.7. Curves were presented using an average value of 0.5. Typical curves are reproduced in Fig. 6 for drawdown ratios of 0.0 and 0.5. Also shown in this figure are points calculated by ADEP. The relationship of $[kt/(H_c n)] \tan^2 \alpha$ to $(h_c - h_o)/(H_c - h_o)$ is seen to be independent of slope angle. This fact was also observed by Newlin and Rossier. The computed points are seen to fall within the range of the factor C except for very large values $[kt/(H_c n)] \tan^2 \alpha$. Accordingly, Figs. 7 and 8 were developed using ADEP. Fig. 7 evaluates h_c for various drawdown ratios after instantaneous drawdown from full pool. Fig. 8 evaluates h_c for drawdown at a constant rate from full pool for $t \le t_f$. For $t > t_f$, the same procedure is followed as indicated previously for the exit point.

DRAWDOWN FROM PARTIAL POOL

If the initial pool ratio, h_i/H_c, is less than unity, the numerical scheme shown in Fig. 3 will not give accurate results for the location of the exit point.

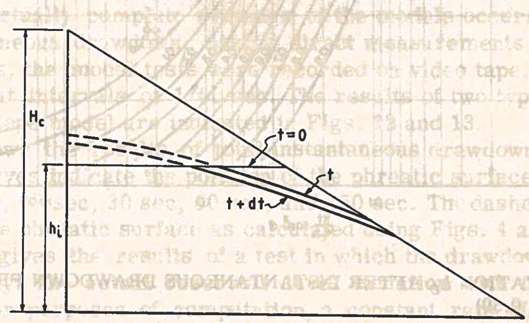

FIG. 9.—LOCATION OF PHREATIC SURFACE A SHORT TIME AFTER DRAWDOWN FROM PARTIAL POOL

Eq. 3 is based upon the assumption that the elevation of the entire length of the phreatic surface from the exit point to the impervious core is changing

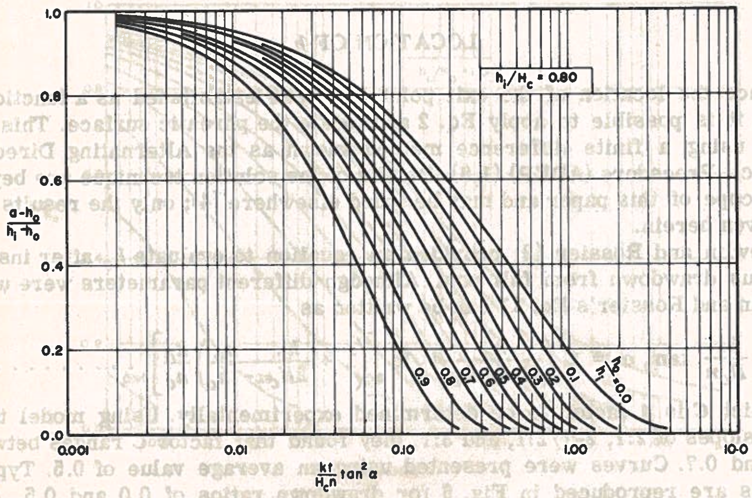

FIG. 10.—ELEVATION OF EXIT POINT AFTER INSTANTANEOUS DRAWDOWN FROM PARTIAL POOL (h_i/H_C = 0.80)

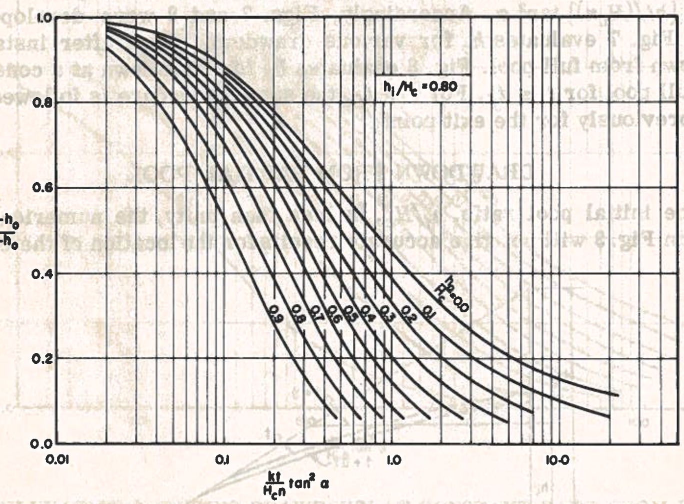

FIG. 11.—ELEVATION h_c AFTER INSTANTANEOUS DRAWDOWN FROM PARTIAL POOL (h_i/H_C = 0.80)

from the first increment of time. However, as shown in Fig. 9, if the initial pool ratio is considerably less than unity a finite time is required before the

effect of the drawdown reaches the impervious core. This phenomenon has been observed in model studies (4), and was accounted for in the finite difference program. Using ADEP, Figs. 10 and 11 were prepared which locate the exit point and evaluate h_c respectively, after instantaneous drawdown from 80 % partial pool.

If the initial pool ratio is only slightly less than unity, Figs. 4 and 5 may be used to determine the time required for the exit point and h_c to reach given elevations. First find the time required to reach the given elevation and then subtract the time which would be required to reach elevation h_i. This procedure may also be used with Figs. 7 and 8, but trial and error is required to match the proper drawdown ratio with the proper time.

LOCATION OF PHREATIC SURFACE

It has been found that after h_c and the exit point have been located, the remainder of the phreatic surface may be reasonably approximated by a parabola subject to the following boundary conditions. At

$$\left.\begin{array}{ll} x = 0 & h_p = h_c \\ x = (H_c - a) \text{ ctn } \alpha & h_p = a \\ x = 0 & \dfrac{dy}{dx} = 0 \end{array}\right\} \qquad (8a)$$

The parabola conforming to these conditions was found to be

$$h_p = h_c - (h_c - a)\left(\frac{x}{D}\right)^2 \qquad (8b)$$

in which $D = (H_c - a)$ ctn α.

EXPERIMENTAL VERIFICATION

To assure the validity of the foregoing analysis, tests were conducted with Hele-Shaw viscous flow models (5,6,10), also referred to as thin slit models. Two models were constructed with slopes of 1:1 (45°) and 2:1 (26.6°), respectively. Virtually complete drainage of the models occurred within 5 min after instantaneous drawdown, making direct measurements impractical. To overcome this, the model tests were recorded on video tape, which produced still frames at intervals of 1/60 sec. The results of two typical model tests with the 2:1 slope model are indicated in Figs. 12 and 13.

Fig. 12 shows the results of total instantaneous drawdown from full pool. The solid curves indicate the position of the phreatic surface in the model at times of 5 sec, 15 sec, 30 sec, 60 sec, and 150 sec. The dashed lines show the position of the phreatic surface as calculated using Figs. 4 and 7 and Eq. 8*b*.

Fig. 13(*b*) gives the results of a test in which the drawdown did not occur instantaneously. The actual reservoir level variation with time is shown in Fig. 13(*a*). For purposes of computation, a constant rate of drawdown was assumed for the first 18 sec with a constant tailwater elevation thereafter. This is shown as a dashed line. In Fig. 13(*b*) the model results are again shown as solid lines, while the calculated phreatic surface locations are again

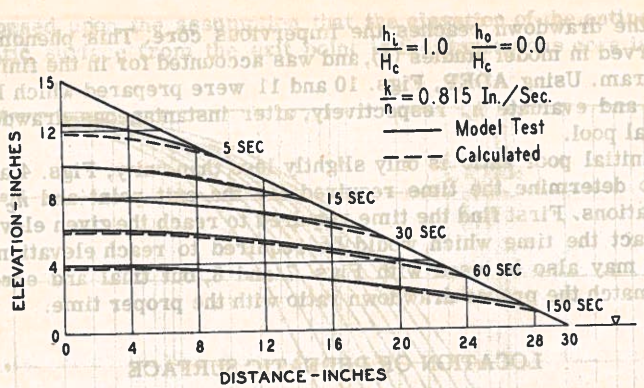

FIG. 12.—MODEL TEST A—INSTANTANEOUS DRAWDOWN

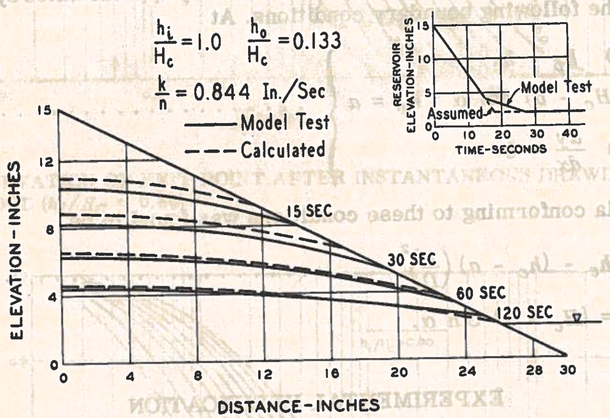

FIG. 13.—(*a*) RESERVOIR LEVEL VARIATION FOR MODEL TEST B; (*b*) MODEL TEST B—DRAWDOWN AT CONSTANT RATE

shown as dashed lines. For the first 18 sec Figs. 5 and 8 were employed. For time greater than 18 sec Figs. 4 and 7 were used.

CONCLUSIONS

It has been demonstrated that the location of the phreatic surface in the shell of a zoned embankment subjected to a rapid drawdown of reservoir level may be predicted by the method of finite differences. The results obtained have been reduced to simple curves and equations which lend themselves readily to hand computation.

ACKNOWLEDGMENTS

The bulk of the research work reported in this paper was performed at Purdue University, School of Civil Engineering. Financial support was obtained from a National Defense Education Act Fellowship, Title IV.

The writers are indebted to R. I. Kaufman for reviewing the manuscript and offering many helpful suggestions.

APPENDIX I.—REFERENCES

1. Allada, S. R., and Quon, D., "Stable, Explicit Numerical Solution of Conduction Equation for Multidimensional Nonhomogeneous Media," *Chemical Engineering Progress Symposium Series*, Vol. 62, No. 64, 1966.
2. Brahma, S. P., and Harr, M. E., "Transient Development of the Free Surface in a Homogeneous Earth Dam," *Geotechnique*, Dec., 1962.
3. Desai, C. S., "Analysis of Transient Seepage Using Viscous Flow Model and Numerical Methods," *Miscellaneous Paper S-70-3*, U.S. Army Engineer Waterways Experiment Station.
4. Dvinoff, A. H., "Response of the Phreatic Surface in Earthen Dams to Headwater Fluctuations," thesis presented to Purdue University, at Lafayette, Ind., in January, 1970, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.
5. Harr, M. E., *Groundwater and Seepage*, McGraw-Hill Book Co., New York, 1962.
6. Hele-Shaw, H. S., "Streamline Motion of a Viscous Film," *Report of the 68th Meeting of the British Association for the Advancement of Science*, London, 1899.
7. Irmay, S., "On the Meaning of the Dupuit and Pavlovskii Approximations in Aquifer Flow," *Water Resources Research*, Vol. 3, No. 2, 1967.
8. Larkin, B. K., "Some Stable, Explicit Difference Approximations to the Diffusion Equation," *Mathematics of Computation*, Vol. 18, No. 86, 1964.
9. Newlin, C. W., and Rossier, S. C., "Embankment Drainage after Instantaneous Drawdown," *Journal of the Soil Mechanics and Foundations Division*, ASCE, Vol. 93, No. SM6, Proc. Paper 5557, November, 1967, pp. 79–95.
10. Polubarinova-Kochina, P. Ya., *Theory of Groundwater Movement* (Moscow, 1952), Trans. J. M. R. DeWiest, Princeton University Press, Princeton, N.J., 1962.

APPENDIX II.—NOTATION

The following symbols are used in this paper:

a = elevation of exit point;
C = experimental parameter;
D = $(H_c - a)$ ctn α;
H_c = elevation of intersection of embankment slope with impervious core;
h = total head at a point;
$\bar{h}$ = mean height of phreatic surface;
h_c = elevation of phreatic surface at impervious core;
h_i = initial elevation of reservoir;
h_o = time dependent drawdown elevation of reservoir;
h_p = elevation of phreatic surface at distance x;
k = isotropic coefficient of permeability;

k_x, k_y = horizontal and vertical coefficients of permeability;
n = effective porosity;
Q = discharge;
T_f = parameter dependent on t_f;
t = time;
t_f = time corresponding to end of drawdown at constant rate;
V = volume;
x, y = rectangular cartesian coordinates;
α = slope angle; and
σ = constant rate of drawdown.

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DYNAMIC MODULI AND DAMPING RATIOS FOR A SOFT CLAY

By William D. Kovacs,[1] A. M. ASCE, H. Bolton Seed,[2] and Clarence K. Chan,[3] Members, ASCE

INTRODUCTION

In recent years, a number of studies have been made to determine the response of soil to various dynamic loadings, such as nuclear blasts, explosions, steady-state machinery, and earthquakes. To evaluate the response of soil to different dynamic loadings, it is necessary to determine the moduli and damping factors of the various soil layers. While many studies have been made to determine these factors for sand, few have been made on the dynamic properties of clay.

Krizek and Franklin (11) analyzed the hysteresis loops produced by harmonic torsional oscillations on a remolded kaolin clay to obtain damping characteristics. In their study, they used a Weissenberg Rheogoniometer to perform the tests. The triaxial test was used by Taylor and Menzies [(17)—sinusoidal load] and by Taylor and Bacchus [(16)—sinusoidal strain] to obtain the damping characteristics of remolded clays from hysteresis loop data. De Graft-Johnson (4) measured damping and elastic moduli of compacted cylindrical samples of kaolinite in axial free-vibration tests. Humphries and Wahls (7) studied the effects of stress history on the moduli of remolded kaolinite and bentonite clay. After being consolidated in a triaxial cell, steady-state torsional vibrations were applied to the top of 1.4-in. diam specimens, and the shear moduli were evaluated by resonant-column techniques. Hardin and Black (5) also used resonant-column techniques to evaluate the shear moduli of extruded specimens of kaolinite.

During an earthquake, the ground motions that develop near the surface of

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